Proceedings of the 4th World Congress on Civil, Structural, and Environmental Engineering (CSEE'19)
Rome, Italy – April, 2019
Paper No. ICGRE 161
DOI: 10.11159/icgre19.161

Analysis of Laterally Loaded Piled Raft Foundation

Stefano Stacul[1], Nunziante Squeglia[1]
[1]Department of Civil and Industrial Engineering, University of Pisa
Largo Lucio Lazzarino 1, Pisa, Italy
stefano.stacul@for.unipi.it; squeglia@ing.unipi.it

Abstract **-** A BEM (Boundary Element Method) based method has been developed for the analysis of laterally loaded piled raft foundation. This method considers the raft-soil contact contribution and all the interactions between the piles, the soil and the raft. The nonlinear soil response is accounted by a hyperbolic modulus reduction curve, while the nonlinear response of reinforced concrete piles is modelled accounting also for the influence of tension stiffening. The behaviour of laterally loaded pile foundation is strongly affected by shallower soil layers, which in turn are frequently influenced by suction. The latter aspect has been considered by implementing the Modified Kovacs model in the proposed BEM method. Moreover, the shadowing effect has been modelled using an approach similar to that described in the Strain Wedge Model. The proposed method saves computational effort compared to more sophisticated FEM (Finite Element Method) or FDM (Finite Difference Method) codes and provides reliable results. The validation of the method in the linear elastic range has been carried out by comparing parametric analysis results with those obtained by using the code APRAF, and a comparison with centrifuge test data is shown to verify its reliability.

Keywords**:** Laterally loaded piles, piled raft, shadowing effect, BEM, tension stiffening, suction.

1. Introduction

Only few experimental experiences and numerical methods [1-10] for laterally loaded piled raft foundation are available. This results in a lack of enough knowledge about the complex piled raft response under lateral loads. Additionally, up to now are not available computationally efficient software and codes for routine use in practice.

Laterally loaded piled raft response is significantly influenced by several key factors, as: the pile-head connection rigidity, the pile-soil relative stiffness, the pile spacing and the pile-soil, raft-soil, pile-pile, raft-pile interactions. Furthermore, full scale tests on pile groups [11] failed to provide definitive information about the influence of the installation method.

Available experimental data demonstrate that the contribution offered by the raft-soil contact is significant and it is also useful in reducing pile bending. The amount of the raft-soil lateral contribution is dependent on the vertical load carried by the raft. However, the technical literature is still poor about experimental and numerical studies [12] aimed to investigate about the influence of the vertical load on the lateral response of pile groups and piled rafts.

The most used analysis methods to study a laterally loaded pile foundation are continuum-based (boundary element method (BEM), finite element method (FEM)) or Winkler-based approaches (i.e. p-y curves [13-15]), and the most frequently used p-y curves are those recommended by the American Petroleum Institute [16].

Despite their big potential in geotechnical engineering applications, FEM suffer from the complexity of the domain discretization, difficulties in choosing the input parameters and are affected by the pile modelling. The high computational costs also prevent their use in parametric studies.

On the contrary BEM approaches describe the soil as an elastic half-space and enable pile-soil-pile interactions to be directly considered, thus making possible to take group effects into account. Nevertheless, BEM methods, while providing a complete solution at the interfaces of the problem domain, require numerical approximations in case of heterogeneous soils.

Most of previously developed BEM codes are limited to lateral load analysis of pile groups [17-19]) in which the raft is rigid and a frictionless contact between the raft and the soil is assumed. More recently some other approaches have been proposed to study also piled raft foundation in which the raft is considered in contact with the ground (Small and Zhang, 2000, 2002, [20-21]; Small et al. 2006 [22]; Zhang and Small, 2000 [23]; Kitiyodom et al. [24, 25]). However, these methods are not able to properly model the soil and pile material nonlinearities, and thus to continuously capture the pile-soil relative stiffness variation.

2. BEM-based method for the analysis of laterally loaded piled raft

A reliable analysis method to study a laterally loaded piled raft should be able to reproduce the most relevant interactions between the soil, the piles and the raft considering also material nonlinearities (soil and pile). The proposed method can realize a complete BEM analysis of the soil continuum in which all the interactions are modelled and represents an extension to the piled raft case of a recently proposed analysis method for laterally loaded pile groups (Stacul and Squeglia, 2018 [26]).

The soil is modelled as a multi-layered elastic half-space and the Mindlin's solution [27] is used to evaluate the pile-pile, pile-soil and raft-pile interactions. The latter solution is strictly valid in case of homogeneous elastic half-space [28], however the approximation proposed in Poulos and Davis (1980) [29] is considered here, thus the soil modulus used in the Mindlin equation is the average between the elastic modulus at the point in which the displacement is computed and the elastic modulus at the point in which a lateral load is applied.

The raft is a thin plate discretized in squared blocks subjected to uniform shear stresses at each block and the Cerutti's elastic solution is considered (Equation 1) to model pile-raft and soil-raft interactions. Also, in this case, the approximation suggested by Poulos and Davis is used.

$$\rho_{ij} = \frac{P_j\left(1+\nu_s\right)}{2\pi E_s R}\left[1+\frac{x^2}{R^2}+\left(1-2\nu\right)\left(\frac{R}{R+z}-\frac{x^2}{\left(R+z\right)^2}\right)\right] \tag{1}$$

Where: ρ_{ij} is the lateral displacement induced at the point i belonging to the half-space and due to the load P_j applied at the point j along the raft-soil interface (Figure 1); E_s and υ_s are the soil elastic modulus and the soil Poisson's ratio, respectively; R, x and z are defined in Figure 1.

The pile is modelled as a beam and its flexibility matrix in case of linear elastic behaviour, is obtained using the elastic beam theory. For reinforced concrete sections the 'moment-curvature-axial load' relationship is obtained taking also into account the influence of tension stiffening (Morelli et al., 2017 [30]). The pile flexibility matrix is thus updated according to the bending-moment reached at each pile-node in the previous step of the analysis procedure [26].

The nonlinear soil response (incremental analysis) is modelled by using a modified formulation of the quasi-hyperbolic elastic modulus reduction curve (Equation 2) proposed by Fahey and Carter in 1993 [31]. The tangent soil elastic modulus is updated at each pile-soil interface point using the Equation (2).

$$\frac{G_{\tan}}{G_{\max}} = \frac{\left({G_{\sec}}/{G_{\max}}\right)^2}{\left[1-R_f\left(1-g\right)\cdot\left(\frac{p}{p_{ult}}\right)^g\right]} \tag{2}$$

Where: G_{tan}, G_{sec} (Equation 3) and G_{max} are the tangent, the secant and the maximum shear modulus of the soil, respectively; R_f and g are the parameters that define the shape of the modulus reduction curve in the formulation proposed in [31]. As fully described in [26,33] an analogy is assumed between the "interface pressure - ultimate soil resistance" ratio and the "shear stress – maximum shear stress" ratio ($p/p_{ult} \approx \tau/\tau_{max}$). The ultimate lateral soil pressure profile is evaluated according to the relationships suggested in the API recommendations [16].

$$\frac{G_{\sec}}{G_{\max}} = 1-R_f\left(\frac{p}{p_{ult}}\right)^g \tag{3}$$

Here, R_f is set equal to 1, while the parameter g ranges between 0.25 and 1, and can be evaluated by a fitting procedure with the load-deflection curve of a single pile obtained in a horizontal load test or with the load-deflection curve obtained with other analysis tools [14, 15, 32].

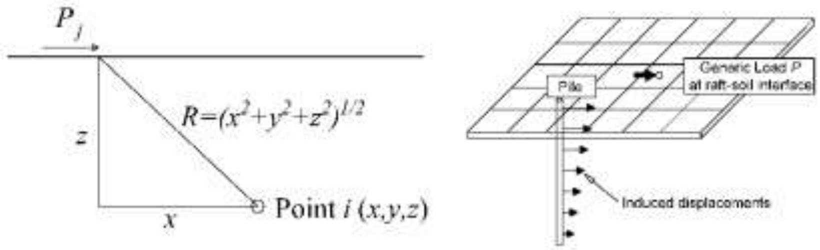

Fig. 1: Cerutti's solution scheme.

The shadowing effect [34] and the influence of suction (in case of partially saturated soil layers) are modelled using an approach similar to that described in Ashour et al. (2004) [34] and the Modified-Kovacs model (Aubertin et al., 2003 [35]), respectively. Details are shown in [26, 33].

The lateral contribution of the raft is activated when a vertical load is applied. At the raft-soil interface the sliding starts when the shear stress at the interface exceeds a value defined with the Equation (4).

$$\tau_f = \sigma_n \tan\delta \tag{4}$$

Where: σ_n is the vertical stress induced at each raft-soil interface element and can be defined after a preliminary vertical load analysis carried out using the Poulos-Davis-Randolph (PDR) method [29, 36, 37]; δ is the angle of friction at the raft-soil interface. Here it is assumed that the vertical load is applied prior to the horizontal load, thus the vertical load analysis is not coupled with the lateral load analysis. The piles-raft vertical load sharing is computed using the PDR method. Once estimated the vertical load rate transferred directly by the raft to the soil the elastic theory can be used to evaluate the increase of both vertical stress state and stiffness of the soil beneath the raft (at each pile-soil interface point), thus accounting, in an approximate way, of the influence of the vertical load on the piled-raft lateral response [33].

The solving scheme (typical of BEM methods) is defined by the compatibility equations (pile-soil and raft-soil displacements) and the equilibrium equations. Once reached the maximum shear stress in a raft-soil interface block the sliding starts and the respective compatibility equation is removed. Both free-to-rotate and fixed-head piles can be studied, nevertheless a different pile-head restraint condition can be considered. The analyses are performed incrementally as described in [26, 33].

3. Validation of the proposed method

3.1. Parametric study

A parametric study has been realized considering a linear elastic response for both the pile and the soil. The results obtained with the proposed method are compared with those by using the code APRAF (Small and Zhang, 2000, 2002 [20, 21]; Small et al. 2006 [22]; Zhang and Small, 2000 [23]), which is based on the finite layer theory (Small and Booker, 1986 [38]).

This study has been carried out considering a square piled raft foundation, with 16 piles (4 rows and 4 columns), resting on a homogenous soil. The Poisson's ratio, the elastic modulus of the soil, the pile diameter (*D*) and the pile slenderness ratio (*L/D*) were set equal to 0.35, 10 MPa, 0.5 m and 30, respectively. The pile-heads are fixed against the rotation.

The influence of the pile-soil stiffness ratio (E_p/E_s) on the load sharing (piles and raft) and on the piled-raft displacement has been investigated assuming a constant value for the pile spacing (*s=5D)*, while E_p/E_s was varied using the following values: 10, 100, 1000, 10000. In the work of Small and Zhang (2000) [20], the normalized lateral displacement of the piled-raft was defined using the Equation (5).

$$I_{u,xx} = \frac{E_s D y}{H} \tag{5}$$

Where y and H are the lateral displacement and the lateral load, respectively. The results (Figure 2) are compared with those by using the code APRAF [20]. As shown in Figure 2 increasing E_p/E_s the normalized lateral displacement decreases while the loading rate carried by the 4x4 pile group increases.

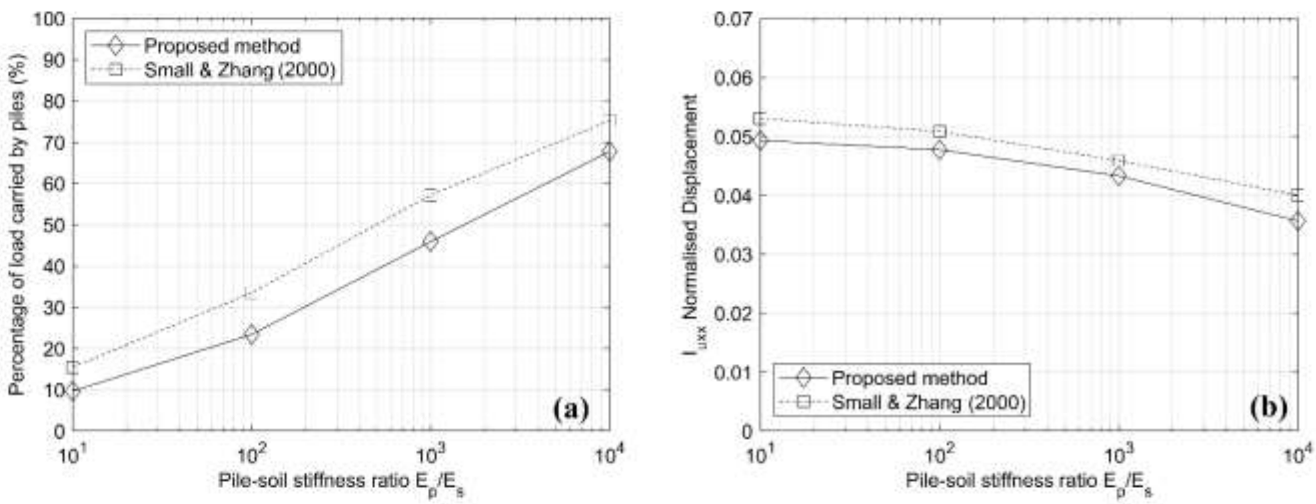

Fig. 2: (a) Effect of pile-soil stiffness ratio on the percentage of load carried by the piles; (b) Effect of pile-soil stiffness ratio on piled raft displacement.

The influence of the pile-spacing on the load sharing and on the lateral displacement has been also studied. In this case the E_p/E_s has been set to 2000, while the pile-spacing was varied using the following values: 2*D*, 3*D*, 4*D*, 6*D* and 10*D*.

The results (Figure 3) are again compared with those inferred by using the code APRAF (Small and Zhang, 2000 [20]). The pile-spacing increase leads to a decrease of both the normalized lateral displacement and the loading rate carried by the pile group.

The differences between the proposed method and APRAF are probably due to the different modelling of the pile-soil-raft interactions. The code APRAF is, in fact, based on a different theory (finite layer theory, Small and Booker, 1986 [38]).

Nevertheless, APRAF results assume a different trend (Figure 3a) for pile spacing values less than 4. In fact, the loading rate carried by the piles seems to tend to an asymptotic value (close to the 80%) for such pile spacings. The APRAF result is surely strange if a pile spacing close to 1 is considered. In fact, in such condition it is expected that approximately the total lateral load should be carried by the piles.

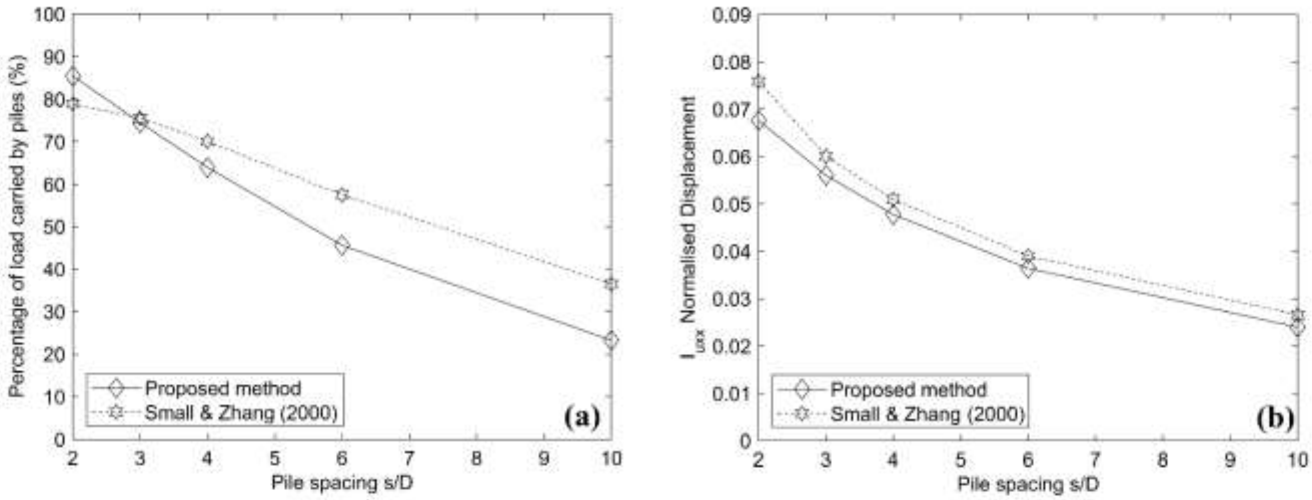

Fig. 3: (a) Effect of pile spacing ratio on the percentage of load carried by the piles; (b) Effect of pile spacing ratio on piled raft displacement.

3.2. Case study

In this section a comparison is shown between centrifuge tests data and results by using the proposed method. The numerical analysis has been carried out not as a back-analysis but as a class A prediction, thus using the actual pile and soil properties based on the information and laboratory tests data provided in Horikoshi and Matsumoto (2003) [3].

In [3] static loading tests were carried out on single pile, raft and piled raft models on air-pluviated dry Toyoura sand by using a geotechnical centrifuge. All these models were loaded in separate tests.

The relative density (D_R) of the sand was about 60% after applying the centrifugal acceleration of 50g. Based on triaxial consolidated drained shear tests (CD) the angle of internal friction, φ', of the Toyoura sand at a relative density of 65% was estimated as 45 degrees, while the shear stiffness at a given shear strain was found to be proportional to the square root of the confining pressure. The measured values of G_{max} at a reference confining pressure equal to 100 kPa was 21.08 MPa.

The properties of the model pile and the corresponding prototype pile are reported in Table 1, while the model raft is a square aluminium raft with width of 80 mm (4 meters at prototype scale). Four piles were installed beneath the raft at a relative spacing of 4 diameters, the raft base was roughened to increase the frictional resistance and a vertical load was applied by using a raft mass before the lateral load test. The pile-heads were fixed against rotation. Based on raft alone lateral load tests it was found a raft-soil interface friction angle of 22.9 degrees. The piled raft model was laterally loaded at a height of 25 mm above the soil surface. Additional details about the centrifuge tests can be retrieved in [3].

Table 1: Properties of model pile and corresponding prototype pile [3].

Item	Centrifuge model	Prototype
Material	Aluminium	Concrete
Outer diameter, D (mm)	10.0	500.0
Wall thickness, t (mm)	1.0	Solid
Length, L (mm)	180.0	9000.0
Cross sectional rigidity, E_pA (GN)	0.002	5.0
Flexural rigidity, E_pI_p (GNm^2)	$2.0\ 10^{-8}$	0.13
Young's modulus, E_p (GN/m^2)	71.0	41.7

3.2.1. Analysis results: Rigid Piled Raft model

The maximum shear modulus profile used in the analysis is that provided in [3]. The Poisson's ratio has been assumed equal to 0.35. The ultimate soil pressure profile has been computed according to the relationship in Reese et al. (1975) [39]. The vertical load distribution between the pile group and the raft prior the lateral loading test was provided in [3], nevertheless, the Authors obtained a similar distribution applying the PDR method.

The results obtained with the proposed method have been compared with the experimental data and with those presented in Kitiyodom et al. (2005) [24] by using the code PRAB in terms of: a) load-displacement curves of the rigid piled raft model and its components (raft and pile group) (Figure 4a) and b) pile bending moment profile at a displacement equal to 12.5 mm (Figure 4b). The code PRAB is a plate-beam-spring model where the soil is modelled with springs and the soil nonlinear response is considered with bi-linear elastic-perfectly plastic springs. PRAB results are the outcomes of a back-analysis procedure in which the soil modulus was varied to obtain the best fit with the centrifuge test data.

The results obtained with the proposed method are in good agreement with the experimental ones and should be remembered that they are the outcomes of a class A prediction, i.e. using the actual pile and soil properties.

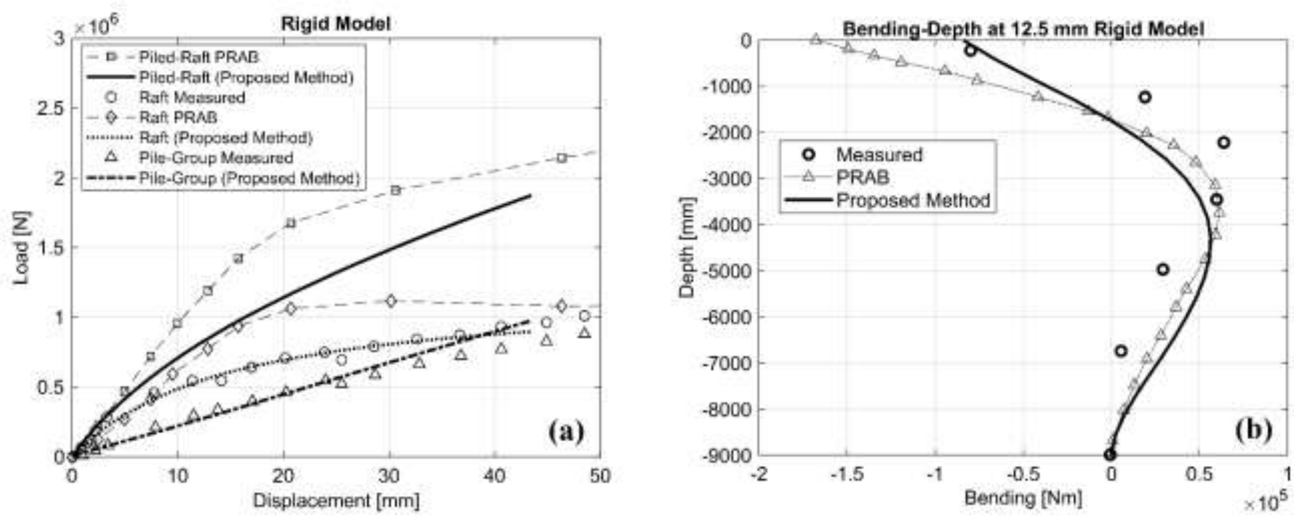

Fig. 4: (a) Computed vs. measured Lateral Load – Displacement curve for the rigid piled raft model (results in prototype scale); (b) Computed vs. measured distributions of bending moments along the pile shaft of an average pile in the rigid piled raft model (results in prototype scale).

4. Conclusions

Most of the computational platforms are specialized either for structural or for geotechnical applications, nevertheless a laterally loaded piled raft foundation represents a complex soil-structure interaction problem which requires a proper modelling of soil and pile material nonlinearities to continuously capture the pile-soil relative stiffness variation.

The proposed BEM-based method is innovative because it can consider these nonlinearities. This method has as additional feature the possibility to model the nonlinear response of reinforced concrete piles accounting also for the influence of tension stiffening. Moreover, since the behaviour of laterally loaded pile foundation is strongly affected by shallower soil layers, frequently interested by partially saturated soil condition, the influence of suction is considered herein using the Modified Kovacs model.

The goodness of the proposed approach has been tested here, in the linear elastic range, by comparing parametric analysis results with those obtained with another code based on the finite layer theory. Additionally, a comparison with centrifuge test data on a laterally loaded piled raft model is shown to verify its reliability. The results obtained highlight the possibility to provide a good forecast of the most representative aspects of the piled raft response.

With the proposed method, the piled raft analysis requires less than 10 min of CPU time to compute the entire load-displacement curve on a laptop with an Intel Core i7 CPU processor (2.20 GHz). Analyses of similar problems by commonly used geotechnical FEM codes require more than 5 hours, using the same hardware setup.

References

[1] R. P. Cunha and H. H. Zhang, "Behavior of piled raft foundation systems under a combined set of loadings," in *10th. Int. Conf. on Piling and Deep Foundations*, 2006, pp. 242-251.

[2] J. Hamada, T. Tsuchiya, T. Tanikawa and K. Yamashita, "Lateral loading tests on piled rafts and simplified method to evaluate sectional forces of piles," *Geotechnical Engineering*, vol. 46, no. 2, pp. 29-42, 2015.

[3] K. Horikoshi and T. Matsumoto, "Performance of piled raft foundations subjected to static horizontal loads," *International Journal of Physical Modelling in Geotechnics*, vol. 3, no. 2, pp. 37-50, 2003.

[4] J. B. Kim and P. L. Sing, "Pile cap soil interaction from full scale lateral load tests", *Journal of Geotechnical Engineering*, ASCE, vol. 105, no. 5, pp. 643-653, 1979.

[5] T. Matsumoto, K. Fukumura, K. Pastsakorn, K. Horikoshi and A. Oki, "Experimental and Analytical Study on Behaviour of model piled rafts in sand subjected to horizontal and moment loading," *International Journal of Physical Modelling in Geotechnics*, vol. 4, no. 3, pp. 01-19, 2004.

[6] T. Matsumoto, H. Nemoto, H. Mikami, K. Yaegashi, T. Arai and P. Kitiyodom, "Load Tests of Piled Raft Models With Different Pile Head Connection Conditions and Their Analyses," *Soils and Foundations*, vol. 50, no. 1, pp. 63-81, 2010.

[7] R. L. Mokwa, "Investigation of the Resistance of Pile Caps to Lateral Loading," PhD thesis, Virginia Polytechnic Institute and State University, Blacksburg, VA, 1999.

[8] R. L. Mokwa and J. M. Duncan, "Experimental Evaluation of Lateral-Load Resistance of Pile Caps," *Journal of Geotechnical and Geoenvironmental Engineering*, vol. 127, no. 2, pp. 185-192, 2001.

[9] Y. Unsever, M. Ozkan, T. Matsumoto, S. Shimono and K. Esashi, "Physical and numerical modelling of pile foundations subjected to vertical and horizontal loading in dry sand," in *Computer Methods and Recent Advances in Geomechanics*, 2014, pp. 963-968.

[10] Y. S. Unsever, T. Matsumoto and M. Y. Özkan, "Numerical analyses of load tests on model foundations in dry sand," *Computers and Geotechnics*, vol. 63, pp. 255-266, 2015.

[11] A. B. Huang, C. K. Hsueh, M. W. O'Neill, S. Chern and C. Chen, "Effects of Construction on Laterally Loaded Pile Groups," *Journal of Geotechnical and Geoenvironmental Engineering*, vol. 127, no. 5, pp. 385-397, 2001.

[12] M. N. Hussien, T. Tobita, S. Iai and M. Karray, "On the influence of vertical loads on the lateral response of pile foundation," *Computers and Geotechnics*, vol. 55, pp. 392-403, 2014.

[13] H. Hirai, "A Winkler model approach for vertically and laterally loaded piles in nonhomogeneous soil," *International Journal for Numerical and Analytical Methods in Geomechanics*, vol. 36, no. 17, pp. 1869-1897, 2012.

[14] M. I. Hoit, M. McVay, C. Hays and P. W. Andrade, "Nonlinear Pile Foundation Analysis Using Florida-Pier," *Journal of Bridge Engineering*, vol. 1, no. 4, pp. 135-142, 1996.

[15] L. C. Reese, S. T. Wang, J. Arrellaga and J. Hendrix, "Computer program GROUP for Windows user's manual, version 5.0," Ensoft, Austin, Texas 2000.

[16] API. Recommended Practice for Planning, Designing and Constructing Fixed Offshore Platforms - Working Stress Design. vol. 24-WSD. 2007.

[17] F. Basile, "Non-linear analysis of vertically loaded piled rafts," *Computers and Geotechnics*, vol. 63, pp. 73-82, 2015.

[18] R. M. Filho, A. V. Mendonça and J. B. Paiva, "Static boundary element analysis of piles submitted to horizontal and vertical loads," *Engineering Analysis with Boundary Elements*, vol. 29, no. 3, pp. 195-203, 2005.

[19] H. G. Poulos, "User's guide to program DEFPIG 3/4 Deformation analysis of pile groups, Revision 6," School of Civil Engineering, University of Sidney 1990.

[20] J. C. Small and H. H. Zhang, "Piled raft foundations subjected to general loadings," in *Developments in Theoretical Geomechanics*, 2000, pp. 57-72.

[21] J. C. Small and H. H. Zhang, "Behavior of Piled Raft Foundations Under Lateral and Vertical Loading," *International Journal of Geomechanics*, vol. 2, no. 1, pp. 29-45, 2002.

[22] J. C. Small, H. H. Zhang and A. L. de Ambrosis. "Analysis of Piled Raft Foundation," Department of Civil Engineering, University of Sydney, NSW, 2006.
[23] H. H. Zhang and J. C. Small, "Analysis of capped pile groups subjected to horizontal and vertical loads," *Computers and Geotechnics*, vol. 26, no. 1, pp. 1-21, 2000.
[24] P. Kitiyodom, T. Matsumoto, K. Horikoshi and T. Watanabe, "Analyses of vertical and horizontal load tests on piled raft models in dry sand," in *Proceedings of the International Conference on Soil Mechanics and Geotechnical Engineering*, 2005.
[25] P. Kitiyodom and T. Matsumoto, "A simplified analysis method for piled raft and pile group foundations with batter piles," *International Journal for Numerical and Analytical Methods in Geomechanics*, vol. 26, no. 13, pp. 1349-1369, 2002.
[26] S. Stacul and N. Squeglia, "Analysis Method for Laterally Loaded Pile Groups Using an Advanced Modeling of Reinforced Concrete Sections," *Materials*, vol 11, no. 2, 2018.
[27] R. D. Mindlin, "Force at a Point in the Interior of a Semi-Infinite Solid," *Physics*, vol. 7, no. 5, pp. 195-202, 1936.
[28] S. Stacul and N. Squeglia. "KIN SP: A boundary element method based code for single pile kinematic bending in layered soil". *Journal of Rock Mechanics and Geotechnical Engineering*, vol. 10, no. 1, pp. 176-187, 2018.
[29] H. G. Poulos and E. H. Davis, "Pile foundation analysis and design," John Wiley & Sons; 1980.
[30] F. Morelli, C. Amico, W. Salvatore, N. Squeglia and S. Stacul, "Influence of Tension Stiffening on the Flexural Stiffness of Reinforced Concrete Circular Sections," *Materials*, vol. 10, no. 6, 2017.
[31] M. Fahey and J. Carter, "A finite element study of the pressuremeter test in sand using a nonlinear elastic plastic model," *Canadian Geotechnical Journal*, vol. 30, no. 2, pp. 348-362, 1993.
[32] S. Stacul, N. Squeglia and F. Morelli, "Laterally Loaded Single Pile Response Considering the Influence of Suction and Non-Linear Behaviour of Reinforced Concrete Sections," *Applied Sciences*, vol. 7, no. 12, 2017.
[33] S. Stacul, "Analysis of piles and piled raft foundation under horizontal load," University of Florence & Technical University of Braunschweig, 2018.
[34] M. Ashour, P. Pilling and G. Norris, "Lateral Behavior of Pile Groups in Layered Soils," *Journal of Geotechnical and Geoenvironmental Engineering*, vol. 130, no. 6, pp. 580-592, 2004.
[35] M. Aubertin, M. Mbonimpa, B. Bussière and R. P. Chapuis, "A model to predict the water retention curve from basic geotechnical properties," *Canadian Geotechnical Journal*, vol. 40, no. 6, pp. 1104-1122, 2003.
[36] H. G. Poulos, "Practical design procedures for piled raft foundations. Design applications of raft foundations," Hemsley J. A. ed., Thomas Telford; 2000, pp. 425-467.
[37] M. F. Randolph, "Design methods for piled groups and piled rafts," in *13th International Conference of Soil Mechanics and Foundations*, 1994, pp. 61-82.
[38] J. C. Small and J. R. Booker, "Finite layer analysis of layered elastic materials using a flexibility approach. Part 2-Circular and rectangular loadings," *International Journal for Numerical Methods in Engineering*, vol. 23, no. 5, pp. 959-978, 1986.
[39] L. C. Reese, W. R. Cox and F. D. Koop, "Field Testing and Analysis of Laterally Loaded Piles on Stiff Clay," in *Offshore Technology Conference*, 1975.