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Simulation of Non-Linear Flight Control Using Backstepping Method

Edouard Finoki, Vahé Nerguizian, Maarouf Saad
Department of Aerospace Engineering, Ecole de Technologie Supérieure
1100 Notre-Dame West Street, Montreal, Canada
edouard@finoki.fr; Vahe.Nerguizian@etsmtl.ca, Maarouf.Saad@etsmtl.ca

Abstract- This paper describes the design and the simulation of a non-linear controller for an aircraft using the backstepping method. The aim is to find the expressions of the elevator deflection in order to control the flight path angle. Backstepping controller uses the non-linear equations of motion of an aircraft, the Lyapunov analysis and the errors between the real and the desired values. The advantage of the backstepping method is to work with cascaded structures. Compared to the PID method, there is no need of tuning gains to ensure the stability. Furthermore, compared to the dynamic inversion there is no linearization and no approximations of the system; it works with the true non-linear system using virtual controls. Compared to other works, this paper deals with very accurate equations of motion and a very detailed non-linear coefficient aerodynamic model. This technique does not control only the angle of attack or the pitch Euler angle but particularly the flight path angle allowing a steady, climb or descent flight. The controller has been implemented in Matlab/Simulink and FlightGear.

Keywords**:** Flight Control**,** Non-Linear Control, Backstepping, Matlab/Simulink, Lyapunov.

1. Introduction

An aircraft is a non-linear system. Controlling it is achieved by moving the ailerons, the elevator, the rudder (Figure 1), and the throttle. In this paper, only the elevator is dealt. The throttle is constant, the ailerons and the rudder are null.

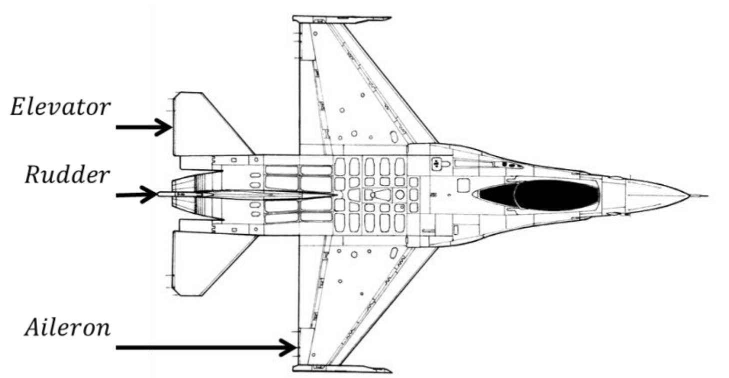

Fig. 1. Actuators: Rudder, Elevator, Aileron

The flight control complexity is due to the fact that for similar input values, the same results are not obtained with respect to the altitude, the speed, the temperature, etc.(Stengel, 2014).

For instance, considering two identical airplanes with the same actuators' deflection and the same throttle, only a difference in altitude will imply two different behaviours. Indeed, higher the altitude is, smaller are the air flow, the friction on the wings, the drag and much smaller is the airspeed (bigger is the ground speed). Thus, the flight path angle will be different due to it is direct dependency on the airspeed. In this paper, we will consider the aircraft as a non-linear model.

In order to resolve this problem in the classical method, a controller is designed on linear control theory due to the wealth of tools for linear design and analysis. This method is to simplify the non-linear aircraft model by several linear models according to the altitude, the speed and the temperature (Nelson, 1997), (Toumes and Johnson, 1998). Thus, this technique uses a multiple modelling approach. In fact, several controllers are used for several equilibrium points. The most well-known methods are PID, gain-scheduled and nested saturation. These methods can be completed with a neural network technique (Steck et al, 1996), (Puttige et al, 2009).

In this paper, due to the non-linear equations of motion of the model, classical method is not possible to implement. Therefore a single controller is used for the non-linear aircraft dynamics. It allows manoeuvres outside the region where the flight dynamics are linear. In this method, the aircraft is stabilized regardless of the altitude, the speed or the temperature. The most well-known non-linear aircraft dynamics control methods are feedback linearization, backstepping and slide mode control, (Landry et al, 2012), (Espinoza et al, 2013).

In this research, the backstepping method is used to control the non–linear system with virtual laws (Borra, 2012), (Lungu, 2012), (HarkegArd and Glad, 2000). As per Brian M. Borra (2012), “Backstepping is a recursive, control-effort minimizing, constructive design procedure that interlaces the choice of a Lyapunov function with the design of feedback control. It allows the use of certain plant states to act as intermediate, virtual controls, for others breaking complex high order systems into a sequence of simpler lower-order design tasks”.

The rest of the paper is organized as follows. Section 2 presents the aircraft model. The backstepping approach is explained in section 3. Simulations and results are shown in section 4. Finally, a conclusion is given in section 5.

2. Aircraft model

In this section, all variables, their symbols and equations of motion are described.

2. 1. Nomenclature

Table.1. Nomenclature

γ	Flight path angle (rad).	δ_e	Elevator deflection (rad).
μ	Bank angle (rad).	V	Airspeed (m/s).
α	Angle of attack (rad).	Lift	Lift force (N).
β	Sideslip angle (rad).	D	Drag force (N).
Q	Pitch rate (rad/s).	T	Thrust (N).
L	Roll moment (N.m).	Y	Side force (N).
M	Pitch moment (N.m).	M_T	Engine momentum (N.m).
N	Yaw moment (N.m).	m	Mass (kg).
C_M	Pitch aerodynamic coefficient.	$\bar{q}$	Dynamic pressure (N/m^2).
C_{M0}	Pitch aerodynamic coefficient.	S	Wing area (m^2).
C_{Mq}	Pitch aerodynamic coefficient.	c	Mean aerodynamic chord (m).
$C_{M\delta_e}$	Pitch aerodynamic coefficient.	$c_{5,6,7}$	Inertial terms.

2. 2. Equations of motion

The following equations express the elevator deflection δ_e given in equation 1 to the flight path angle (equation 11). (Borra, 2012), (Landry et al, 2012). Figure 2 shows the main three variables used for the backstepping method (γ, α and Q).

The pitch aerodynamic coefficient is:

$$C_M = C_{M_0}(\alpha,\beta) + C_{M_q}(\alpha)\frac{c}{2V}Q + C_{M_{\delta_e}}(\alpha)\delta_e \quad (1)$$

The pitch moment is:

$$M = \bar{q}ScC_M \quad (2)$$

The derivate of pitch rate is:

$$\dot{Q} = (c_5P - c_7M_T)R - c_6(P^2 - R^2) + c_7M \quad (3)$$

$$\dot{Q} = f_P(\gamma,\alpha,Q) + g_P(\gamma,\alpha,Q)\delta_e \quad (4)$$

$$f_P(\gamma,\alpha,Q) = (c_5P - c_7M_T)R - c_6(P^2 - R^2) + c_7qSc\left(C_{M_0}(\alpha,\beta) + C_{M_q}(\alpha)\frac{c}{2V}Q\right) \quad (5)$$

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$$g_P(\gamma,\alpha,Q) = c_7qScC_{M_{\delta_e}}(\alpha) \quad (6)$$

The derivate of angle of attack is:

$$\dot{\alpha} = \frac{-Lift}{mV\cos\beta} + \frac{1}{mV\cos\beta}(-T\sin\alpha + mg\cos\gamma\cos\mu) + Q - \tan\beta(P\cos\alpha + R\sin\alpha) \quad (7)$$

$$\dot{\alpha} = f_\alpha(\gamma,\alpha) + g_\alpha(\gamma,\alpha)Q \quad (8)$$

$$f_\alpha(\gamma,\alpha) = \frac{-Lift}{mV\cos\beta} + \frac{1}{mV\cos\beta}(-T\sin\alpha + mg\cos\gamma\cos\mu) - \tan\beta(P\cos\alpha + R\sin\alpha) \quad (9)$$

$$g_\alpha(\gamma,\alpha) = 1 \quad (10)$$

The derivate of flight path angle is:

$$\dot{\gamma} = \frac{\sin\mu}{mV}(-D\sin\beta - Y\cos\beta) + \frac{1}{mV}(T\cos\alpha\sin\beta\sin\mu - mg\cos\gamma) + \frac{\cos\mu}{mV}(Lift + T\sin\alpha) \quad (11)$$

$$\dot{\gamma} = f_\gamma(\gamma) + g_{\gamma c}(\gamma)\cos\alpha + g_{\gamma s}(\gamma)\sin\alpha \quad (12)$$

$$f_\gamma(\gamma) = \frac{\sin\mu}{mV}(-D\sin\beta - Y\cos\beta) + \frac{1}{mV}(\cos\mu Lift - mg\cos\gamma) \quad (13)$$

$$g_{\gamma c}(\gamma) = \frac{T\sin\beta\sin\mu}{mV} \quad (14)$$

$$g_{\gamma s}(\gamma) = \frac{\cos\mu}{mV}T \quad (15)$$

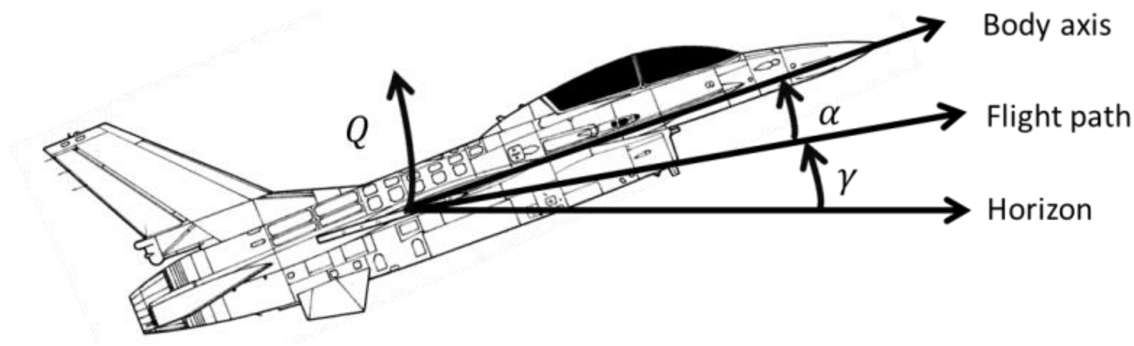


Fig. 2. Flight path angle γ, Angle of attack α, and Pitch rate Q.

3. Backstepping control

3. 1. Controller design for the flight path angle

The objective is to find the elevator expression that corresponds to the desired flight path angle. The approach is to create an error and derivate it in order to zero it out. The process is summarized in figure 3:

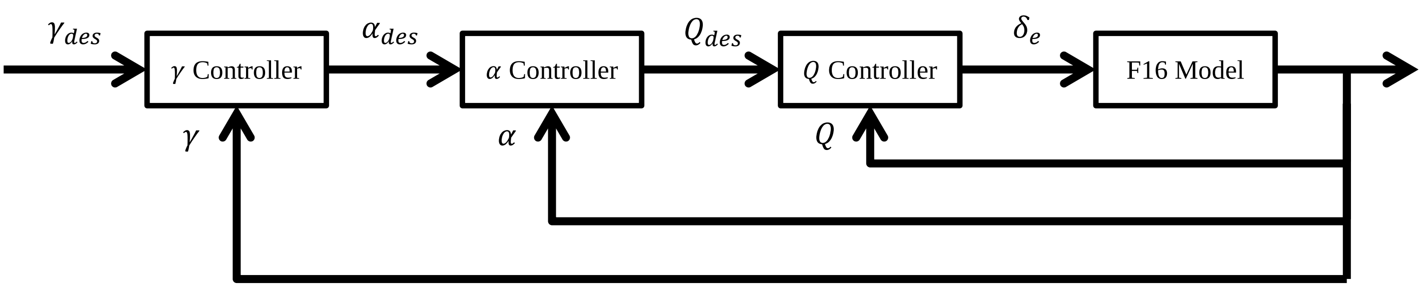


Fig. 3. Controller block diagram

The first step is to derivate the flight path angle in order to get the desired angle of attack. Both of these angles are described in figure 2.

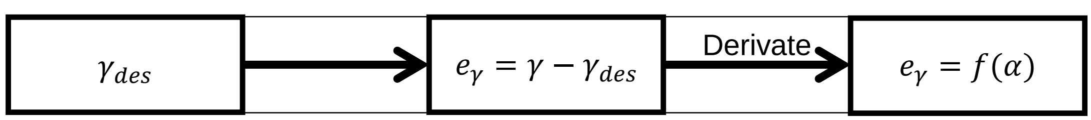


Fig. 4. Flight path angle

$$e_\gamma = \gamma - \gamma_{des} \tag{16}$$

$$\dot{e_\gamma} = \dot{\gamma} \tag{17}$$

$$\dot{e_\gamma} = -k_\gamma e_\gamma + k_\gamma e_\gamma + f_\gamma(\gamma) + g_{\gamma c}(\gamma) cos\alpha + g_{\gamma s}(\gamma) sin\alpha \tag{18}$$

If α was an input of this system, it should be chosen such as:

$$k_\gamma e_\gamma f_\gamma(\gamma) + g_{\gamma c}(\gamma) cos\alpha + g_{\gamma s}(\gamma) sin\alpha = 0 \tag{19}$$

Thus the equation (18) would be linear:

$$\dot{e_\gamma}' = -k_\gamma e_\gamma \tag{20}$$

A Lyapunov function is defined to stabilize the system (18). The system is stable if the derivate of Lyapunov function $\dot{V}_\gamma(e_\gamma)$ is negative:

$$V_\gamma(e_\gamma) = \frac{1}{2} e_\gamma^2 \tag{21}$$

$$\dot{V}_\gamma(e_\gamma) = e_\gamma . \dot{e_\gamma} \tag{22}$$

$$\dot{V}_\gamma(e_\gamma) = -k_\gamma e_\gamma^2 < 0 \ \ k_\gamma > 0 \tag{23}$$

The derivate of the Lyapunov function $\dot{V}_\gamma(e_\gamma)$ is definite negative. Thus, the system $\dot{e_\gamma} = -k_\gamma e_\gamma$ is asymptotically stable. The system (18) can be written:

$$\dot{e_\gamma} = -k_\gamma e_\gamma + e_\alpha \tag{24}$$

With the error e_α that converges to zero.

$$e_\alpha = k_\gamma e_\gamma f_\gamma(\gamma) + g_{\gamma c}(\gamma) cos\alpha_{des} + g_{\gamma s}(\gamma) sin\alpha_{des} \tag{25}$$

And the desired angle of attack is:

$$\alpha_{des} = 2\,\mathrm{atan}\left(\frac{g_{\gamma c}(\gamma) \pm \sqrt{g_{\gamma c}(\gamma)^2 + k_\gamma e_\gamma f_\gamma(\gamma)^2 - g_{\gamma s}(\gamma)^2}}{k_\gamma e_\gamma f_\gamma(\gamma) + g_{\gamma s}(\gamma)}\right) \tag{26}$$

The second step is to derivate the angle of attack in order to get the desired pitch rate:

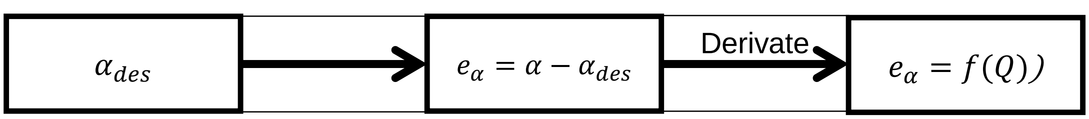


Fig. 5. Angle of attack

$$e_\alpha = \alpha - \alpha_{des} \tag{27}$$

$$\dot{e_\alpha} = \dot{\alpha} - \dot{\alpha}_{des} \tag{28}$$

$$\dot{e_\alpha} = -e_\gamma - k_\alpha e_\alpha + k_\alpha e_\alpha + e_\gamma + f_\alpha(\gamma,\alpha) + g_\alpha(\gamma,\alpha)Q - \dot{\alpha}_{des} \tag{29}$$

If Q wax an input of this system, it should be chosen such as:

$$k_\alpha e_\alpha + e_\gamma + f_\alpha(\gamma,\alpha) + g_\alpha(\gamma,\alpha)Q - \dot{\alpha}_{des} = 0 \tag{30}$$

Thus the equation (29) would be linear:

$$\dot{e_\alpha}' = -k_\alpha e_\alpha \qquad (31)$$

A Lyapunov function is defined to stabilize the system (29). The system is stable if the derivate of Lyapunov function $\dot{V_\alpha}(e_\gamma, e_\alpha)$ is negative:

$$V_\alpha(e_\gamma, e_\alpha) = \frac{1}{2}e_\gamma^2 + \frac{1}{2}e_\alpha^2 \qquad (32)$$

$$\dot{V_\alpha}(e_\gamma, e_\alpha) = e_\gamma . \dot{e_\gamma} + e_\alpha . \dot{e_\alpha} \qquad (33)$$

$$\dot{V_\alpha}(e_\gamma, e_\alpha) = -k_\gamma e_\gamma^2 - k_\alpha e_\alpha^2 + e_\alpha(\dot{e_\alpha} + e_\gamma + k_\alpha e_\alpha) < 0 \;\; k_\alpha > 0 \qquad (34)$$

In order to have a negative derivate Lyapunov function $\dot{V_\alpha}(e_\gamma, e_\alpha)$, it is necessary to cancel the non-negative terms:

$$\dot{e_\alpha} + e_\gamma + k_\alpha e_\alpha = 0 \qquad (35)$$

Thus, the desired pitch rate becomes:

$$Q_{des} = \frac{\dot{\alpha}_{des} - k_\alpha e_\alpha - f_\alpha(\gamma, \alpha) - e_\gamma}{g_\alpha(\gamma, \alpha)} \qquad (36)$$

The system (29) can be written:

$$\dot{e_\alpha} = -k_\alpha e_\alpha + e_Q - e_\gamma \qquad (37)$$

With the error e_Q that converges to zero.

$$e_Q = k_\alpha e_\alpha + f_\alpha(\gamma, \alpha) + g_\alpha(\gamma, \alpha) Q_{des} - \dot{\alpha}_{des} \qquad (38)$$

The third step is to derivate the pitch rate in order to get the desired elevator deflection:

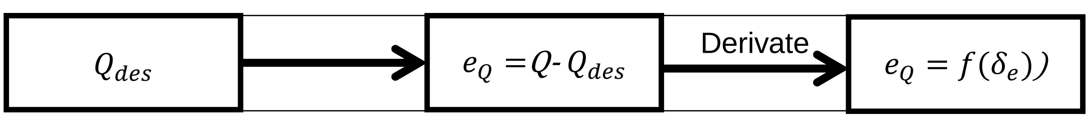


Fig. 6. Pitch rate

$$e_Q = Q - Q_{des} \qquad (39)$$

$$\dot{e_Q} = \dot{Q} - \dot{Q}_{des} \qquad (40)$$

$$\dot{e_Q} = -k_Q e_Q + k_Q e_Q + f_P(\gamma, \alpha, Q) + g_P(\gamma, \alpha, Q)\delta_e - \dot{Q}_{des} \qquad (41)$$

If δ_e wax an input of this system, it should be chosen such as:

$$k_Q e_Q + f_P(\gamma, \alpha, Q) + g_P(\gamma, \alpha, Q)\delta_e - \dot{Q}_{des} = 0 \qquad (42)$$

Thus the equation (41) would be linear:

$$\dot{e_Q} = -k_Q e_Q \tag{43}$$

A Lyapunov function is defined to stabilize the system (41). The system is stable if the derivate of Lyapunov function $\dot{V_Q}(e_\gamma, e_\alpha, e_Q)$ is negative:

$$V_Q(e_\gamma, e_\alpha, e_Q) = \frac{1}{2}e_\gamma^2 + \frac{1}{2}e_\alpha^2 + \frac{1}{2}e_Q^2 \tag{44}$$

$$\dot{V_Q}(e_\gamma, e_\alpha, e_Q) = e_\gamma.\dot{e_\gamma} + e_\alpha.\dot{e_\alpha} + e_Q.\dot{e_Q} \tag{45}$$

$$\dot{V_Q}(e_\gamma, e_\alpha, e_Q) = -k_\gamma e_\gamma^2 - k_\alpha e_\alpha^2 - k_Q e_Q^2 + e_\alpha e_\gamma - e_\alpha e_\gamma + e_\alpha e_Q + k_Q e_Q^2 + e_Q.\dot{e_Q} < 0 \quad k_Q > 0 \tag{46}$$

In order to have a negative derivate Lyapunov function $\dot{V_Q}(e_\gamma, e_\alpha, e_Q)$, it is necessary to annul the non-negative terms:

$$e_Q(\dot{e_Q} + e_\alpha + k_Q e_Q) = 0 \tag{47}$$

The system (41) can be written:

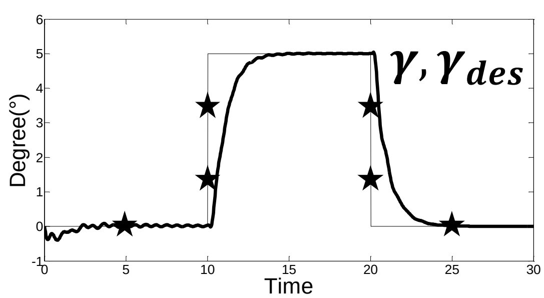
$$\dot{e_Q} = -k_Q e_Q \tag{48}$$

The desired elevator deflection is:

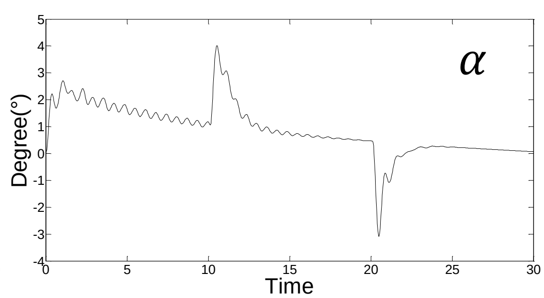
$$\delta_e = \frac{\dot{Q}_{des} - k_Q e_Q - f_P(\gamma,\alpha,Q) - e_\alpha}{g_P(\gamma,\alpha,Q)} \tag{49}$$

4. Simulations and Results

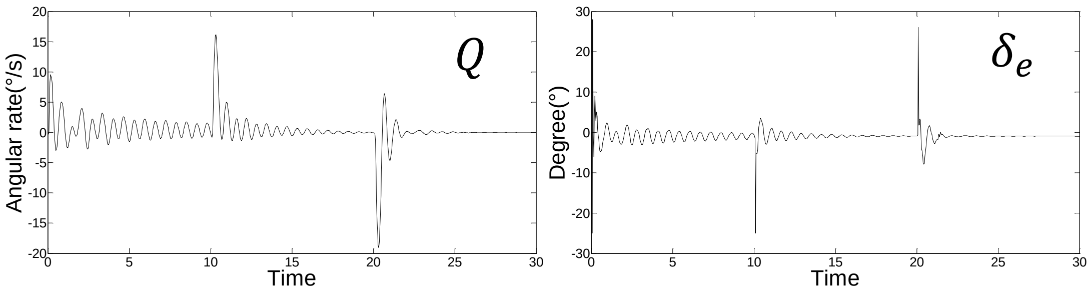
Simulation was completed with Matlab/Simulink. Results are shown in Figure 7. The simulation duration time is 30s. Figure 7.a shows the desired value, identified with stars and the real value that follows the reference signal. The flight path angle oscillates the first two seconds, then takes 5 seconds to reach the 5° degrees, and finally takes 5 seconds to return to 0°. Changes in the flight path angle leads to changes in the angle of attack (figure 7.b) that causes modifications in the pitch rate (figure 7.c). In order to follow the desired flight path angle, the elevator deflection (figure 7.d) oscillates the first two seconds, deflects to the bottom at 10 seconds, and deflects to the top at 12 seconds.



(a)



(b)



(c) (d)

Fig.7. (a): Flight path angle and desired flight path angle, (b): Angle of attack, (c): Pitch rate, (d): elevator deflection.

5. Conclusion

In this paper, backstepping method is applied to control the flight path angle.

The process derivates first the desired flight path angle in order to get the desired angle of attack, then, it derivates the desired angle of attack to obtain the desired pitch rate, and finally, derivates the desired pitch rate in order to have the elevator deflection through the aerodynamic coefficient and the pitch moment. The control is achieved by acting the elevator. The throttle is constant, the ailerons and the rudder are null.

As indicated in the introduction, the classical techniques need a linearization. This is not the case for backstepping method that works for every condition at any moment. As seen in the results, the backstepping method is very efficient to control a non-linear system. It avoids gain adjustment, and adjustment is made only to improve the performances.

Acknowledgements

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