Coordination Control with Collision Avoidance for Multiagent Systems with Underactuated Agent Dynamics

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Abstract - A coordinated control framework for multiagent systems using sliding mode control approach was developed in (Ghasemi and Nersesov, 2013). The framework addresses a general class of underactuated agents and presents a specialization to the agents represented by wheeled mobile robots. In this paper, we extend the results of (Ghasemi and Nersesov, 2013) to incorporate the obstacle/collision avoidance algorithm into the overall coordination control routine. Obstacle avoidance is achieved by surrounding the stationary as well as moving obstacles by elliptical or other convex shapes that serve as stable periodic solutions to planar systems of ordinary differential equations and using transient trajectories of those systems to navigate the agents around the obstacles. We experimentally validate the efficacy of our theoretical approach using two wheeled mobile robots that reach and maintain a desired formation in the presence of the obstacles while avoiding collisions.

Keywords: multiagent systems, underactuated systems, sliding mode control, collision avoidance.

1. Introduction

Research on analysis and control design for networks of mobile agents has grown overwhelmingly over the past few decades. Some of the most relevant references to the results presented in this paper include (Porfiri et al., 2007; Ren and Beard, 2008; Bullo et al., 2009) where the authors develop a variety of control algorithms for network consensus. The most common manoeuvres that a group of mobile agents may perform are flocking, cyclic pursuit, (virtual) leader following, and rendezvous. However, many of the existing results use single- or double-integrator agent models to validate the developed approaches without taking into account complex internal dynamics of individual agents including various kinematic constraints such as, for example, non-holonomic constraints for wheeled mobile robots or second-order non-holonomic constraints for surface vessels.

In (Ghasemi and Nersesov, 2013), we consider coordinated motion of an ensemble of agents characterized by general underactuated nonlinear dynamics and develop cooperative control algorithm using sliding mode control approach. We use graph theoretic notions to characterize the formation and in the directed graph we identify a spanning tree (Bullo et al., 2009) to characterize the minimum set of communication links among the agents required to ensure a weakly connected directed graph. Decentralized controllers for individual agents are based on the sliding mode control technique (Utkin, 1977) and use only local information from the agent itself and a neighboring agent from the spanning tree that communicates its state...
In this paper, we extend the results of (Ghasemi and Nersesov, 2013) to incorporate an obstacle avoidance technique that is based on encircling the obstacles by elliptical shapes which serve as stable limit cycles of planar systems of ordinary differential equations (Soltan et al., 2011). We identify a safety zone around an obstacle that is of similar elliptical shape which encloses the limit cycle such that when an agent enters this zone, the individual control strategy for this agent switches to the transient control strategy that drives this agent towards the limit cycle. Due to uniqueness of solutions to the systems of ordinary differential equations, the agent position never crosses into the limit cycle thus guaranteeing safe bypassing of an obstacle. Once the obstacle is cleared, the individual control strategy switches back to the cooperative control strategy aimed at maintaining the desired formation for the multiagent system. The developed framework also addresses moving obstacles including agents themselves which ensures collision avoidance. We validate our results experimentally using two wheeled mobile robots performing a coordinated motion.

2. Mathematical Preliminaries

In this section, we present some basic definitions from the graph theory. A directed graph or digraph \( G \) of order \( n \) is a pair \((\mathcal{V}, \mathcal{E})\), where \( \mathcal{V} \) (vertex set) is a set of \( n \) elements called vertices (or nodes) and \( \mathcal{E} \) (edge set) is a set of ordered pairs of vertices called edges. For \( u, v \in \mathcal{V}(G) \), the ordered pair \((u, v) \in \mathcal{E}(G)\) denotes an edge from \( u \) to \( v \). We assume that \( u \neq v \) for any \((u, v) \in \mathcal{E}(G)\), that is, the graph has no self-loops. If \((u, v) \in \mathcal{E}(G)\) implies \((v, u) \in \mathcal{E}(G)\) for any \((u, v) \in \mathcal{E}(G)\), then the graph is called undirected graph or simply graph.

A path on graph \( G \) of length \( m \) from \( u_0 \in \mathcal{V}(G) \) to \( u_m \in \mathcal{V}(G) \) is an ordered set of distinct vertices \( \{u_0, \ldots, u_m\} \) such that any pair of consecutive vertices \((u_{i-1}, u_i) \in \mathcal{E}(G)\) for all \( i = 1, \ldots, m \). A directed path on a digraph \( G \) of length \( m \) from \( u_0 \in \mathcal{V}(G) \) to \( u_m \in \mathcal{V}(G) \) is an ordered set of distinct vertices \( \{u_0, \ldots, u_m\} \) such that any directed pair of consecutive vertices \((u_{i-1}, u_i) \in \mathcal{E}(G)\) for all \( i = 1, \ldots, m \). A graph \( G \) is connected if there exists a path between any two of its vertices. A digraph \( G \) is strongly connected if there exists a directed path between any two of its vertices. Also, a digraph \( G \) is weakly connected if it is not strongly connected and its associated undirected graph obtained by removing directions on edges is connected.

A directed tree is a weakly connected digraph containing one source such that the in-degree of its other vertices is one. A directed spanning tree or simply a spanning tree of a digraph is a spanning subgraph that is a directed tree (Bullo et al., 2009).

3. Obstacle and Collision Avoidance

A general coordinated control framework for multiagent systems using sliding mode control approach was developed in (Ghasemi and Nersesov, 2013). The framework addresses a general class of underactuated systems whose individual dynamics are given by

\[
\begin{align*}
\dot{x}_{1i}(t) &= f_{1i}(t, x_i(t)), & x_{1i}(0) &= x_{1i0}, & i = 1, \ldots, p, & t \geq 0, \\
\dot{x}_{2i}(t) &= f_{2i}(t, x_i(t)) + B_{2i}(t, x_i(t))u(t) + g_i(t, x_i(t)), & x_{2i}(0) &= x_{2i0},
\end{align*}
\]

where \( x_{1i} \in \mathbb{R}^{n-m}, x_{2i} \in \mathbb{R}^m, x_i \triangleq [x_{1i}^T, x_{2i}^T]^T \in \mathbb{R}^n \) are the state variables of the \( i \)th agent, \( u_i(t) \in \mathbb{R}^m, t \geq 0 \), is the control input for the \( i \)th agent, \( f_{1i}(t, x_i) \in \mathbb{R}^{n-m} \) and \( f_{2i}(t, x_i) \in \mathbb{R}^m \) represent the internal dynamics of the \( i \)th agent, \( B_{2i}(t, x_i) \in \mathbb{R}^{m \times m} \) is invertible, and \( g_i(t, x_i) \in \mathbb{R}^m \) represents the vector of bounded uncertainties and disturbances affecting the dynamics of the \( i \)th agent.

In this paper, we extend the results of (Ghasemi and Nersesov, 2013) to address obstacle and collision avoidance in the context of coordinated motion and specialize these results to the multiagent systems composed of wheeled mobile robots whose individual dynamics are given in (Ghasemi and Nersesov, 2013). In
particular, we develop an obstacle avoidance algorithm using the idea presented in (Soltan et al., 2011). In this approach, every obstacle, including moving obstacles, is enclosed by an ellipsoid which also serves as a stable limit cycle for a planar system of ordinary differential equations. The idea is as follows, when an agent appears in a close vicinity of an obstacle, the decentralized cooperative control algorithm developed in (Ghasemi and Nersesov, 2013) switches to the obstacle avoidance algorithm driving the agent towards the periodic orbit enclosing the obstacle. Eventually, as soon as the obstacle is cleared, the agent switches back to the coordination algorithm leading the agent to its desired path. We assume that the agents are initially outside the periodic orbits enclosing all obstacles such that our algorithm ensures obstacle avoidance for all times. Note that each agent can be considered as an obstacle for other agents, therefore our obstacle avoidance algorithm also ensures collision avoidance.

To elucidate our obstacle avoidance algorithm, consider an individual agent \( i \) whose dynamics are given by (1), (2) and define the error states with respect to the \( j \)th obstacle as

\[
\dot{z}_{ij}(t) \triangleq x_i(t) - \bar{x}_j(t), \quad j \in \mathcal{O}, \quad t \geq 0,
\]

where \( z_{ij} \in \mathbb{R}^{n-m} \), \( \bar{x}_j \in \mathbb{R}^{n-m} \) is the center position of the \( j \)th obstacle, and \( \mathcal{O} \) is a set containing all obstacles in the workspace. Accordingly, the error dynamics of the \( i \)th agent is given by

\[
\begin{align*}
\dot{\bar{x}}(t) &= f_i(t, \bar{x}_i, x_2)(t) + B_2(t, \bar{x}_i, \bar{x}(t))u_i(t) + g_i(t, \bar{x}_i, \bar{x}(t)), \quad x_2(0) = x_{20},
\end{align*}
\]

Next, we present a general structure for the sliding surface and the corresponding sliding mode control such that the closed-loop dynamics restricted to the sliding surface exhibit periodic behavior. Consider a vector function \( \sigma_{ij} : \mathbb{R}_+ \times \mathbb{R}^{(n-m)} \times \mathbb{R}^m \times \mathbb{R}^{n-m} \rightarrow \mathbb{R}^m \) given by

\[
\sigma_{ij}(t, \bar{x}_i, x_2, \bar{x}(t)) \triangleq S_{1ij}(t, \bar{x}_i) + S_{2ij}(t, \bar{x}_i) - 2S_{1ij}(t, \bar{x}_i)\bar{x}_i(t) + \sigma_i(t, z_{ij}),
\]

where \( S_{1ij} : \mathbb{R}_+ \times \mathbb{R}^{(n-m)} \rightarrow \mathbb{R}^{m \times (n-m)} \) is invertible, \( S_{2ij} : \mathbb{R}_+ \times \mathbb{R}^{(n-m)} \rightarrow \mathbb{R}^{m \times (n-m)} \), \( \sigma_i : \mathbb{R}_+ \times \mathbb{R}^{(n-m)} \rightarrow \mathbb{R}^m \). We define the \( i \)th sliding surface as the null space of \( \sigma_{ij}(\cdot, \cdot, \cdot, \cdot) \), that is,

\[
\mathcal{S}_i \triangleq \{(t, \bar{x}_i, x_2, \bar{x}(t)) \in \mathbb{R}_+ \times \mathbb{R}^{(n-m)} \times \mathbb{R}^m \times \mathbb{R}^{n-m} : \sigma_{ij}(t, \bar{x}_i, x_2, \bar{x}(t)) = 0\}.
\]

The sliding mode control law \( u_i \) is calculated by setting \( \sigma_i(t, \bar{x}_i, x_2, \bar{x}(t)) = 0, \ j \in \mathcal{O}, \) for the nominal system and adding a signum function to ensure finite-time convergence to the sliding surface. Specifically, using (4), (5) and (6), we set

\[
\begin{align*}
\dot{\bar{x}}(t) &= -B_2^{-1}(t, \bar{x}_i)S_{1ij}(t, \bar{x}_i)[S_{1ij}(t, \bar{x}_i)f_i(t, \bar{x}_i, x_2, \bar{x}(t)) + \dot{S}_{1ij}(t, \bar{x}_i)x_2(t) - \dot{S}_{2ij}(t, \bar{x}_i)\bar{x}_i(t) + \dot{\bar{x}}(t, \bar{x}_i) + \bar{K}_i\text{sign}(\sigma_{ij}(t, \bar{x}_i, x_2, \bar{x}(t)))]
\end{align*}
\]

where

\[
\text{sign}(\sigma_{ij}(t, \bar{x}_i, x_2, \bar{x}(t))) \triangleq \begin{bmatrix} 
\text{sign}(\sigma_{ij1}(t, \bar{x}_i, x_2, \bar{x}(t))) \\
\vdots \\
\text{sign}(\sigma_{ijn}(t, \bar{x}_i, x_2, \bar{x}(t))) 
\end{bmatrix}^T,
\]

\[
\sigma_{ijk}(\cdot, \cdot, \cdot, \cdot), \ k = 1, \ldots, m, \text{ is the } k\text{th component of } \sigma_{ij}(\cdot, \cdot, \cdot, \cdot), \text{ and } \bar{K}_i \triangleq \text{diag}[\bar{k}_{ij1}, \ldots, \bar{k}_{ijn}],
\]

\[
\bar{k}_{ijk} = \bar{\kappa}_{ijk} + \sup_{(t, \bar{x}_i, x_2, \bar{x}(t)) \in \mathbb{R}_+ \times \mathbb{R}^{n-m} \times \mathbb{R}^m \times \mathbb{R}^{n-m}} \|g_i(t, \bar{x}_i, x_2, \bar{x}(t))\|_\infty.
\]
with \( \tilde{\lambda}_{ij} > 0, i = 1, \ldots, p, j \in \mathcal{G}, k = 1, \ldots, m. \)

Next, we show that with the feedback control law (8), the trajectories of the closed-loop system (4), (5) converge to the sliding surface (7) in finite time. To see this, consider a Lyapunov function candidate given by

\[
V_i(\sigma_j(t, \tilde{z}_{ij}, x_{2i}, \tilde{x}_j(t))) = \sigma_j^T(t, \tilde{z}_{ij}, x_{2i}, \tilde{x}_j(t))\sigma_j(t, \tilde{z}_{ij}, x_{2i}, \tilde{x}_j(t)), \quad \sigma_j(t, \tilde{z}_{ij}, x_{2i}, \tilde{x}_j(t)) \in \mathbb{R}^m. \tag{11}
\]

Note that \( V_i(\sigma(t, \tilde{z}_{ij}, x_{2i}, \tilde{x}_j(t))) = 0, (t, \tilde{z}_{ij}, x_{2i}, \tilde{x}_j(t)) \in \mathcal{S}_i, \) and \( V_i(\sigma(t, \tilde{z}_{ij}, x_{2i}, \tilde{x}_j(t))) > 0, (t, \tilde{z}_{ij}, x_{2i}, \tilde{x}_j(t)) \notin \mathcal{S}_i. \) Furthermore, it can be shown that the Lyapunov function along the trajectories of the closed-loop system satisfies

\[
\dot{V}_i(\sigma(t, \tilde{z}_{ij}, x_{2i}, \tilde{x}_j(t))) \leq -\sqrt{2} \min_{j=1,\ldots, m} \{ \lambda_{ij} \} V_j^{1/2}(\sigma_j(t, \tilde{z}_{ij}, x_{2i}, \tilde{x}_j(t))), \quad \sigma_j(t, \tilde{z}_{ij}, x_{2i}, \tilde{x}_j(t)) \in \mathbb{R}^m. \tag{12}
\]

Thus, it follows from Theorem 4.1 in (Nersesov et al., 2009) that the sliding surface \( \mathcal{S}_i \) is finite-time stable. Furthermore, while on the sliding surface, the closed-loop dynamics for the \( i \)th agent are given by

\[
\dot{\tilde{z}}_{ij}(t) = W_i(t, \tilde{z}_{ij}, x_{2i}, \tilde{x}_j(t)) - \dot{\tilde{x}}_j(t), \quad \tilde{z}_{ij}(0) = \tilde{z}_{j0}, \quad j \in \mathcal{G}, \quad t \geq 0, \tag{13}
\]

\[
x_{2i}(t) = S_{1i}^{-1}(t, \tilde{z}_{ij}) \left[ S_{2i}(t, \tilde{z}_{ij}) \dot{\tilde{x}}_j(t) - S_3(t, \tilde{z}_{ij}) \tilde{z}_{ij}(t) + \tilde{\sigma}_j(t, \tilde{z}_{ij}) \right], \tag{14}
\]

and with the proper choice of the sliding surface structure (7), we can ensure that the reduced-order closed-loop dynamics (13) restricted to the surface exhibit stable limit cycles, thus guaranteeing obstacle avoidance.

The strategy for the obstacle avoidance along with the coordination control for the network of agents and the leader is described as follows. Consider the \( j \)th obstacle, where \( j \in \mathcal{G} \). We define a prohibited region \( \mathcal{P}_j \subset \mathbb{R}^{n-m} \) and a protected region \( \mathcal{D}_j \subset \mathbb{R}^{n-m} \) enclosing the \( i \)th obstacle, such that \( \mathcal{P}_j \subset \mathcal{D}_j \). The safe region of the \( j \)th obstacle is then defined as \( \mathcal{R}_j \triangleq \mathbb{R}^{n-m} \setminus \mathcal{D}_j \). We also define the \( j \)th periodic orbit as the boundary of the \( j \)th prohibited region, that is, \( \Lambda_j \triangleq \partial \mathcal{P}_j \).

If the \( i \)th agent is in the protected region of the \( j \)th obstacle, that is, \( \tilde{z}_{ij} \in \mathcal{D}_j \), and if the direction of its velocity vector is towards \( \mathcal{P}_j \), then the control strategy switches from the coordination control algorithm given in (Ghasemi and Nersesov, 2013) to the obstacle avoidance control algorithm given by (8). Therefore, the agent starts to act independently from the rest of the network and automatically all incoming connections from other agents are deactivated, that is in the connectivity graph \( G \), all edges \((k, i) \in \mathcal{E}(G) \) for all \( k \notin \mathcal{V}(G) \) are disconnected. Next, for the resulting connectivity graph, we define a vertex set \( \mathcal{V}_i \subset \mathcal{V}(G) \) containing all agents for which there is at least one directed path from the leader and a vertex set \( \mathcal{V}_i \subset \mathcal{V}(G) \) containing all agents for which there is no directed path from the leader. Note that if \( i \in \mathcal{V}_i \) and for all \( u \in \mathcal{V}_L \) and \( v \in \mathcal{V}_i \), \((u, v) \notin \mathcal{E}(G) \). Next, we define an edge set \( \mathcal{E}_L \subset \mathcal{E}(G) \) containing all pairs \((u, v) \in \mathcal{E}(G) \), where \( u, v \in \mathcal{V}_L \). Respectively, we define an edge set \( \mathcal{E}_i \subset \mathcal{E}(G) \) containing all pairs \((u, v) \in \mathcal{E}(G) \), where \( u, v \in \mathcal{V}_i \). Thus, the resulting connectivity graph \( \tilde{G} \) is partitioned to an up-stream sub-graph \( \tilde{G}_L \triangleq \{ \mathcal{V}_L, \mathcal{E}_L \} \) with the leader as its source and a down-stream sub-graph \( \tilde{G}_i \triangleq \{ \mathcal{V}_i, \mathcal{E}_i \} \) with the \( i \)th agent as its source (Figure 1).

For any of these sub-graphs, we identify the possible spanning trees \( \tilde{G}_L^* \) and \( \tilde{G}_i^* \) respectively. Thus, the coordination control algorithm for \( \tilde{G}_L^* \), respectively \( \tilde{G}_i^* \), ensure coordination among up-stream agents with respect to the leader, respectively among down-stream agents with respect to the \( i \)th agent. As soon as the obstacle is cleared, that is, the obstacle is not between the agent and its current desired position, then all the deactivated connections are restored and the control strategy switches back to the coordination control algorithm given in (Ghasemi and Nersesov, 2013) which ensures coordination among all agents with respect to the leader.

In case when the \( j \)th obstacle is an agent itself, then there is a possibility that the \( i \)th agent appears to be an obstacle for the \( j \)th agent too. Therefore, the down-stream subgraph \( \tilde{G}_{ij} \) contains both the \( i \)th and the
(a) Graph $G$

(b) Sub-graphs $\tilde{G}_L$ and $\tilde{G}_i$

Fig. 1. Information flow topology change due to obstacle avoidance.

$j$th agents as its sources. We might arbitrarily partition the subgraph $\tilde{G}_{ij}$ into $\tilde{G}_i$ and $\tilde{G}_j$ with the $i$th and the $j$th agents as their sources, respectively. Then, we can use the coordination control algorithm for any of the subgraphs, $\tilde{G}_L$, $\tilde{G}_i$, and $\tilde{G}_j$ to avoid collisions.

As an example of the obstacle avoidance algorithm, consider the network of wheeled mobile robots with individual dynamics given in the previous section. We consider planar elliptical periodic orbits characterizing the prohibited boundaries for all obstacles in $\mathcal{O}$. We define the error states for the $i$th agent with respect to the $j$th obstacle as

$$z_{ij}(t) \triangleq [\tilde{x}_{ij}(t), \tilde{y}_{ij}(t), \tilde{\theta}_{ij}(t)]^T = [x_i(t) - \tilde{x}_j(t), y_i(t) - \tilde{y}_j(t), \theta_i(t) - \tilde{\theta}_j(t)]^T, \quad j \in \mathcal{O}, \quad t \geq 0, \quad (15)$$

where $[\tilde{x}_j, \tilde{y}_j]^T$ is the center position of the periodic orbit enclosing the $j$th obstacle, and $\tilde{\theta}_j \in \mathbb{R}$ is an arbitrary orientation assigned to the $j$th obstacle. Note that the definition of $\tilde{\theta}_j$ does not affect the behavior of the system on the periodic orbit and for the simplicity, we assume it is zero for all times. Accordingly, the error dynamics of the $i$th agent are given by

$$\dot{z}_{ij}(t) = v_{ai}(t) \cos \tilde{\theta}_{ij}(t) - d \omega_i(t) \sin \tilde{\theta}_{ij}(t) - \dot{x}_j(t), \quad \tilde{z}_{ij}(0) = \tilde{z}_{0ij}, \quad t \geq 0, \quad (16)$$

$$\dot{\tilde{x}}_{ij}(t) = v_{ai}(t) \sin \tilde{\theta}_{ij}(t) + d \omega_i(t) \cos \tilde{\theta}_{ij}(t) - \dot{y}_j(t), \quad \tilde{z}_{ij}(0) = \tilde{z}_{0ij}, \quad (17)$$

$$\dot{\tilde{\theta}}_{ij}(t) = \omega_i(t), \quad \tilde{\theta}_{ij}(0) = \tilde{\theta}_{0ij}, \quad (18)$$

$$\dot{v}_{ai}(t) = \frac{m_i d}{m_i} \omega_i^2(t) + \frac{1}{m_i r_i} (\tau_{1i}(t) + \tau_{2i}(t)) + g_{1i}(t, \eta_{1i}, \eta_{2i}), \quad \dot{v}_{ai}(0) = v_{0ai}, \quad (19)$$

$$\dot{\omega}_i(t) = - \frac{m_i d}{l_i} \omega_i(t) v_{ai}(t) + \frac{a_i}{2 l_i r_i} (\tau_{2i}(t) - \tau_{1i}(t)) + g_{2i}(t, \eta_{1i}, \eta_{2i}), \quad \dot{\omega}_i(0) = \omega_0, \quad (20)$$

Next, define functions $h_{ij}: \mathbb{R}_+ \times \mathbb{R}^3 \rightarrow \mathbb{R}$ as

$$h_{ij}(t, z_{ij}) \triangleq \frac{1}{r_{1j}} (\cos \phi_j(t) \tilde{x}_{ij}(t) + \sin \phi_j(t) \tilde{y}_{ij}(t))^2 + \frac{1}{r_{2j}} (- \sin(t) \phi_j \tilde{z}_{ij}(t) + \cos \phi_j(t) \tilde{z}_{ij}(t))^2 - 1, \quad i = 1, \ldots, p, \quad j \in \mathcal{O}, \quad t \geq 0, \quad (21)$$

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where $r_{1j}, r_{2j} \in \mathbb{R}_+$, $j \in \mathcal{O}$ are the semi-major and the semi-minor axes of the elliptical periodic orbit enclosing the $j$th obstacle, respectively, and $\phi_j \in \mathbb{R}$, $j \in \mathcal{O}$, is the orientation of the semi-major axis with respect to horizontal axis (Figure 2). Accordingly, we define the periodic orbit as the null space of $h_{ij} \langle \cdot, \cdot \rangle$, that is,

$$
\Lambda_{ij} \triangleq \{ (t, \tilde{z}_{ij}) \in \mathbb{R}_+ \times \mathbb{R}^3 : h_{ij}(t, \tilde{z}_{ij}) = 0 \}. 
$$

Finally, the structure of the sliding surface (6) is chosen as follows, for $i = 1, \ldots, p$ and $j \in \mathcal{O}$,

$$
S_{1ij}(t, \tilde{z}_{ij}) = \begin{bmatrix}
\cos \tilde{\theta}_{ij}(t) & -d \sin \tilde{\theta}_{ij}(t) \\
\sin \tilde{\theta}_{ij}(t) & d \cos \tilde{\theta}_{ij}(t)
\end{bmatrix},
$$

$$
S_{2ij}(t, \tilde{z}_{ij}) = \begin{bmatrix} 1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \end{bmatrix},
$$

$$
\bar{\sigma}_{ij}(t, \tilde{z}_{ij}) = -H_{ij}(t, \tilde{z}_{ij}) + \bar{C}_{ij}(t) \tilde{z}_{ij}(t) h_{ij}(t, \tilde{z}_{ij}),
$$

where

$$
H_{ij}(t, \tilde{z}_{ij}) \triangleq \begin{bmatrix}
-\tilde{z}_{ij} \dot{\phi}_j(t) + \frac{\psi_j(t)}{r_{1j}} \tilde{h}_{1j}(t) \tilde{z}_{ij} - \tilde{h}_{2j}(t) \tilde{z}_{ij}(t) \\
\tilde{z}_{ij} \dot{\phi}_j(t) + \frac{\psi_j(t)}{r_{1j}^2} \tilde{h}_{3j}(t) \tilde{z}_{ij} - \tilde{h}_{1j}(t) \tilde{z}_{ij}(t)
\end{bmatrix},
$$

$$
\tilde{h}_{1j}(t) \triangleq (r_{1j}^2 - r_{2j}^2) \sin \phi_j(t) \cos \phi_j(t),
$$

$$
\tilde{h}_{2j}(t) \triangleq r_{1j}^2 \cos^2 \phi_j(t) + r_{2j}^2 \sin^2 \phi_j(t),
$$

$$
\tilde{h}_{3j}(t) \triangleq r_{1j}^2 \sin^2 \phi_j(t) + r_{2j}^2 \cos^2 \phi_j(t),
$$

$\psi_j(\cdot) \in \mathbb{R}_+$ is a monotonically increasing positive function of time which describes the rotation of the agent around the periodic orbit, and

$$
\bar{C}_{ij}(t) \triangleq \begin{bmatrix}
\tilde{c}_{1ij}(t) & 0 & 0 \\
0 & \tilde{c}_{2j}(t) & 0
\end{bmatrix},
$$

with $\tilde{c}_{1ij}(\cdot), \tilde{c}_{2j}(\cdot) \in \mathbb{R}_+$ are monotonically increasing positive functions of time which define the convergence rate toward the periodic orbit. Note that $\psi_j, \tilde{c}_{1ij}$ and $\tilde{c}_{2ij}$ are in fact some bounded time-varying gains allowing a smooth transition to the periodic orbit (Soltan et al., 2011).
Thus, the sliding mode controller (8) guarantees finite time convergence to the sliding surface; while, on the sliding surface, the reduced-order dynamics exhibit a stable limit cycle (Soltan et al., 2011) which ensures obstacle avoidance.

4. Experimental Validation
We validate the above results using two experimental wheeled mobile robots shown in Figure 3. The robot wheels are driven by two DC motors which are controlled by a Pandaboard single-board computer running a Linux\textsuperscript{\textcopyright} operating system and an Arduino\textsuperscript{TM} microcontroller board. The control program is built in The MathWorks\textsuperscript{TM} Simulink\textsuperscript{R} environment. Through the Simulink\textsuperscript{R} Real-Time Workshop\textsuperscript{R}, the program is compiled and built as a real-time executable for the Pandaboard target (Fabian and Clayton, 2012). The communication between the PC and the Pandaboard is done via wireless connection. We use a black and white 640 × 480-resolution COHU-2672 camera equipped with a Data Transitions DT3120 frame grabber to capture overhead image frames in order to identify robot positions that will be used in our feedback measurements. The camera is installed 2.86 m above the floor. To ensure that there is no delay throughout the experiment, the frames rate is set at 2fps. Since the captured image is distorted, a calibration process is used to allow the true real-world position of every point in the image to be calculated.

We consider the coordination problem in which the first robot is leading the formation while communicating its state information with the second robot. The objective is to keep robots in parallel formation with respect to the y axis. The leader agent is set to move along a sinusoidal path with a constant velocity according to $y_L(t) = 0.2 \sin(\pi x_L(t))$, $(x_L^2(t) + y_L^2(t))^{\frac{1}{2}} = 0.05$ m/sec, $t \geq 0$. Two stationary obstacles are placed on the way of the robots with their center positions located at $[1, -0.2]^T$ and $[1.5, 1.2]^T$ and their orientations being $2\pi/3$ and $\pi/3$, respectively. Figure 4 (a) shows the position phase portrait with the robots orientations and Figure 4 (b) shows the time history of the control torques acting on both wheels of each robot with $\tau_{1i}$ and $\tau_{2i}$ being the control torques acting on the left and the right wheels of the $i$th robot, respectively.

5. Conclusion
In this paper, we extended the cooperative control technique for multiagent systems developed in (Ghasemi and Nersesov, 2013) by incorporating it with the obstacle/collision avoidance algorithm. Obstacle avoidance is achieved by surrounding the stationary as well as moving obstacles by elliptical or other convex shapes that serve as stable periodic solutions to planar systems of ordinary differential equations and using
transient trajectories of those systems to navigate the agents around the obstacles. Experimental validation demonstrated the efficacy of our theoretical approach.

References


