A Novel 6 DOFs Parallel Robot Used in the Airline Assembling

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Abstract – In this paper, a novel 6 Degree of Freedom (DOF) parallel manipulator was presented used for airplane assembling. Because of many advantages, parallel robot is perfect used in this special application. A variant of Stewart-liked configuration was adapted as the main structure. After inverse kinematics analysis, stiffness analysis, accuracy analysis, dexterity analysis, multiobjective optimizations were conducted. This paper present a different multiobjective Differential evolution (DE) algorithm to handle the nonlinear, multivariable, multi-model objective function. This fresh algorithm considers nondominance and crowding distance at the same time on the selection stage. At the same time, it maintains a extra archive to store the best solution and tailor its size at each iteration.

Keywords: Parallel robot, Differential Evolution, airline assembling.

1. Introduction

Robotics has fascinated numerous scientists over the past six decades. Parallel robot, usually called parallel manipulator, which is opposite to the serial robot, features a closed-loop of a mechanical chain. Due to its high stiffness, accuracy and payload, parallel robot begins to emerge in many fields, such as fast pick-and-place in the factory (Ilian Bonev, 2001), fine poisoning devise due to its high resolution and accuracy (Comin F, 1995) and parallel robot-based machine-tools (Dan Zhang, 2009).

Aviation industry has grown rapidly in recent years. It is very important for one country and it usually stands for its highest level of industry. However, in the commercial airplane manufacturing line, final assembling is still manually because of huge weight and volume of the airplane section. Much time has been wasted due to this traditional adjustment method. This paper proposes a novel parallel robot used to assemble airplane sections faster.

Kinematic performance is the foundation both for parallel and serial manipulator. It mainly involves stiffness, singularity, accuracy and dexterity. Manually adjust the geometrical parameters of the robot can be a difficult and time-consuming work. For kinematic optimization of parallel manipulator, many works has been done in this field. Dan used genetic algorithm to optimize the global stiffness and workspace of a parallel tool heads (Zhang D et al., 2004). Zhen used genetic algorithm and artificial neural networks as optimization method to optimize the stiffness and dexterity of the parallel manipulator. Because of the complexity of the objective function, artificial neural network was used to evaluate the fitness value. He also used the Pareto-optimal method to get a Pareto-optimal frontier set of Multi-objective optimization whose parameters are conflict to each other (Zhen Gao et al., 2010). Chen used Particle Swarm Optimization algorithm to optimize the maximum pay-load capacity of a reconfigurable motor-driven parallel kinematic manipulator (Chun-Ta Chen, 2012). Toz employ Particle Swarm Optimization to get the dexterous workspace of an asymmetric Generalized Stewart-Gough platform (Metin Toz, 2013). Weihmanna took a modified differential evolution (MDE) approach which enhanced the search ability and thus prevent premature to improve the force performance of a planar 3RRR parallel manipulator (Lucas Weihmann et al., 2012).

Differential Evolution (DE) algorithm is a simple and powerful stochastic optimization algorithm and was first proposed by Storn and Price in 1995 (R. Storn et al., 1995). It can solve global optimization problem regardless of linear, differentiable and continuous of the objective function. In recent years,
many multiobjective differential evolution algorithms (MODE) appeared in order to handle the real world problems, such as JADE (Jingqiao Zhang, 2009), GDE (Moraglio et al., 2010). A novel DE and dominance based multiobjective algorithm was presented in this paper so as to get a better convergence in nonlinear, multivariable, multi-optimal problems.

The rest of the paper is organized into following sections. Section 2 introduces the basic structure and geometric modelling of the proposed parallel robot. Kinematic index of the manipulator also analysed in this section. Section 3, DE and its multiobjective optimization method are presented in detail. Two objectives optimization and three objectives optimization result are offered in section 4. Finally, section 5 present s conclusion and future work.

2. Geometric Modelling and Kinematic Analysis
In order to deal with this problem, this paper presents a novel parallel robot structure used for airplane assembling. After analysing the situation of airline assembling, some conclusions can be obtained:

1. 6 DOFs is required in order to adjust two airplane parts into one line (both orientation and position).
2. Accuracy is very important
3. It can support high weight (one section may be 200 tons.)
4. Small workspace but high resolution.

These three conclusions can just meet by the parallel robots advantages: accurate, high payload and resolution.

Our design is based on above analysis.

2.1. Structure Design
In order to design a 6 DOFs parallel robot, we have many choose. According to Dan’s research (D. Zhang, 2000), for 2 to 6 legs, as long as each leg owns 6 DOFs, one gets 6 DOFs for the whole parallel robot.

And because of our application situation: crowd industry environment, we must promise it is simple and controllable, so we choose UPS as each legs configuration. Putting the prismatic joint in the centre can increase the compactness and it is very easy to realize by electric or hydromantic actuator.

However, there are many configurations that own 6 DOFs with UPS in each leg, such as classical Stewart and Double-triangle. Because of its shape of cross section of the airplane section, its impossible to place a huge airplane section on a traditional Stewart parallel manipulator. Then, another three structures are considered. At last, we take the structure with the largest stiffness distribution.

2.2. Geometric Modeling
The Stewart-like manipulator contains a moving platform and a fixed basement. For the moving platform, it has six attached points and for the basement it owns only 4 points because B1 and B6, B3 and B4 merge into one point, respectively. Each leg start form the basement are Hooke joint, prismatic joint and spherical joint.

![Proposed 6-UPS parallel robot.](image)
The reference coordinate in the fixed platform, \( O_{xyz} \), called fixed coordinate. On the other hand, the coordinate on the moving platform, \( P_{x'y'z'} \), called moving platform. As showed in the Fig. 1, the length and width of the two platforms are \( a \), \( b \) and \( c \), \( d \), respectively. The attached points on the moving platform represented by \( P_i \), the attached points on the fixed platform represented by \( B_i \), with \( i = 1, 2, \ldots, 6 \). \( O \) and \( P \) are the centre of fixed base and moving platform. The coordinate of point \( P \) represents in the coordinate \( O_{xyz} \). The orientation of the coordinate \( P_{x'y'z'} \), expressed by the rotation matrix \( Q \) with Euler angles \( \phi \), \( \theta \) and \( \phi \).

**2.3. Inverse Kinematic**

The inverse kinematic is used for getting the leg length when given a position and orientation of end-effector. The inverse kinematic problem is simple by using the vector method. According to the geometric relation, one has:

\[
\begin{align*}
\begin{cases}
    r_1 &= \left[ \frac{c}{2}, -\frac{d}{2}, 0 \right]^T \\
    r_2 &= \left[ \frac{c}{2}, 0, 0 \right]^T \\
    r_3 &= \left[ \frac{c}{2}, \frac{d}{2}, 0 \right]^T \\
    r_4 &= \left[ -\frac{c}{2}, \frac{d}{2}, 0 \right]^T \\
    r_5 &= \left[ \frac{c}{2}, 0, 0 \right]^T \\
    r_6 &= \left[ -\frac{c}{2}, -\frac{d}{2}, 0 \right]^T
\end{cases}
\end{align*}
\] (1)

\[
\begin{align*}
\begin{cases}
    B_1 &= \left[ 0, -\frac{d}{2}, 0 \right]^T \\
    B_2 &= B_1 \\
    B_3 &= \left[ 0, -\frac{d}{2}, 0 \right]^T \\
    B_4 &= \left[ 0, -\frac{d}{2}, 0 \right]^T \\
    B_5 &= B_4 \\
    B_6 &= \left[ 0, -\frac{d}{2}, 0 \right]^T
\end{cases}
\end{align*}
\] (2)

\[ P = [x, y, z]^T \] (3)

Where, \( i = 1, 2, \ldots, 6 \); \( B_i \) is the coordinate of attached points on the base expressed in the fixed coordinate; \( r_i \) is the coordinate of attached points on the moving platform expressed in the \( P_{x'y'z'} \).
coordinate. And the angle $\theta_{bi}$ and $\theta_{pi}$ are define as follow: According to the coordinate transformation, can obtain the following expression:

$$P_i = P + Qr_i$$  \hspace{1cm} (4)

Where $P_i$ is the coordinate of attached points on the moving platform expressed in the fixed coordinate; Subtract $B_i$ of both side of Eq. (4):

$$P_i - B_i = P + Qr_i - B_i$$  \hspace{1cm} (5)

Taking Euclidean norm of both side of Eq. (5), one has:

$$\|P_i - B_i\| = \|P + Qr_i - B_i\| = \rho_i$$  \hspace{1cm} (6)

Where $\rho_i$ is the length of the $i$th leg. Thus the inverse kinematic problem of parallel robot can be solved with the following expression:

$$p_i^2 = (P_i - B_i)^T(P_i - B_i)$$  \hspace{1cm} (7)

where $i = 1,2,...,6$.

2.4. System Stiffness Analysis

Stiffness measure the ability of a body to resist deformation from the external force or other actions. Generally, one use the stiffness matrix expresses its stiffness at a given point in its workspace. The principle diagonal element means the value of stiffness at $[x,y,z,\theta_x,\theta_y,\theta_z]$. There are mainly two methods to model the stiffness: firstly, combine elements and nodes of the structure model as whole. The second method uses the Jacobian matrix to calculate the stiffness. This paper will use the second method.

For parallel manipulator, the velocity between joint and end effector can written in the form of:

$$\dot{\theta} = J\dot{x}$$  \hspace{1cm} (8)

Where $\dot{\theta}$ is the vector contains velocity of joint and $\dot{x}$ is the Cartesian velocity of end effector. And $J$ is the Jacobian matrix. So for the infinitesimal displacement, the following equation also exists:

$$\Delta \theta = J\Delta x$$  \hspace{1cm} (9)

According to the force equilibrium of parallel robot:

$$F = J^Tf$$  \hspace{1cm} (10)

Where $f$ is the vector of force from the actuator, $F$ is the vector of force and torque at the end-effector. According to the Hooke’s law, for the force from actuator, one has:

$$f = K_f \Delta \theta$$  \hspace{1cm} (11)

Where $K_f$ is a diagonal matrix of the joint stiffness. Plug Eq. (9) into Eq. (11):

$$f = K_f J \Delta x$$  \hspace{1cm} (12)

Substitute Eq. (12) into Eq. (10):
Thus, the stiffness matrix of parallel mechanism in Cartesian coordinate is given:

$$K_c = J^T K_f J$$

(14)

Where $K_f$ is the joint stiffness matrix of parallel robot. Here suppose that each actuators as an elastic component and model as linear spring. Just for simplification, all actuators have the same stiffness, that is $k = k_1 = k_2 = \cdots = k_6$, then the Eq. (14) reduced to:

$$K_c = K_f^T J$$

(15)

2.5. System Accuracy Analysis

Many methods have been proposed to deal with the accuracy of the manipulator. For example, DH method (Jian Wang, 1993) take into account position errors, actuator errors and inaccuracies of the link joint in order to evaluate the effect of the manufacturing tolerances of a Stewart platform. However, different robot owns different structure and configuration; DH method cannot cover all situation. Timo Ropponen (Timo Ropponen, 1995) presented an inverse kinematic differential method which can get a closed-form solution of the end-effector. The closed-form method based on the position errors of the joints, actuator errors and backlash.

Consider the $i$th close-loop of the manipulator, one can get the relation:

$$P_i + Qr_i - B_i = l_i u_i$$

(16)

Where, $P_i$ stand for the end-effector position expressed in the fixed platform; $Q$ is the rotation matrix from the fixed coordinate to the moving coordinate; $r_i$ is the position of $P_i$ expressed in the moving platform and $B_i$ is the joint point in the basement. $l_i$ is the link length that contains all the errors and $u_i$ is the unit vector of $P_i$ and $B_i$. Take differential of both side of Eq. (16):

$$dP_i + r_i dQ + Qdr_i - dB_i = u_i dl_i + l_i du_i$$

(17)

Multiply $u_i^T$ for both sides of Eq. (17):

$$u_i^T dP_i + u_i^T r_i dQ + u_i^T Q dr_i - u_i^T dB_i = u_i^T u_i dl_i + u_i^T l_i du_i$$

(18)

The first part of the right becomes:

$$u_i^T u_i dl_i = dl_i$$

(19)

Reduce the second term and eliminate the last term by $\delta u$ and $\delta Q$; Take replace:

$$\delta l_i = dl_i; \delta P_i = dP_i; \delta r_i = dr_i; \delta B_i = dB_i; C_i = Q r_i$$ and rearrange Eq. (18).

$$dl_i = [u_i^T (C_i \times u_i)^T] \cdot [\delta P \ \delta \theta] + [u_i^T Q - u_i^T] \cdot [\delta C_i \ \delta B_i]$$

(20)

It can be simplified into:

$$\delta L = J_x \delta D + J_p \delta m$$

(21)

Where:
\[
\delta L = \begin{bmatrix} \delta l_1 \\ \delta l_2 \\ \vdots \\ \delta l_6 \end{bmatrix}, \quad J_x = \begin{bmatrix} u_1^t (Q r_1 \times u_1)^t \\ u_2^t (Q r_2 \times u_2)^t \\ \vdots \\ u_6^t (Q r_6 \times u_6)^t \end{bmatrix}, \quad J_p = \begin{bmatrix} u_1^t Q - u_1^t & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & u_1^t Q - u_1^t & \cdots & 0 & 0 \\ 0 & 0 & 0 & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & u_6^t Q - u_6^t & 0 \end{bmatrix}.
\]

\[
\delta m = \begin{bmatrix} \delta C_1 \\ \delta B_1 \\ \vdots \\ \delta C_6 \\ \delta B_6 \end{bmatrix}, \quad \delta D = \begin{bmatrix} \delta \theta \end{bmatrix}, \quad \delta C_i = [\delta C_{ix} \delta C_{iy} \delta C_{iz}], \quad \delta B_i = [\delta B_{ix} \delta B_{iy} \delta B_{iz}].
\]

Where \( i = 1, 2, \ldots, 6. \)

Thus, the position error at the end effector \( \delta D \) is obtained:

\[
\delta D = J_x^{-1} \delta L - J_x^{-1} J_p \delta m
\]  
(22)

Where \( J_x \) and \( J_p \) are error transfer matrix which are \( 6 \times 6 \) and \( 6 \times 36 \) matrix, repetitively; \( \delta L \) is a \( 6 \times 1 \) matrix stands for errors of each link; \( \delta m \) is the error matrix of the link joints both at platform and basement.

### 2.6. System Dexterity Analysis

The dexterity of a parallel manipulator suggests the ability of isotropy. And it expressed in the condition number of the Jacobian matrix. In mathematics, the condition number of matrix means the health condition of a matrix. The larger condition number, the worst condition and they are called ill-conditioned matrix. If the condition number close to infinity, it means the Jacobian reaches to singularity. The dexterity is defined as:

\[
Dexterity = Cond(J)
\]  
(23)

Where \( Cond(J) \) is the condition number of the Jacobian matrix and it defined as:

\[
Cond(J) = ||J|| \cdot ||J^{-1}||
\]  
(24)

Where \( || \cdot || \) means Frobenius norm, and \( ||J|| = \sqrt{tr(J^T J)} \). So the dexterity can be written as:

\[
Cond(J) = \sqrt{tr(J^T J)} \cdot \sqrt{tr((J^{-1})^T (J^{-1}))}
\]  
(25)

### 3. Design Optimization

As for designer, dimensional synthesis is very important. Specifically for parallel robot, the total goal is to maximize the stiffness and minimize the dexterity and accuracy. Many methods based on DE were proposed to handle the multiobjective problems. Among them, the fitness assignment, diversity and elitism are three most important aspects for designing the multiobjective algorithm.

In classical DE, selection is made solely depend on the fitness value, as long as the value of trial vector small than the target vector. In the multiobjective situation, dominance relation is widely used as fitness assignment method. However, this cannot guarantee their diversity when using the traditional selection method because only two vectors cannot judge their distance.

Instead of generate one trial vector at each iteration, this paper generate NP number trial vector once a time, and then combine the initial population into 2NP number population. After sorting by the dominance and distance principle, the best NP individual will become the new generation.
Here is the question, how to realize dominance and diversity in the selection? In the selection, if the number of Rank1 individuals larger than NP, we just choose the first NP number; if the number of Rank1 smaller than NP, we only choose the upper larger distance individual.

### 3.1. Mutation Method

Many methods have been presented during the mutant stage. Just for convenience, the notation "DE/a/b/c" is used, where "a" stands for the vector be mutated; "b" means the number of used difference vector; "c" is the combine scheme, binomial or exponential (S. Das, 2011). Some well-known mutation methods are as follows: DE/rand/1/bin:

\[ v_i^{g+1} = v_i^g + F(v_{r1}^g + v_{r2}^g + v_{r3}^g) \]  

DE/best/1/bin:

\[ v_i^{g+1} = v_{best}^g + F(v_{r1}^g + v_{r2}^g + v_{r3}^g) \]  

DE/rand-to-best/1/bin:

\[ v_i^{g+1} = v_i^g + F_1(v_{r1}^g + v_{r2}^g + v_{r3}^g) + F_2(v_{r4}^g + v_{r5}^g + v_{r6}^g) \]

Formula Eq. (26) is the standard random mutant scheme which leads to global search or exploration. Eq. (27) introduces the best individual in to mutant vector which favour local search or exploitation. In this paper, at early stage, when little ”good solutions” are found during the iteration, Eq. (26) will be used to make sure DE exploration large areas. Later on, more “good solutions” are found and stored in the archive; Eq. (27) will be adapted as the mutant method.

### 3.2. Non-dominance Rank

The dominance concept was proposed firstly by F.Y. Edgeworth in 1881 (F.Y. Edgeworth, 1881). The dominance relation indicates the performance between two solutions in the multiobjective situation. In this paper, we use the dominance depth strategy to deal with fitness assignment problem.

### 3.3. Diversity Method

The loss of diversity are observed in many P-metaheuristics (Talbi, 2009). So it is very important to keep diversity in the population. Many methods have been proposed to preserve diversity in the population. Among them, crowding distance method was chosen because it is simple and did not need other added parameter which may require experience to tune. The crowding distance define a follow, for the solution locate at the first and the last one in one dominance rank, it has an infinity distance; for the other solution, it calculate by the distance of his nearest right and left neighbours. It has been proved that Crowding distance is highly competitive compare with other algorithms (Deb K., 2002).

### 3.4 Elitism

Elitism is a secondary population contains high-quality solutions and it is very vital for multiobjective algorithm (Eckart Zitzler et al., 2000). If it participates in the process of generate new solution, then it called active elitism, otherwise, it called passive elitism. Different from other elitisms, propose method maintain NP number of best elite solution. These solutions choose from Rank 1 solution according to the dominance depth principle at each generation. The size always keep at NP number by the criteria of both dominance and crowding distance as the number of elite solutions increase at each generation. This method makes sure that all NP solutions are always best as the iteration goes on.
4. Simulation Results

As it has mentioned in the previous section, the optimization of kinematic index of parallel robot is a multiobjective problem which can be expressed as following mathematical equation:

\[ \min f(a, b, c, d, e) = [f_1, f_2, \ldots, f_n]^T \]  \hspace{1cm} (29)

Where \( a, b, c, d, e \) are the design parameters, \( f_n \) are the objective functions. The optimization result will compare with the result of Genetic Algorithm Toolbox in Matlab. Other parameters are setting as: population size: 100, mutant factor: 0.5, crossover rate: 0.9, Maximum iteration: 500.

4.1 Optimization Stiffness and Dexterity

The optimization problem can be presented as follows:

\[
\begin{align*}
f_1 &= \min \left( \frac{1}{\text{mean}(K_{C1})} + \text{std}(K_{C1}) + \frac{1}{\text{mean}(K_{C2})} + \text{std}(K_{C2}) \right) \\
f_2 &= \min(\text{Cond}(f))
\end{align*}
\]  \hspace{1cm} (30)

Where \( K_{C1} \) and \( K_{C2} \) are the first and last three elements of the principal diagonal of stiffness matrix Eq. (15). The first three elements stands for the stiffness value of three direction: \( x, y, z \); the last three elements means the stiffness value of three angle direction: \( \theta_x, \theta_y, \theta_z \). Because (1) the scale of the first three and the last three elements is different; (2) As in this application, all stiffness in all directions are very important, this paper minimize their mean value and standard deviation separately. Smaller condition number means better dexterity. The result was showed below.

![Optimization stiffness and dexterity by General DE.](image)

Fig. 4. Optimization stiffness and dexterity by General DE.
4.2 Assembling Simulation

Three laser emitter are fixed on the one section and thee accepotor are fixed on the another section which was need to be adjusted. Before assembling, make sure three pair of laser emitter and acceptor is on the right predetermined point. The process is like this, make the first laser pass through its acceptor by manipulate the robot. When both three pair of them is all on the one line, two sections are on the same line.

5. Conclusions

A novel 6 DOFs parallel robot was presented to deal with the assembling problem in the airplane final assembly stage. After kinematic analysis such as stiffness, dexterity and accuracy, optimization was conducted by proposed DE algorithm. The result proved that proposed DE outperformed general multiobjective DE and NSGA || on three objectives optimization cases. At last, assembling simulation was given to exam the correctness of proposed parallel manipulator structure.

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