# Maximizing Wind Farm Energy Production in Presence of Aerodynamic Interactions

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**Abstract** - Wind energy is an attractive alternative to fossil fuels. However, as many other types of renewable energy sources, efficient control strategies development implies many challenges due to the dynamic and unpredictable behaviour of the energy source. More specifically, the variability of the wind velocity and the aerodynamic interactions between wind turbines reduce the electrical power production of wind farms. Extremum-seeking control (ESC) is one way to reduce the power losses due to the wake effect in wind farm. In this paper, the multi-unit optimization (MUO) method has been used in order to maximize in real-time the extracted power of the wind farm of 6 wind turbines taking in consideration the wake effect. The use of MUO method is made possible by the definition of a novel objective function which considers a normalized power regardless of wind speed inputs. Simulation results show that using the MUO method lead to a fast convergence to the optimal operational point of the wind farm even in presence of a disturbed wind.

Keywords: Extremum-seeking control, Multi-unit optimization, wake effect, wind farm.

# 1. Introduction

The worldwide wind power capacities keep on growing and become more and more attractive. However, even if the wind capacities had become more important in the recent years, the electrical power produced by wind farms is often sub-optimal and wind industry still faces many challenges to improve their power productivity.

One of these challenges consists of establishing efficient control and optimization strategies able to deal with disturbances such as the variability of the wind and the aerodynamic interactions between wind turbines. Since these disturbances are changing over time, they cause the optimal power point to change as well. Thus, tracking that optimal power point in real time became an important objective in wind energy industry and consequently, many control and optimization methods have been developed for that purpose. However, only few of them take the aerodynamic interaction between turbines in consideration. Consequently, the total power extracted by a wind farm is most of the time sub-optimal. As mention by Pao and Johnson (2009), taking the aerodynamic interaction in consideration in the control strategies could increase the electrical power produced by wind farms.

Many researches have been done on real-time optimization techniques also known as Maximum power point trackers (MPPT). One of the well-known techniques used for tracking the maximum power in wind turbines applications is the perturbation-observation (PO) method (González et al., 2010; Molina and Mercado, 2008). However, main drawbacks of the PO method are its slow convergence to the optimum and the mechanical stress caused on the turbine by the oscillations introduced as perturbations. The gradient approximation method has been proposed in order to increase the convergence speed of the PO method (Hong et al., 2009). Still, this method introduces significant oscillations in the system. A novel MPPT algorithm is proposed by Zou (2012) based on the characteristic power curve of the wind turbine. This method allows a fast convergence speed and adds no oscillation. However, an accurate tracking of the optimal power point requires the exact model of the wind turbine.

All previously referred researches have been done on standalone wind turbine for which no aerodynamic interaction has been considered. Johnson and Fritsch (2012) have made a simulation taking the aerodynamic interactions in consideration and have used the perturbation method to optimize the global wind farm power production. Because of the wake effect created by upwind turbines, the disturbances in the wind increase for the downwind turbines. Faster are the upwind turbines rotation speed, higher will be the wake effect and lower will be the power extracted from downwind turbines. It has been shown that operating upwind wind turbines in a suboptimal power point increases the overall power of the wind farm by increasing the downwind turbines extracted power. However, the slow convergence speed of the PO method is still an issue and reduces the efficiency of the wind farm especially when the latter is subject to a highly turbulent wind.

In the present paper, a new approach is proposed in order to increase the total power of a six wind turbines wind farm. The proposed strategy is based on the multi-unit optimization (MUO) method (Srinivasan, 2007). This real-time optimization method had already been tested as MPPT in a wind turbine simulation (Mehenna and Woodward, 2012). The capacity of the MUO method to converge rapidly to the optimum allowed a precise tracking of the optimal power point. Besides this main advantage, the MUO method does not require the wind turbine model and does not cause any oscillations of the system. Nevertheless, the good performance of the MUO method is based on a strong assumption which requires identical wind turbines. Consequently, considering a whole wind farm, this assumption implies that all the turbines need to be submitted to the same wind input. Due to the variability of the wind at different places over a wind farm, the probability to have the same wind for each wind turbines is low. Taking that in consideration, in this paper, a new objective function based on a normalized power is formulated for the optimization problem. This new objective function makes each wind turbines identical regardless of the wind speed and its variations. The good performance of the MUO method applied to this optimization problem is confirmed by MATLAB/Simulink simulations.

The present paper is organized as follow. In section 2, an overview of the wind farm configuration, the wind turbine mechanical model and the wake effect model considered in this work will be presented. Section 3 describes the optimization problem based on the normalized power and presents the MUO method used for the simulation. In section 4, the simulation results are presented and discussed. Finally, further work and conclusions regarding the simulation and results will be set forth in section 5.

# 2. Presentation of the Wind Farm Model

The wind farm configuration considered for the simulations is shown in Fig. 1. This configuration is inspired from the one used by Johnson and Fritsch (2012), but some modification has been applied to fit the MUO method. Thus, in the present work, six turbines have been used separated in three columns. Upwind turbines wake downwind turbines of the same columns and this aerodynamic interaction causes a decrease of the total power extracted from the wind.

## 2. 1. Wind Turbine Model

The wind turbine has been modelled using the Blade Element model. A detailed presentation of this model is given by Bianchi (2007). The main equations will be presented in this section. First, the power available  $(P_w)$  in the wind for a given area (A) is expressed by:

$$P_{w} = \frac{1}{2} \rho A V_{w}^{3} \tag{1}$$

Where  $\rho$  is the air density in kilograms per cubic meter and  $V_w$ , the wind speed in meters per second. According to the Betz Law, the maximum power that can be extracted from a wind turbine rotor is 59% of the power available in the wind. A power coefficient (C<sub>p</sub>) is added to (1) to represent the amount of mechanical power that can be extracted from the wind. This coefficient is function of the pitch angle  $\beta$  and the tip speed ratio  $\lambda$  defined by (2).

$$\lambda = \frac{R\omega}{V_w} \tag{2}$$



Fig. 1. Wind farm configuration.

Where *R* is the blade radius in meters and  $\omega$  is the turbine angular speed in radians per second. The mechanical power is then defined as follow:

$$P_m = P_w C_p(\lambda, \beta) = \frac{1}{2} \rho A V_w^3 C_p(\lambda, \beta)$$
(3)

According to Soleimanzadeh and Wisniewski (2011), Cp can be expressed as follow:

$$C_p = \chi_{00} + \chi_{10}\beta + \chi_{01}\lambda + \chi_{20}\beta^2 + \chi_{11}\beta\lambda + \chi_{02}\lambda^2 + \cdots$$
(4)

As mentioned by Bianchi (2007), for a maximum extraction of wind power,  $\beta$  should be near 0°. Consequently, Equation (4) can be rewritten as:

$$C_p = \chi_{00} + \chi_{01}\lambda + \chi_{02}\lambda^2 + \dots + \chi_{0l}\lambda^l$$
(5)

Values of  $\chi_{00} \dots \chi_{0l}$  depend of the wind turbine characteristics. For this simulation,  $\chi_{00} \dots \chi_{0l}$  have been taken from the Aeolus MATLAB/Simulink toolbox (Grunnet et al., 2010).

#### 2. 2. Wake Effect Model

The wake effect Model provided by Aeolus has been used (Grunnet et al., 2010). This model has three components divided as follow: the wake center line, the wake expansion and the wake deficit. The wake center line defines the direction of the wake. The wake expansion defines the area around the wake center line for which the deficit will be applied.

However the model has been simplified to allow a better understanding of the impact of the wake effect on the extracted power of each column. The wake center has been assumed constant and the distance between two turbines of the same column have been set large enough such that the downwind turbine's rotor is completely in the wake area. Thus, the wake created by one column does not affect others. The deficit, noted  $\alpha$ , applied on the wind speed input of the downwind turbine is described as follow:

$$\alpha = 1 - \frac{1}{2}C_t (1 + \frac{d}{(4R)})^{-1}$$

$$V_d = \alpha V_u$$
(6)
(7)

Where  $C_t$  is the thrust coefficient of the upwind turbine, d is the distance in meters between two turbines of the same column,  $V_d$  is the downwind turbine's wind speed in meters per second and  $V_u$  is the

upwind turbine's wind speed in meters per second. Such as (5),  $C_t$  can be expressed as follow when  $\beta$  is near 0° (Soleimanzadeh and Wisniewski, 2011):

$$C_t = \kappa_{00} + \kappa_{01}\lambda + \kappa_{02}\lambda^2 + \dots + \kappa_{0l}\lambda^l$$
(8)

With  $\kappa_{00} \dots \kappa_{0l}$  values taken from Aeolus. A delay has been introduced in the model in order to simulate the dynamic wind inflow of each downwind turbine as shown in the following equations (Johnson and Fritsch, 2012):

$$V_u(t) = \overline{V}_u(t - \tau(t)) \tag{9}$$

$$\tau(t) = \frac{d}{V_u(t)} \tag{10}$$

Where  $\tau$  is the time delay in seconds. As show in (10), the delay is function of the distance between turbines of the same column and the wind speed of the upwind turbine.

# 3. Optimization Problem and Multi-unit Method

The optimization problem addressed in this paper is based on the following assumptions regarding the wind farm and wind model:

- Structures of turbines of the wind farm are identical.
- Turbulences remain unchanged when wind evolves in the field and there are no other disturbances between the downwind turbines and the upwind turbines except the wake effect.
- The wind speed in front of the upwind turbine is known with a perfect accuracy.
- The wind turbine model is static except for the delay between the upwind and the downwind turbines.

#### 3. 1. Optimization Problem

In the context of real-time optimization of wind turbine power, most of the time, the objective function is chosen to be the power output delivered by the wind turbines. In the present application, the total power extracted from column n which contains two turbines will be noted  $P_{totn}$ . Thus, from (3) and (7),  $P_{totn}$  can be expressed as follow:

$$P_{totn} = \frac{1}{2} \rho A V_{wn}^3 \left( C_{p1}(\lambda_1) + \alpha(n)^3 C_{p2}(\lambda_2) \right)$$
(11)

As mentioned previously, the main advantage of the MUO method is its fast convergence to the optimum. However, this rapid convergence is dependent of a strong assumption: all units in the system to optimize have to be identical. In the context of the wind application, considering the variability of the wind speed over a field, this assumption cannot be verified if the output power defined in (11) is considered as the objective function. Consequently, in this paper, a new objective function is proposed based on a normalized power ( $N_p$ ) which is made independent of the wind speed. The normalized power of each column ( $N_{pn}$ ) is obtain by removing  $V_w$  of (11) as follow:

$$N_{pn} = \frac{P_{totn}}{V_{wn}^3} = \frac{1}{2} \rho A \left( C_{p1}(\lambda_1) + \alpha(n)^3 C_{p2}(\lambda_2) \right)$$
(12)

With the arrival of the Light detection Ranging devices (LIDAR) (Harris et al., 2006), it is possible to measure with a better accuracy the wind speed in front of a wind turbine. In the present paper, the LIDAR system will be assumed to be ideal i.e., i) the wind speed will be read with a perfect accuracy and ii) no reading delay will be added.

Thus, the power optimization problem addressed in the present work can be expressed as follow:

$$\max_{\lambda_1 \lambda_2} N_p = \sum_{n=1}^3 N_{pn} \tag{13}$$

Where Np is the total normalized power of the wind farm. The resulting static characteristics are shown in fig. 2. The maximum normalized power point is 3154 kilograms per meter and is reached when  $\lambda_1$  and  $\lambda_2$  are respectively 7.65 and 8.05.



Fig. 2. Static characteristics of the optimization problem: normalized power in function of the tip speed ratios of the upwind turbine and the downwind turbine.

## 3. 2. Multi-unit Optimization Method

Several extremum-seeking control (ESC) methods exist and the MUO method is part of it. In the MUO approach, the gradient is estimated by finite difference i.e. by introducing a constant offset between the inputs of units and then, by subtracting the corresponding outputs one from each other. Thus, the MUO method requires m + 1 identical units with m being the number of inputs of a unit to be controlled (Srinivasan, 2007). In fig. 3, the structure of MUO applied on the wind farm considered in this paper is shown.

Here, the system is composed of 3 units (columns) and two inputs (tip speed ratios). The MUO control laws applied to the normalized power optimization problem for this system are given by:

$$\dot{\lambda}_{i1} = K_{mui} \left( N_{pi+1} - N_{p1} \right) / \Delta \lambda_i \tag{14}$$

$$\lambda_{ij} = \lambda_{i1} + \Delta \lambda_i \tag{15}$$

With i = 1, 2 being the upwind and downwind turbines respectively and j = 2, 3 being the column number.



Fig. 3. Structure of the multi-unit optimization method applied to the wind farm.

The main advantage of this ESC method is that the gradient estimation is not based on a temporal perturbation but on a constant offset between the tip speed ratios ( $\Delta\lambda_i$ ). Consequently, the adaptation can be performed at the same speed as the dynamic of the wind farm itself. However, the MUO method is based on a strong assumption that every unit has to be identical. In the paper of Woodward (2009), it is shown that MUO can be used with not identical units but similar ones when using a corrector to adjust the differences between them. Unfortunately, the corrector's adaptation laws are based on a temporal perturbation which slows down the convergence. However, this method is still faster than the PO method when differences between units are low. Still, the best performance is obtained when units are identical. In the present work, turbines are assumed identical in a structural point of view. Moreover, the normalized objective function makes the static curves identical from disturbances point of view, i.e. even if the wind speed is different from one row to another. Results from the application of the MUO method to track the maximum normalized power point shown in fig. 2 will be presented and discussed in the following section.

# 4. Simulation and Results

In order to evaluate the performance of the proposed approach, simulations have been made with different wind profile inputs. The simulation parameters used are shown in Table. 1.

|   | Name                    | Code               | Value       | Unit |   | Name         | Code             | Value       | Unit                 |
|---|-------------------------|--------------------|-------------|------|---|--------------|------------------|-------------|----------------------|
| 1 | Air density             | ρ                  | 1.205       | Kg/m | 2 | Blade radius | R                | 44.64       | m                    |
| 3 | Distance                | d                  | 500         | m    | 4 | Adaptation   | K <sub>mu1</sub> | 0.001       | m/kg*s <sup>-1</sup> |
|   | between two<br>turbines |                    |             |      |   | gains        | K <sub>mu2</sub> | 0.001       |                      |
| 5 | C <sub>P</sub>          | χ00                | 1.23554e-1  |      | 6 | CT           | $\kappa_{00}$    | 6.85833e-2  |                      |
|   | coefficients            | <b>X</b> 01        | -1.36642e-1 |      |   | coefficients | $\kappa_{01}$    | -1.85769e-1 |                      |
|   |                         | <b>X</b> 02        | 9.42599e-2  |      |   |              | $\kappa_{02}$    | 1.13119e-1  |                      |
|   |                         | X03                | -1.45057e-2 |      |   |              | $\kappa_{03}$    | -1.88182e-2 |                      |
|   |                         | χ04                | 1.01827e-3  |      |   |              | $\kappa_{04}$    | 1.40564e-3  |                      |
|   |                         | <b>X</b> 05        | -3.36326e-5 |      |   |              | $\kappa_{05}$    | -4.98839e-5 |                      |
|   |                         | <b>X</b> 06        | 4.18951e-7  |      |   |              | $\kappa_{06}$    | 6.77083e-7  |                      |
| 7 | Tip speed               | $\Delta \lambda_i$ | 0.01        |      |   |              |                  |             |                      |
|   | ratios offset           |                    |             |      |   |              |                  |             |                      |

Table 1. Simulations parameters used for this paper

The first simulation was performed with constant but different wind speeds for each column of the wind farm: the wind speeds for columns 1, 2 and 3 were respectively 8 m/s, 7 m/s and 6 m/s and the initial tip speed ratio for each turbine was fixed at 8.1. As shown in Fig. 4.a), the three columns reach their optimal power after approximately 3500 seconds.

In Fig. 4.b), results of the second simulation are shown. This time, simulation was performed under a disturbed wind speed with the same initial tip speed ratio for each turbine as simulation 1. The disturbed wind speed was simulated by adding a white noise to the constant wind speeds input used in the first simulation, i.e. for each column's wind input, the mean wind speed is 8 m/s, 7 m/s and 6 m/s respectively. The variance of the white noise was chosen to be 0.01. As a result, the normalized power is still constant regardless of the disturbed wind speeds. Again, the maximum power point is found approximately after 3500 seconds. The normalized power is therefore not affected by the wind disturbances



Fig. 4. a) Simulation results obtained considering three different constant wind speeds for each column. b) Simulation results obtained considering three different disturbed wind speeds for each column.

The results presented in this section show that the normalized power allows reaching the optimal power with the MUO method even in the case where the three columns receive different wind speeds. Results show that, based on assumptions considered in this paper, the performance of the proposed approach is not affected by the difference between wind speeds of each column. This is a direct consequence of the normalized optimization problem considered in the extremum-seeking control scheme.

## 5. Conclusion

In this paper, the application of the MUO method to optimize in real-time the electrical power of a wind farm has been presented. The introduction of a normalized power performance index in the optimization problem lead to an optimal solution even in presence of different wind speeds. Moreover, simulation results showed that the maximum power was found regardless of the disturbances in the wind. However, this simulation has been made with strong assumptions such as i) the reading of the wind speed in front of turbines is ideal and ii) each turbine are structurally identical. As future work, uncertainty in wind speed measurements will be considered by introducing the model of a LIDAR system in the simulation. Moreover, in order to improve the model, a generator and a drive train will be added. Those changes will then allow analysing the proposed method in a more realistic environment.

#### Acknowledgements

We want to give special thanks to the Natural Sciences and Engineering Research Council of Canada for financially supporting this research.

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