

Stress Analysis of The Effects of Mortar Thickness in Masonry Structures Using an Anisotropic Model

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Abstract - The aim of this study is to investigate the effect of mortar/brick thickness ratio, brick configuration on the masonry stresses and on masonry structure modelling. Different mortar/brick thickness ratios are micro-modelled by using stress- strain relationship. The results of these models are used to obtain the material properties of anisotropic macro-models in vertical and horizontal directions. The sample unreinforced masonry structures are simulated by the SAP2000 software using anisotropic material properties. In general masonry structures are composite materials that consists of brick and mortar. The main goal in modelling is to simulate a model that behaves close to the real structure as much as possible. Micro-modelling of masonry structure by considering its components individually is more realistic but not practical for large scale structures. In order to overcome this problem, macro-modelling is used in literature. Macro-model is obtained by smearing out the material properties of the micro-model to an anisotropic continuum material. Currently, most engineers simply analysis masonry structures with isotropic modelling in practice. However, presence of head and bed mortar joints in the construction of masonry causes the masonry to be anisotropic. Anisotropic modelling of a large scale structure does not need unreasonable effort as in micro-modelling and can be used instead of isotropic modelling easily in practice. Considering the horizontal and vertical loads in the plane, some wall types with rectangular cross-section, rectangular and arches cross-sections cavities were modeled as anisotropic and the effect of anisotropic material was investigated. In terms of displacements, an isotropic model and anisotropic models were found to be close to each other. However, when examined in terms of stresses, the maximum and minimum stresses in the masonry structure were changed.

Keywords: Unreinforced masonry, mortar/unit thickness ratio, micro model, macro model, homogenization, composite materials, stress analysis.

1. Introduction

The main purpose in modeling is to create a model that behaves as close as possible to the actual structure. Masonry is a composite material composed of brick and mortar. Micro-modeling, taking into account the individual components, is not practical for a more realistic but wide structure. To overcome this problem, the macro-model can be used to obtain an anisotropic continuity in the material properties of its components.

Therefore, the wall elements shown in Figure 1 cause the wall to be anisotropic, since they have different material properties [1]. In this study, the modulus of elasticity in the wall structure was obtained by taking into consideration the change in the thickness of the mortar [2]. Material constants in both directions were obtained by using the stress - strain relations of the typical composite wall element in the plane stress effect.

An example of this type of masonry structure is shown in Figure 2 by the author of this study during the restoration of a minaret in Istanbul where the mortar / brick thickness ratio is close to 1. Therefore, the effect of mortar thickness and anisotropic modeling on the brick wall was investigated.

Only the horizontal component increased of the mortar thickness is shown figure 3. The micro-model is given in figure 3. The length of this modeled composite element was taken as L , width W , brick thickness h_u , mortar thickness h_m . Since the plane element is taken into consideration, the material constants are changed only in the direction 2 and in the direction 1, 3 has the same characteristics as shown fig. 3.

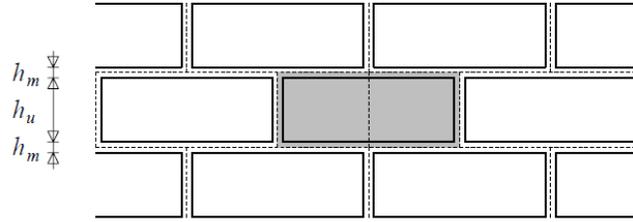


Fig. 1: Typical masonry wall model.



Fig. 2: Example of masonry minaret wall.

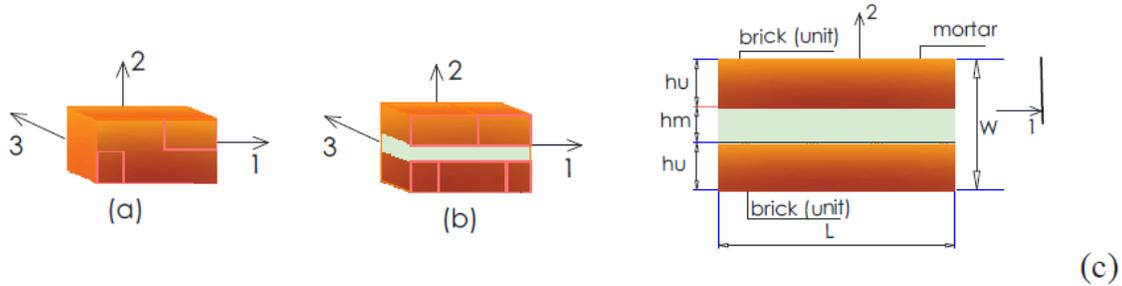


Fig. 3: (a) horizontal and vertical mortar thickness fixed, (b) horizontal mortar thickness increased, (c) micro-wall model.

2. Micro Model in Composite Material

In the stress-strain relations in linear elastic theory; there are generally 36 independent material constants. These material constants are reduced to 21 in anisotropic material, 13 in monoclin material (3 in symmetrical), 9 in orthotropic material, 5 in transverse isotropic material and 2 in isotropic material. In the case of plane stress, the transverse isotropic material has 4 constant independent stress-strain relations (E_1 , E_2 , ν_{12} , G_{12}). In this study, 4 independent material constants are obtained from stress-strain relations. Relations between stress-strain in anisotropic medium for plane stress state using Generalized Hooke Laws can be written as:

$$\varepsilon_1 = \frac{1}{E_1} \sigma_1 - \frac{\nu_{12}}{E_1} \sigma_2, \quad \varepsilon_2 = \frac{1}{E_2} \sigma_2 - \frac{\nu_{12}}{E_1} \sigma_1, \quad \gamma_{12} = \frac{\tau_{12}}{G_{12}} \quad (1)$$

Here, $E_1, E_2, \nu_{12}, G_{12}$ are the engineering constants of the material.

The micromechanical theories developed for conventional composites can be used to predict such properties. The assumptions regarding the both mortar and brick are linearly elastic isotropic materials [3-5].

2.1. Calculation of Longitudinal Elastic Modulus (E_1)

In the loading in direction 1, the deflections (elongation ratios) are obtained from the conditions of equality (geometric conformity condition) Fig 4 a. The total force of equilibrium equations F_1 , the force of the F_u brick, the force of the mortar F_m , the total cross-sectional area of A_1 , the cross-sectional area of the brick, the cross-sectional area of the mortar, the ratio of the V_u brick area to the total area, the ratio of the V_m mortar area to the total area, the stress on the axis of σ_1 , direction 1. The stress in the σ_u brick is the stress in the σ_m mortar.

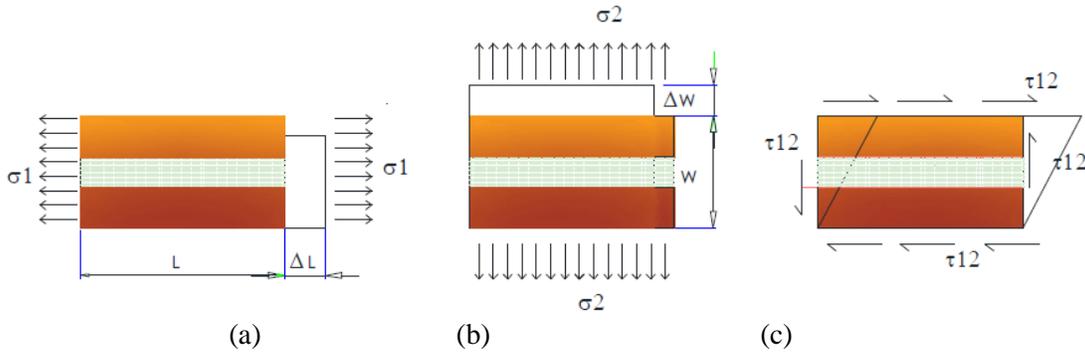


Fig. 4: Strain effect element in (a) direction 1, (b) direction 2 and (c) shear effect.

Equilibrium equation in direction 1 written as:

$$\sigma_1 A_1 = \sigma_m A_m + \sigma_u A_u, \quad E_1 \varepsilon_1 A_1 = E_m \varepsilon_m A_m + E_u \varepsilon_u A_u \quad (2)$$

where $F_1 = F_m + F_u$, $\sigma_i = E_i \varepsilon_i$ and $F_i = \sigma_i A_i$.

Material constant in direction 1 when considering equality of elongation rates can be calculated as:

$$E_1 = E_m V_m + E_u V_u \quad (3)$$

The ratio of brick and mortar to total area is the $V_m = A_m/A_1$, $V_u = A_u/A_1$, ($V_m + V_u = 1$).

2.2. Calculation of Transverse Elastic Modulus (E_2)

Calculation of the material constant E_2 is made by loading it in 2 directions, the condition of the stresses in the materials (from the condition of continuity) and the sum of the displacements ($\Delta_2 = \Delta_m + \Delta_u$) in the 2 direction as shown in Fig. 4 b.

$$\varepsilon_2 W = \varepsilon_m W V_m + \varepsilon_u W V_u \quad (4)$$

In the form of E_2 material constant in the direction 2 obtained as:

$$\frac{1}{E_2} = \frac{V_m}{E_m} + \frac{V_u}{E_u} \quad (5)$$

2.3. Calculation of Poisson's Ratio (ν_{12})

In the case of the ratio of the deformations in the loading in direction 1 $\nu_{12} = -\frac{\varepsilon_2}{\varepsilon_1}$ and the sum of vertical displacements Fig. 4 $\Delta_W = \Delta_{mw} + \Delta_{uw}$ as shown in Fig. 4

$$\varepsilon_1 \nu_{12} W = \varepsilon_m W V_m \nu_m + \varepsilon_u W V_u \nu_u \quad (6)$$

Poisson's ratio obtained as:

$$\nu_{12} = \nu_m V_m + \nu_u V_u \quad (7)$$

2.4. Calculation Shear Modulus (G_{12})

Due to the sum of deformations due to shear stress $\Delta_L = \Delta_m + \Delta_u$ as shown Fig. 4, shear modulus G_{12} obtained as:

$$\gamma W = W V_m \gamma_m + W V_u \gamma_u \quad (8)$$

$$\frac{1}{G_{12}} = \frac{V_m}{G_m} + \frac{V_u}{G_u} \quad (9)$$

The transition from the micro-model to the macro-model can be calculated using these constants in the actual structure after obtaining the corresponding elastic modulus values of the anisotropic continuity representations called macro models.

In this study, the mortar modulus of elasticity is 2000Mpa, the brick modulus of elasticity is 20,000Mpa, the mortar poisson ratio is 0.25 and the brick Poisson's ratio is 0.15. E_1 , E_2 , ν_{12} , G_{12} are calculated used to Eqn. (3), (5), (7), (9) and results given Table 1. E_1 and E_2 diagram according to mortar thickness as shown Fig .5.

Table 1: Material coefficients according to the total thickness of the mortar thickness.

	1	2	3	4	5	6
V_u	1	0,8	0,6	0,4	0,2	0
V_m	0	0,2	0,4	0,6	0,8	1
E_1 (MPa)	20000	16400	12800	9200	5600	2000
E_2 (MPa)	20000	7143	4348	3125	2439	2000
ν_{12}	0,15	0,17	0,19	0,21	0,23	0,25
G_{12} (MPa)	8696	2924	1757	1256	978	800

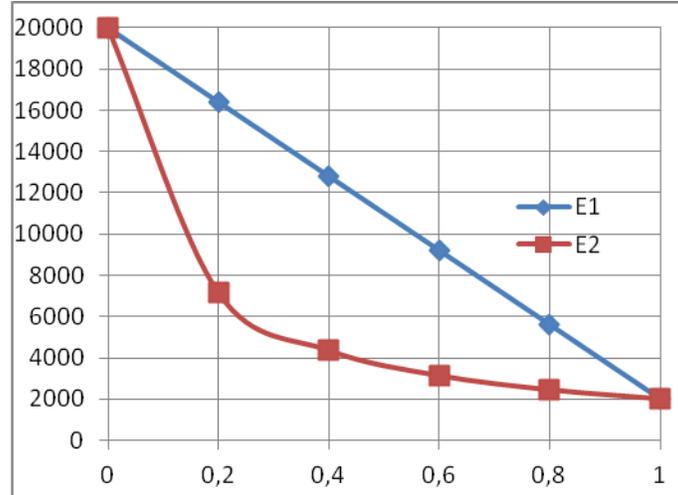


Fig 5: Elastic modulus E_1 , E_2 , values containing material data of macro models are given in Table 1.

3. Typical Walls

Typical walls were taken into consideration and a typical wall of 4.0m wide and 3.0m high .025m thick was considered. Isotropic and anisotropic solutions were made with Sap2000 program [6]. 1.0MPa normal stress, 1.0MPa shear stress was applied on the upper surface of the wall as loading and isotropic anisotropic solutions were compared in some parts of the wall.

Isotropic walls were constructed by considering the E_2 value and were used to compare the anisotropic wall models (Fig. 6). Wall cavity / total area: (a) wall cavity ratio = 0, (b) wall cavity ratio = 0.185, (c) wall cavity ratio = 0.615. This is the rule of mixtures for composite materials based on the idea of idealizing two linear bows of brick and mortar in series. ($E_2 = E_1$).

Wall total area / gap area ratio σ_{max} , σ_{min} brick thickness / mortar thickness ratio was calculated according to $V_m/V_u = 0.2/0.8$ and $V_m/V_u = 0.6/0.4$ σ_{max} , σ_{min} was obtained as follows. According to the ratio of $V_m/V_u = 0.6/0.4$ in the anisotropic environment, the σ_{max} diagram in the effect of horizontal load is given in figure 7. In addition, σ_{max} values obtained from isotropic and anisotropic solutions are given in table 2.

In anisotropic environment, σ_{min} diagram is shown in figure 8 according to the ratio of $V_m/V_u = 0.6/0.4$. In addition, σ_{max} values obtained from isotropic and anisotropic solutions are given in table 3.

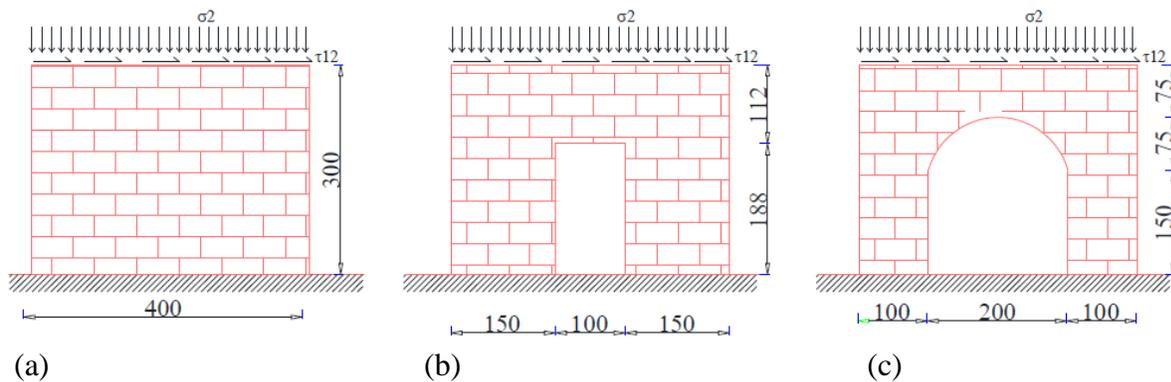


Fig. 6 Wall models (a) filled wall, (b) rectangular cavity, (c) circular/rectangular cavity.

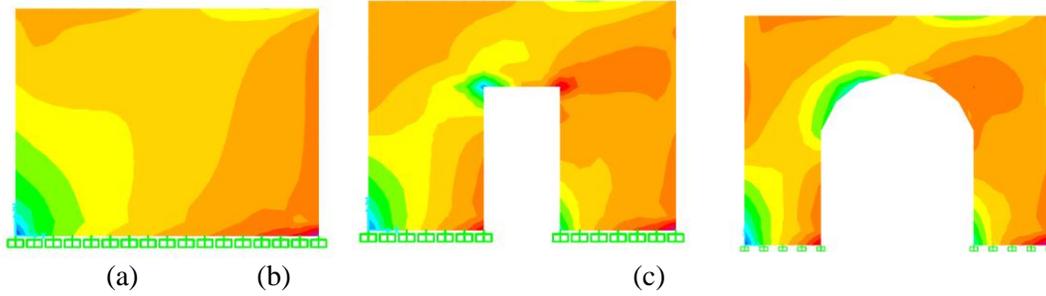


Fig. 7: σ_{\max} diagrams in horizontal load effect $V_m/V_u = 0.6/0.4$.

Table 2: Material coefficients according to the total thickness of the mortar thickness.

		horizontal load	a2	b2	c2	a4	b4	c4
σ_{\max} Mpa	anisotrop		7.153	11.586	18.241	7.792	12.99	19.31
	isotrop		6.642	11.001	18.054	6.691	11.06	18.12
	ratio	anisotrop/isotrop	1.077	1.053	1.010	1.165	1.175	1.066
U1 mm	anisotrop		2.230	3.645	6.413	5.078	8.164	14.23
	isotrop		2.259	3.934	7.198	5.182	9.012	16.48
	ratio	anisotrop/isotrop	0.99	0.93	0.89	0.98	0.91	0.86
U2 mm	anisotrop		1.131	1.269	1.386	2.549	2.821	3.039
	isotrop		1.170	1.397	1.618	2.675	3.191	3.695
	ratio	anisotrop/isotrop	0.97	0.91	0.86	0.95	0.88	0.82

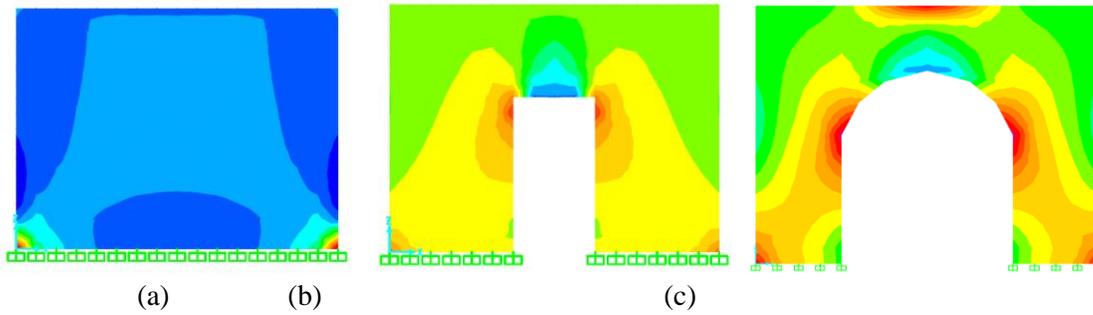


Fig. 8: σ_{\min} diagrams in vertical load effect $V_m/V_u = 0.6/0.4$.

Table 3: Material coefficients according to the total thickness of the mortar thickness

		vertical load	a2	b2	c2	a4	b4	c4
σ_{\min} Mpa	anisotrop		1.425	2.852	4.64	1.634	2.89	4.622
	isotrop		1.284	2.965	4.933	1.303	2.971	4.941
	ratio	anisotrop/isotrop	1.110	0.962	0.941	1.254	0.973	0.935
U2 mm	anisotrop		0.414	0.616	1.182	0.941	1.385	2.614
	isotrop		0.415	0.637	1.287	0.948	1.455	2.94
	ratio	anisotrop/isotrop	1.00	0.97	0.92	0.99	0.95	0.89

4. Conclusion

In this study, micro-modelled walls are homogenized to anisotropic continuum wall models and their material properties are presented in Table 3 and Fig. 6. It is observed that the Elastic modulus of anisotropic wall models in vertical direction E_2 is always higher than the one in horizontal direction E_1 for the three wall configurations considered herein.

In vertical direction, the Elastic modulus values E_2 of anisotropic walls A and B1 are around E_1 is 2.94 times larger than their isotropic counterparts E_2 . Therefore, we can conclude that rigidity of anisotropic model in vertical direction is always higher than that of its isotropic counterpart but this deviation increases with increasing mortar thickness equals brick thickness.

Recalling the fact that most of masonry structure failures take place due to lateral loading, therefore the rigidity in horizontal direction is important to be investigated. Comparing data for anisotropic wall models a, b, c given in Table 2 and in Table 3, it is observed that the effect of mortar thickness increased.

The wall structures are simulated using anisotropic and isotropic wall data of (a), (b), (c), and the displacements at the top level are presented in Table 2 and Table 3. Comparison of a, b, and c structure model displacements given in Table 2 shows that model c displaced around vertical displacement u_2 is 0.82 times larger than their isotropic counterparts.

The anisotropic / isotropic ratio decreases as the cavity ratio increases. The effect of anisotropy solution increases as the rate of mortar increases.

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