Estimating Mobile Robot Odometer Error with LSE Method

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Abstract - In this paper, a scheme which is composed of two Kalman Filters is used to estimate the pose of a mobile platform whose kinematic model of odometer is subjected to systematic errors. Systematic errors of kinematic model of odometer which are considered to be soft-failure are modelled as an unknown disturbance to plant model of Kalman Filter. Using the fact that bias pattern in filter innovations can be expressed as a linear function of these constant disturbance inputs, they are estimated via LSE and output of Kalman Filter is improved using these estimated unknown inputs.

Keywords: Kalman filter, least square estimation, mobile robot, odometer, systematic errors.

1. Introduction

Localization of a mobile robot is a fundamental problem in robotics research. Several solutions are proposed for this specific problem; many of them depend on sensor fusion to cope with limitations of a single sensor modality; accelerometer, gyroscope, compass, odometer. When these sensors are used together, it is evident that ever growing error accumulates in the estimated position. One can reason that when sensors which monitors internal state of mobile robot are well calibrated, the frequency of absolute measurements which are required to reset these additive errors, may be reduced to a minimum.

In this paper, position of a differential-drive mobile robot is estimated with a Kalman Filter (KF) scheme which is robust against constant disturbances. The robust characteristic of this filter-scheme can be best explained with a reference to a regular Kalman Filter. Regular KF diverges when biased errors are present in plant model and it cannot track state of system, on the contrary the KF scheme that is used in this paper, is able to recover from false pose belief under similar conditions. In this paper, this KF scheme which is described somewhere else, in reference Bar-Shalom et.al, is specially tailored to handle biased errors which stems from inaccurate kinematic model of odometer. In this KF-scheme, these systematic/biased errors are modelled as an additional unknown disturbance inputs to a regular KF. Estimation of these unknown disturbance inputs is based on the fact that when same input commands i.e. unknown disturbance is issued multiple times in a time window, biased terms in measurement innovation can be expressed as a linear function of these unknown disturbance inputs which are constant for that specific time window. Hence this problem can be viewed as a Least Squares Estimation (LSE) problem and it can be solved to estimate unknown disturbance inputs. Then, original command input vector is adjusted with a correction term computed with LSE as shown by Bar-Shalom et.al. Thus two KF-scheme is able track states of plant safely for long period of times, and it needs less absolute measurements than a regular KF.

The main contribution of this paper is to extend the KF scheme described in reference Bar-Shalom et.al. such that it can handle inaccurate kinematic model of odometer of a mobile platform. Error sources which are expressed in the form of scale factors are studied and an approximate parametric form of the input gain matrix of these unknown input commands are derived. Finally, plant models developed in this study and measurement model is combined under the framework e.g. the so-called two KF scheme. The final form of solution is a particular realization of what is described in reference Bar-Shalom et.al, to deal with odometer inaccuracy, specifically. This final solution and the problem defined here describes a special test environment for odometer error detection and identification.
The rest of the paper is organized as follows. In part two, related works are reviewed. In part three, localization problem and the goal of this study is defined. In part four, a solution to the problem which is described in part three, is sought. In part five, simulation results are discussed. Finally, in part six, conclusions and discussions are presented.

2. Related Works

Classification of odometer calibration attempts can be made as follows: Off-line / On-line calibration methods; Geometry based calibration methods; Sensor fusion based calibration methods; another classification can be done according to the type of error considered: calibration of Systematic/Non-systematic errors. In reference, Chong and Kleeman (1997) odometer error model is developed and in reference Gianluca et.al. (2005), off-line calibration method is developed for odometer. In reference, Mondal et.al. (2008), Terminal Iterative Learning Control is used to estimate the systematic error parameters of a mobile robot. In reference Martinelli et.al. (2007) Augmented Kalman filter is used to localize a mobile robot and to estimate the systematic error parameters simultaneously. In references, von der Hardt et.al. (1998), Rudolph (2003), a multi-sensor system i.e. odometer, gyroscope and compass, is calibrated by using redundant data. In reference Kelly (2004) analytical expression is derived for propagation of odometer errors and a calibration technique is developed. In reference Goel et.al. (2000), Hashimoto et.al. (2003), a multi-model approach is taken to model hard failure, noise failure and scale failure of sensors of a mobile robot. In reference Hashimoto (2001) KF is used within a framework of Interacting Multiple Models to identify and detect hard failure of sensors of a mobile robot. The KF scheme that is used in this study can be considered as a tool which detects and corrects the soft-failure i.e. scale failure of the odometer.

3. Problem Definition: Expression for Soft-Failure of Mobile Robot Odometer

In this section, localization problem described; mobile robot is localized using bearing and range measurements to landmarks and it is assumed that landmark correspondence is known. This scenario is used to localize a mobile robot and estimate the odometry errors simultaneously. For this purpose, a simple plant model is used and state space model of the plant is expressed as linear combination of states, command input, command noise and disturbance inputs. Discrete-time state-space model that matches the continuous-time model of plant are given in Equation (1) where sampling time is $\Delta t = t_{k+1} - t_k$ and $k$ denotes time step.

$$
\begin{bmatrix}
    x_{k+1} \\
    y_{k+1} \\
    \theta_{k+1}
\end{bmatrix} =
\begin{bmatrix}
    x_k \\
    y_k \\
    \theta_k
\end{bmatrix} + \Delta t \begin{bmatrix}
    v_k \cos \phi_k \\
    v_k \sin \phi_k \\
    \omega_k
\end{bmatrix} + T_k n_u + \phi_k = \theta_k + \frac{\Delta \theta_k}{2}
$$

(1)

$x_k = [x_k, y_k, \theta_k]^T$ is the state vector of the mobile robot; $\{x_k, y_k\}$ are the x- and y-coordinates and $\{\theta_k\}$ is the heading angle of the mobile robot at time $k$. $n_u$ is a zero-mean Gaussian noise with covariance matrix $R$. Displacement of mobile robot is measured with respect to a local coordinate system attached to mobile platform. The transformation matrix that maps this displacement to the global Cartesian coordinate can be written as:

$$
T_k = \begin{bmatrix}
    \cos \phi_k & 0 \\
    \sin \phi_k & 0 \\
    0 & 1
\end{bmatrix}
$$

(2)

Velocity of mobile platform can be written as:

$$
T_k = \begin{bmatrix}
    \cos \phi_k & 0 \\
    \sin \phi_k & 0 \\
    0 & 1
\end{bmatrix}
$$

(2)
Angular velocity of mobile robot can be written as
\[
\omega_k = \frac{[1 + \mu_R] v_{R,k} - [1 + \mu_L] v_{L,k}}{[1 + \mu_D] D}
\]  
(3)

\( \mu_R, \mu_L, \mu_D \) are the scale factor of right, left wheel and distance between wheels, respectively. Further, these translational and angular speeds can be decomposed into a nominal component \([v, w]_k\) and error component \([dv, dw]_k\). Nominal speed components are computed using nominal values of the geometry of the mobile platform and error components represents disturbance input due to deviations in mobile platform dimensions. Translational velocity can be written as:

\[
v_k = v_k + dv_k
\]  
(5)

where nominal translational velocity of the tracking point is computed with

\[
v_k = \frac{v_{R,k} + v_{L,k}}{2}
\]  
(6)

Deviations from the nominal velocity can be expressed in terms of input command and scale factors which are presently unknown. Translational disturbance velocity is:

\[
dv_k = \frac{\mu_R v_{R,k} + \mu_L v_{L,k}}{2}
\]  
(7)

Similarly, angular velocity of the mobile base is divided into two components:

\[
\omega_k = \omega_k + d\omega_k
\]  
(8)

Nominal angular velocity is given below:

\[
\omega_k = \frac{v_{R,k} - v_{L,k}}{D}
\]  
(9)

Deviation of angular velocity is computed as:

\[
d\omega_k = \frac{[\mu_R - \mu_D] v_{R,k} - [\mu_L - \mu_D] v_{L,k}}{[1 + \mu_D] D}
\]  
(10)

Vector \( U_k \) is defined as speed components in \( x/y \) - and \( \theta \)-directions at time \( k \), it can be written as

\[
U_k = \begin{bmatrix}
v_k \cos \phi_k \\
v_k \sin \phi_k \\
\omega_k
\end{bmatrix}
\]  
(11)
Translation and angular speed values computed with real geometry parameters are substituted into $U_k$ and each elements of $U_k$ are examined in the sequel.

$$U_{k,i} = (v_k + dv_k) \cos \left[ \theta_k + \Delta t \left( \frac{w_k + dw_k}{2} \right) \right]$$

Using first-order Taylor series expansion and ignoring higher order error terms, one can write a simple equation for the first element of vector $U_k$: $U_{k,1}$.

$$U_{k,1} = v_k \cos \phi_k - \left( \frac{\Delta t}{2} v_k \sin \phi_k \right) dw_k + (\cos \phi_k) dv_k$$

Similarly, one can write the following equation for $U_{k,2}$

$$U_{k,2} = v_k \sin \phi_k + \left( \frac{\Delta t}{2} v_k \cos \phi_k \right) dw_k + (\sin \phi_k) dv_k$$

$U_{k,3}$ can simply be written as:

$$U_{k,3} = w_k + dw_k$$

Vector $U_k$ can be decomposed into a nominal component and an error component $\{U, dU\}_k$:

$$U_k = U_k + dU_k$$

where nominal component is:

$$U_k = \begin{bmatrix} v_k \cos \phi_k \\ v_k \sin \phi_k \\ w_k \end{bmatrix}$$

and error component can be written in matrix form as:

$$dS_k = dU_k \Delta t = G_k dU_k$$

where input gain matrix $G$ of disturbance input can be written as:

$$G_k = \begin{bmatrix} \Delta t \cos \phi_k - \frac{\Delta t^2}{2} v_k \sin \phi_k \\ \Delta t \sin \phi_k + \frac{\Delta t^2}{2} v_k \cos \phi_k \\ 0 \end{bmatrix}$$

$\phi_k = \theta_k + \frac{w_k \Delta t}{2}$

Disturbance input vector can be written as:
The state vector \( \Delta x_k \) is computed using plant model with nominal parameters is written as:

\[
\Delta x_k = \begin{bmatrix} \Delta x_k \\ \Delta y_k \\ \Delta \theta_k \end{bmatrix}
\]

Plant model of regular filter e.g. filter which does not include terms resulting from error in odometer modelling can be written as

\[
\Delta x_{k+1} = F \Delta x_k + T_n u
\]

Plant model of the proposed filter which takes into account the error in odometer model can be written as

\[
\Delta x_{k+1} = F \Delta x_k + G \Delta z
\]

The measurement model derived by Thrun et.al is used in this paper; range and bearing measurements to \( i \text{th} \) point landmark at time \( k \) with known correspondence. One can refer to Thrun et.al to see the algorithm for localization of mobile robot with range and bearing measurements to nearby point landmark with known correspondence. Note that the plant model that is being used here is different from the one derived by Thrun et.al, so the plant model and Jacobian of the plant model given by Thrun et.al. must be replaced before implementing the algorithm.

4. Estimation of Unknown Odometer Errors

In this section, equations of the linearized KF scheme are presented. While linearized KF scheme is being designed, the style, notation and equations in reference Bar-Shalom et.al are adopted. Note that when Extended Kalman Filter is used in place of a Kalman Filter, all plant and measurements matrices must be replaced with the corresponding Jacobian of those nonlinear equations. However, these Jacobian matrices must be evaluated along the nominal trajectory computed with the only real filter; that is the regular Extended Kalman Filter and error state vector is propagated as a separate data sequence without updating the nominal trajectory. Otherwise, the derivation of the linearized KF scheme is not possible, because many terms in the recursion formula of state equation for both regular and filter scheme are considered to be equal to each other. In the following discussion underscore symbol of system matrices are dropped for simplicity. Consider the system with state equation given in (22). The observations are

\[
\Delta z_{k+1} = H \Delta x_{k+1} + n_z
\]

\( n_z \) is zero mean Gaussian noise, with covariance matrix \( Q \). When the state of the plant is estimated with regular filter, the plant model of linearized Kalman Filter is given in equation [22]. \( x_k \) denotes the state which is estimated by using regular filter. The recursion of these state estimation for one cycle, can be written as

\[
\Delta \hat{x}_{k+1}^- = \Phi_k \Delta \hat{x}_k^- + T_n u + F_k K_k \Delta z_k
\]

where

\[
\Phi_k = F_k \left[ I - K_k H_k \right]
\]
At onset time of time window, \((c - s)\), define: \(\hat{x}_{c-s} = \hat{x}_{c-s}\). Recursions of state estimation for multiple cycles, until time \((k + 1)\) can be written as

\[
\Delta \hat{x}_{k+1} = \left[ \prod_{m=0}^{k-(c-s)} \Phi_{k-m} \right] \Delta \hat{x}_{c-s} + \sum_{j=c-s}^{k} \left[ \prod_{m=0}^{k-(j+1)} \Phi_{k-m} \right] F \cdot K^\prime \cdot \Delta z_j + T \cdot n_u
\]

\(k = (c - s), \ldots, (c - 1)\) \quad (27)

If the unknown disturbance inputs are available to the filter, recursion for multiple cycles until time \((k + 1)\) can be written as:

\[
\Delta \hat{x}_{k+1} = \left[ \prod_{m=0}^{k-(c-s)} \Phi_{k-m} \right] \Delta \hat{x}_{c-s} + \sum_{j=c-s}^{k} \left[ \prod_{m=0}^{k-(j+1)} \Phi_{k-m} \right] F \cdot K^\prime \cdot \Delta z_j + G \cdot du_j + T \cdot n_u
\]

\(k = (c - s), \ldots, (c - 1)\) \quad (28)

It is observed that the recursions for these filters are similar to each other except the additional term due to unknown disturbance input due to systematic errors in odometer model. The measurement innovation of the filter with unknown disturbance input is a zero mean white sequence given as:

\[
(\Delta v_{k+1} = \Delta z_{k+1} - H_{k+1} \Delta \hat{x}_{k+1}^\prime)_{\text{UDI}}
\]

The measurement innovations of the regular filter are

\[
(\Delta v_{k+1} = \Delta z_{k+1} - H_{k+1} \Delta \hat{x}_{k+1}^\prime)_{\text{RG}}
\]

When Equation (29) and (30) are compared, it is seen that the innovations (30) of the regular filter can be expressed as a white noise sequence plus a constant term due to the unknown disturbance inputs.

\[
(\Delta v_{k+1})_{\text{RG}} = (\Delta v_{k+1})_{\text{UDI}} + H_{k+1} \sum_{j=c-s}^{k} \left[ \prod_{m=0}^{k-(j+1)} \Phi_{k-m} \right] G \cdot du_j
\]

\((\Delta z_{k+1})_{\text{UDI}}\) and \((\Delta z_{k+1})_{\text{RG}}\) are considered to be equal because both filter uses the same range/bearing sensor measurement and both of them are linearized around the same nominal trajectory. Equation (31) can be imagined as a convolution integral where the effect of unknown input is shifted from the time it is applied to the time step where its output can be observed.

\[
\sum_{j=c-s}^{k} h(k - j) u(j) = \sum_{j=c-s}^{k} \left[ \prod_{m=0}^{k-(j+1)} \Phi_{k-m} \right] G \cdot du_j
\]

where

\[
h(k - j) = \left[ \prod_{m=0}^{k-(j+1)} \Phi_{k-m} \right] G \quad u(j) = du_j
\]

When disturbance input is assumed to be constant over the interval, \([c - s, \ldots, c]\), that is to say:

\[
du_k = du \quad k = (c - s), \ldots, (c - 1)
\]

Yields
\[
(\Delta u_{k+1})_{RG} = \Psi_{k+1} du + (\Delta u_{k+1})_{LE}
\]
\[
\Psi_{k+1} = H_{k+1} \sum_{j=0}^{i} \left[ \prod_{m=0}^{k-j-1} \Phi_{k-m} \right] G_j
\]

(35)

Measurement innovation of regular filter \((\Delta u_{k+1})_{RG}\) can be written as a linear function of unknown disturbance input \(du\) with additive white noise \((\Delta u_{k+1})_{LE}\). Based on (35), the disturbance input can be estimated via Least Squares from

\[
y = \Psi du + n_y \quad y = \begin{bmatrix} \Delta u_1 \\ \vdots \\ \Delta u_r \end{bmatrix}, \quad \Psi = \begin{bmatrix} \Psi_1 \\ \vdots \\ \Psi_r \end{bmatrix}
\]

(36)

where \(y\) is the stacked measurement vector, and \(\Psi\) is the measurement matrix and where \(r\) is the total number of landmark that is observed in the sliding time window. Noise \(n_y\), whose components are the innovations, is zero mean with block diagonal covariance matrix. The batch form estimate of disturbance input and the resulting covariance matrix are

\[
du = [\Psi^T S^{-1} \Psi]^{-1} \Psi^T S^{-1} y \quad L = [\Psi^T S^{-1} \Psi]^{-1}
\]

(37)

The method used above which estimates the unknown disturbance inputs resulting from odometer errors is approximate. Because the nonlinear KF scheme is linearized around the trajectory computed with regular linearized KF. It is obvious that regular linearized KF and linearized KF scheme trajectories don’t match. Therefore, all Jacobian matrices e.g. Jacobian matrices of filter scheme are approximate so an iterative approach with the following equations are proposed.

\[
F_{k,j} = F(u_{k,j}, x_k) \quad T_{k,j} = T(u_{k,j}, x_k) \quad G_{k,j} = G(u_{k,j}, x_k) \quad u_{k,j+1} = u_{k,j} + du_{k,j} \quad \frac{\Delta du_{k,j}}{|u_{k,j}|} \leq \varepsilon
\]

(38)

Table 1. Simulation Parameters.

<table>
<thead>
<tr>
<th>Prm</th>
<th>MV</th>
<th>Prm</th>
<th>RV</th>
<th>MV</th>
<th>Prm</th>
<th>RV</th>
<th>MV</th>
</tr>
</thead>
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<tr>
<td>RW-D</td>
<td>10.0</td>
<td>RW-S</td>
<td>1.020</td>
<td>1.000</td>
<td>OLN</td>
<td>1/20</td>
<td>1/10</td>
</tr>
<tr>
<td>LW-D</td>
<td>10.0</td>
<td>LW-S</td>
<td>1.000</td>
<td>1.000</td>
<td>OAN</td>
<td>1/30</td>
<td>1/20</td>
</tr>
<tr>
<td>B-D</td>
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<td>B-S</td>
<td>1.020</td>
<td>1.000</td>
<td>LRN</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>L-R</td>
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<td></td>
<td></td>
<td></td>
<td>LAN</td>
<td>pi/90</td>
<td>pi/60</td>
</tr>
</tbody>
</table>

L/L/L/R : Linear/Laser/Left/Right  W : Wheel  O : Odometer
A : Angular  B : Base  S : Scale
D/D : Diameter/Distance  N : Noise
R/R/M/V : Range/ Real/Model/Value

All units are in centimeters
Fig. 1. A typical simulation; **blue**: commanded trajectory; **black line**: true trajectory, **full circles**: landmarks **green** RG-KF output; Color lines KF-UDI.

5. Simulations

A Matlab program is written to simulate the method on computer. Simulation parameters and a typical simulation result is given in Table-I and Figure-1 respectively. Odometer noise is modelled as fraction of travelled distance. Analysis of simulation parameters reveals that KF scheme is effective when range and bearing sensor is not very accurate. When range/bearing sensor is not accurate, the bias patterns in measurement innovation persist. This helps the filter estimating the correction term. Otherwise, bias pattern on measurement innovation diminishes and LSE method is not as successful as it is expected. This suggests that the filter scheme is most effective when sensing device’s accuracy is low and landmarks are sparse. Note that past trajectories are also corrected with this filter scheme. Furthermore, by collecting different mobile robot commands and estimating the disturbance input one can proceed to compute the odometer scale factors online.

6. Conclusion

In this paper a linearized KF scheme is used to track the poses of a mobile platform. Systematic errors of odometer which are ignored in mathematical model is estimated online in batch form and then compensated by the linearized KF scheme. This eliminates the frequent need of resetting of the error which accumulates in position estimation and need for an accurate calibration of the sensors. This work can be extended to compute the scale factor online when adequate number of diverse disturbance inputs is collected. Finally the batch estimation algorithm can be replaced with a sequential estimation algorithm which suits for online odometer error estimation better.

References


Thrun S., Burgard W., Dieter F., “Probabilistic Robotics” MIT Press.