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Dynamic Behaviour of Oscillating Bubbles inside a Nonlinear Thixotropic Fluid in Existence of Magnetic Field

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Abstract – Bubbles inside fluids respond to acoustically imposed pressure forces and reveal their full potential when periodically driven by sound waves. These bubbles oscillations could not be easily captured experimentally especially for high frequency excitations. The effect of an imposed magnetic field on nonlinear oscillations of acoustically forced spherical gas bubbles in a time-dependant viscosity fluid (thixotropic fluid) is investigated in this paper. These behaviours are of great interest as the bubbles should adapt their dynamical properties with a shear thinning surrounded medium with some lag time of reaction in addition to nonlinearities being imported due to the magnetic field force on the bubbles. For this study the basic governing equations of the bubbles oscillations has been modified to add the magnetic body force effects. The Moore's model is also used to model the Thixotropic properties of the medium and various structural parameters were considered to understand the dynamic behaviour of the bubbles. By using the RK4 method for numerical solution of the set of defined governing equations, the combined effects of structure build-up, structure break-down and magnetic field on bubble radius variations are investigated. The results show that both the gas bubbles oscillation in thixotropic fluids and structural changes (viscosity changes) of these kinds of fluids could be effectively controlled by external magnetic fields. The relevance and importance of these bubble dynamics to biomedical ultrasound applications and light emissions by sonoluminescen and many other cement paste industries and etc. could be obviously addressed as the possible applications of this study.

***Keywords*:** Thixotropic Fluid, Magnetic Field, Acoustic excitation, Bubbles nonlinear oscillations.

1. Introduction

Due to a review by Barnes [1], the time-dependent response of a microstructure which is itself changing with time is one of the most challenging issues for rheologists that are studying the viscoelastic behavior of fluids. The rheological manifestation of flow-induced structural changes is a variable viscosity. If the changes are reversible and time dependent, the effect is called thixotropy. The most recent review by Mewis and Wagner [2] reemphasizes the importance of time-dependent viscosity fluids flow in various engineering and bio-medical applications. Among common thixotropic fluid systems, one can mention drilling muds, foodstuffs, paints, concretes, semi-solids, and physiological fluids. There are also a number of experimental and theoretical studies on the thixtropic fluid flows (Assad et al. [3]; Wallevik [4]; Derksen & Prashant [5]; Ardakani et al. [6] and Najmi et al. [7]) that are published recently due to the important applications of these kinds of fluids in various industries.

Thixotropic structures progressively break down on shearing and slowly rebuild at rest and the time-scales involved can range from many minutes in the case of breakdown to many hours in rebuilding. The microstructure breakdown is counterbalanced, to some extent, by the action of the Brownian forces which tries to reform these microstructures as soon as they are broken down (Schalek et al. [8]).

The single gas bubble dynamics was the issue for numerous researches since its first observation by Lord Raleigh [9]. The considerable fundamental studies were done by Plesset, Prosperetti and others and the interested readers are referred to our previous work by Aliabadi & Taklifi [10] for more details of these studies. The study of forced oscillations of gas bubbles in Non-Newtonian media was first issued by Yang et al. [11] in 1996 and since then bubble oscillations are studied both experimentally and numerically with some rheological models. For instances Allen et al. [12, 13] investigated the bubble growth dynamics in linear and non-linear viscoelastic fluids, Smolianski et al. [14] developed a two dimensional code which show the dynamics of gas bubbles rising in a viscous liquid in different flow regimes. Aliseda and Engebrecht [15] discussed the dynamics of micro-bubbles under the effect of ultrasonic pressure waves, which has major applications in new drug delivery concepts.

In addition in the case of bubble dynamics in visco-elastic fluid flows Brujan [16] found that the most interesting effect of the non-Newtonian properties of the liquid flow is the reduction of cavitation damage and noise. Albernaz et al. [17] investigated the bubble dynamics in a Maxwell fluid with extensional viscosity and they found that a degree of elasticity in the ambient liquid for the linear viscoelastic model, increases the degree of instability in the oscillatory motion of the bubble which is in contrast with Brujan's results. Also recent works have been exploring different models to study nonlinear bubble oscillations (Amini Kafiabad and Sadeghy [18]; Naude and Mendez [19]; Foteinopoulou and Laso [20]).

Nomenclature

τ	stress tensor	C	Constant parameter
σ_s	fluid static surface tension	R_0	equilibrium bubble radius
σ	fluid electrical conductivity	λ	relaxation time
$u(r,t)$	velocity	η_0	zero shear rate viscosity
ρ	fluid density	De	Deborah number
k	polytropic exponent	Re	Reynolds number
$k_1 . k_2$	two material properties	We	Weber number
t	time	p	pressure
$R(t)$	instantaneous bubble radius	ω	acoustic frequency
B	magnetic field	α	ratio of pressure amplitudes

For the case of gas bubble dynamics in thixotropic fluids, Ahmadpour et al. [21]studied the rise of second harmonics in forced oscillation of gas bubbles in thixotropic fluids, which show that at sufficiently high pressure amplitudes, typical of medical ultrasound applications, a second harmonics may be observed in the bubble's response. As it is stated in abovementioned paper as the time-dependent viscosity adds significantly to the difficulty level of any analysis involving thixotropic fluids, these kinds of flows have not been investigated to the same extent as is the case with other non-Newtonian fluid systems (e.g., viscoelastic fluids) and also most of these limited works concentrate on shear flows however the bubble oscillations problem is an extensional flow and so current study would be considerable for thixotropic flows.

MHD effects on bubble flows also are limited to a number of studies which are done in last decade (Merrouche et al. [22]; Shibasaki et al. [23]).

In current study a straightforward semi-analytical equation is developed to investigate the magnetic field effects on bubble dynamics and collapse in a thixotropic media. So a modified Rayleigh-Plesset equation in terms of the bubble radius considering external magnetic field effect is obtained by using the Navier-Stokes equation. In parallel the constitutive structural equation of Moore's model is considered, which is coupled to the previously developed modified Rayleigh-Plesset equation for the bubble. The effect of magnetic field on forced oscillations of bubbles in a thixotropic fluid is studied. The results show interesting changes in radius variations and stress variations with respect to time for pulsating bubbles.

2. Governing Equations

In the present study it is assumed that the material outside the gas bubble wall is incompressible, and the spatially uniform conditions are assumed to exist within the bubble. Also the bubble remains spherical. For the discussion of the pulsation of a single bubble whose centre remains fixed in space, as occurs here, the convective term of material derivative of particle velocity is zero (Leighton [24]). The magnetic field is constant.

Considering the abovementioned assumptions and using the method developed in by Aliabadi & Taklifi [10] the modified Rayleigh-Plesset equation including the external magnetic field could be expressed as follows:

$$R\ddot{R}+\frac{3\dot{R}^2}{2}+\frac{\sigma B^2}{\rho}R\dot{R}=\frac{1}{\rho}\left[\rho_{g_0}(\frac{R_0}{R})^{3k}-(p_0+p_A\sin\omega t)-\frac{2\sigma_s}{R}+\int_r^\infty\left(\frac{\tau_{rr}-\tau_{\theta\theta}}{r}\right)dr\right] \tag{1}$$

Where σ_s the fluid static surface tension, k is polytropic exponent and ω is acoustic frequency.

By defining the non-dimensional time, radial spatial variable, radius, and stress as (Allen et al. [13]):

$$\bar{t}=\omega t;\qquad \bar{r}=r/R_0;\qquad \bar{R}=R/R_0;\qquad \bar{\tau}=\tau\frac{R_0}{\eta_0}\sqrt{\rho/\rho_0} \tag{2}$$

We obtain the Deborah number as follows:

$$De=\lambda\omega \tag{3}$$

Then the Reynolds number is given by:

$$Re=\frac{\rho\omega R_0^2}{\eta_0} \tag{4}$$

Finally the Equation (1) is rewritten in non-dimensional form as,

$$\begin{aligned}R\ddot{R}+\frac{3\dot{R}^2}{2}+\frac{\sigma B^2}{\rho}R\dot{R}\\ =\frac{p_0}{\rho\omega^2R_0^2}\left[\left(1+\frac{2\sigma_s}{p_0R_0}\right)\left(\frac{1}{R}\right)^{3k}-\left(\frac{2\sigma_s}{p_0R_0}\right)\left(\frac{1}{R}\right)-(1+\alpha\sin(t))\right]\\ +\frac{1}{Re}\frac{1}{\omega R_0}\sqrt{\frac{p_0}{\rho}}\int_r^\infty\left(\frac{\tau_{rr}-\tau_{\theta\theta}}{r}\right)dr\end{aligned} \tag{5}$$

For the spherical bubble problem surrounded by a thixotropic fluid obeying Moore's model (Moore [25]), the stress tensor,τ_{ij}, is related to the rate-of-deformation tensor by the relationship $\tau_{ij}=2d_{ij}$. So the viscosity function depends on a structural parameter (κ) by following relation (Malaga & Rallison [26]):

$$\eta(t)=\eta_0+C\kappa(t) \tag{6}$$

And the constitutive equation for structural parameter of a thixotropic fluid based on Moore's model is (Malaga & Rallison [26]):

$$\frac{D\kappa}{Dt}=-k_1\dot{\varepsilon}\kappa+k_2(1-\kappa) \tag{7}$$

Where $\dot{\varepsilon}$ is the rate of extension (or, more precisely, the square root of the second invariant of the rate-of-deformation tensor) and for purely-radial flow it is defined as:

$$\dot{\varepsilon} = 2\sqrt{3}\left|\frac{R^2\dot{R}}{r^3}\right| \tag{8}$$

D/Dt is the material derivate, and k_1 and k_2 are the structure break-down and structure build-up parameters, respectively. From this equation it can be concluded that, the greatest rate of build-up occurs when there is no structure left in the fluid. And, there is no build-up when there is complete structure. We also note that, for any value of the shear rate, there is no structural breakdown when there is no structure in the fluid and that the greatest rate of breakdown occurs when there is complete structure (Ahmadpour et al. [21]). By substituting the stress terms from Moore's model into Eq. 5, and after some mathematical manipulations and considering Weber number as $We = \frac{2\sigma_s}{p_0 R_0}$, one would obtain:

$$\begin{aligned} &R\ddot{R} + \frac{3\dot{R}^2}{2} + \frac{\sigma B^2}{\rho} R\dot{R} \\ &= \frac{p_0}{\rho\omega^2 R_0^2}\left[(1+We)\left(\frac{1}{R}\right)^{3k} - We\left(\frac{1}{R}\right) - (1+\alpha\sin(t))\right] \\ &+ \frac{1}{Re}\frac{1}{\omega R_0}\sqrt{\frac{p_0}{\rho}}\int_R^\infty \eta(t)\frac{R^2\dot{R}}{r^4}\,dr \end{aligned} \tag{9}$$

Radiation damping is not considered because the constitutive equations used are based on the incompressible assumption.

3. Numerical Solution

In order to solve the equations 7 and 9 simultaneously Lagrangian coordinates are introduced to immobilize the boundary (Allen et al. [13]). Following this approach, we introduce the transformation:

$$y = r^3 - R^3(t) \tag{10}$$

Thus the equations 7 and 9 yield into:

$$\begin{aligned} &\frac{dR}{dt} = U. \\ &\frac{dU}{dt} = \left[-\frac{3}{2}U^2 + \frac{p_0}{\rho\omega^2 R_0^2}\left((1+We)\left(\frac{1}{R}\right)^{3k} - We\left(\frac{1}{R}\right) - (1+\alpha\sin(t))\right)\right]\frac{1}{R} + \frac{12R\dot{R}}{Re}\left(\frac{1}{\omega R_0}\sqrt{\frac{p_0}{\rho}}\right) \\ &\quad\times\int_0^\infty\left(\frac{\eta(y.t)}{(y_i+R^3)^{4/3}}\right)dy - \frac{\sigma B^2}{\rho}U \\ &\frac{Dk}{Dt} = -k_1 2\sqrt{3}\left|\frac{R^2\dot{R}}{r^3}\right|\kappa + k_2(1-\kappa) \end{aligned} \tag{11}$$

The initial conditions are taken as:

$$\begin{gathered} R(0) = 1 \\ \lambda(0.y_i) = 0 \\ \dot{R}(0) = 0 \end{gathered} \tag{12}$$

A 4th order Runge-Kutta method was used for time integration and Simpson's Rule was used to compute the spatial integral at each time step. During the coupled solution of the ordinary differential equations and integro-differential equation, several computation problems are observed. These problems including the time step of solution, optimized spatial resolution in y-direction and their effects on stability and accuracy of simulation were monitored and possible optimizations are used in the numerical integration processes. Finally a variable step size method was used at each time step and grid distribution of the spatial coordinate y, was modified by a try and error method to preserve the boundary conditions for structural parameter. The model is also verified by the results obtained by Ahmadpour et al. [21], when the magnetic field was not imposed. Also the accuracy of the used numerical method has been discussed extensively in the authors' previous work [10].

4. Results & Discussion

The sets of integro-differential and ordinary differential Eq. (11) considering the initial conditions as Eq. (12), are solved numerically and in order to validate the developed code, the results are checked for the Newtonian fluid case available in Allen et al. [13].

The results show that by imposing a constant magnetic field in radial direction to the fluid domain in the case of constant $p_0/\rho\omega^2R_0^2 = 2\,.\,\mathrm{De} = 3.\,\sigma = 0.01$ and for $\alpha = 2$ and thixotropic ratio $k_1/k_2 = 1$ when the magnetic field strength varies between 0~0.002 it could be seen that by increasing the magnetic strength (B) the oscillations are being damped effectively (Fig.1). Also as it is obvious in Fig.1 the frequency of oscillations is increased by rising the magnetic field strength and so the peak of the oscillations is shifted toward time and so the probability fast bubble break up increases however the magnitude of bubble size expansion is being limited. In the case of strong radial magnetic field the oscillations are completely damped and so the bubble collapse risk is being controlled.

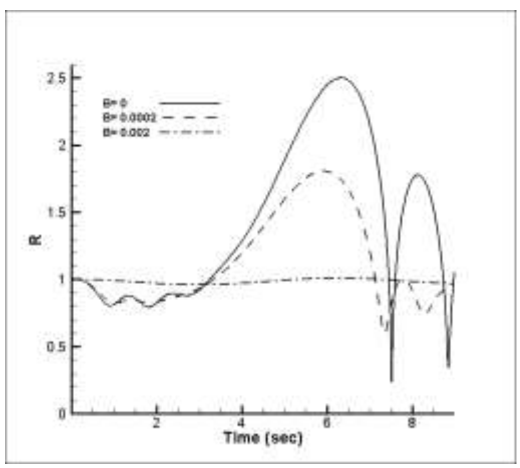

Fig. 1: Bubble size variations due to acoustic motivations for different magnitudes of magnetic fields and $\alpha = 2.\,k_1/k_2 = 1$.

Fig. 2 shows the effects of external magnetic field on the variations of structural parameter by time. This figure illustrates the fact that the structural parameter could be controlled by external magnetic field. The higher magnetic field strength values causes larger structural parameters (near unity) which mean that there is no microstructure left intact in the fluid system. So the more consolidated structural parameter causes more stable bubble which is in contrast to the acoustic motivation which is being imposed externally.

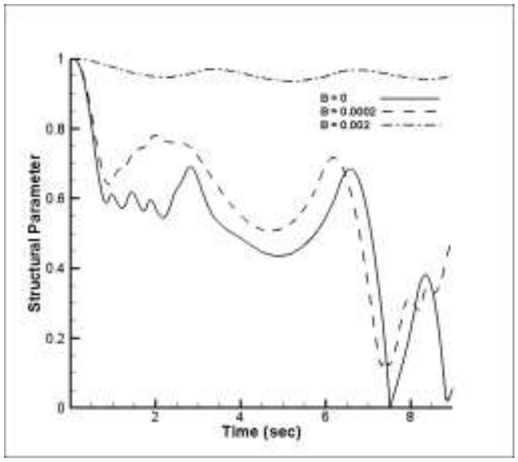

Fig. 2: Structural parameter variations due to acoustic motivations for different magnitudes of magnetic fields and$\alpha = 2. k_1/k_2 = 1$.

Fig. 3 depicts the acoustically motivated dimensionless bubbles size variations in a thixotropic media for different values of magnetic field strength for the case of constant $p_0/\rho\omega^2 R_0^2 = 2$. $De = 3. \sigma = 0.01$ and for $\alpha = 2$ and thixotropic ratio $k_1/k_2 = 1000$. It could be observed that the same damping effect of external magnetic field is present but the point is that from the second harmonic the oscillations are stronger than the case $k_1/k_2 = 1$. The other important point is that by imposing external magnetic field and increasing its value this difference between second harmonics decreases, so the magnetic field weakens the effect of thixotropic ratio variations which would let us isolate the bubble oscillatory behaviour from this parameter.

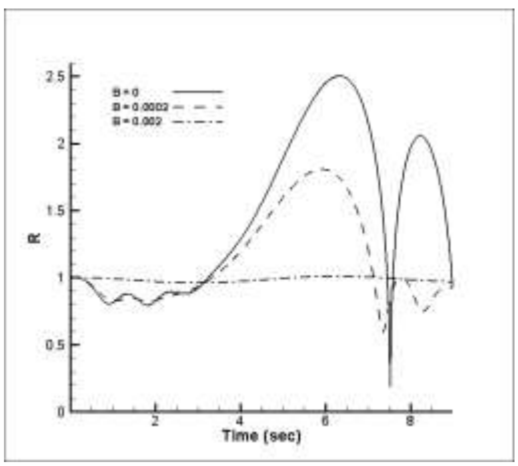

Fig. 3: Bubble size variations due to acoustic motivations for different magnitudes of magnetic fields and$\alpha = 2. k_1/k_2 = 1000$.

The structural parameter's curves versus time are shown in Fig.4 for the case $k_1/k_2 = 1000$. This figure again illustrates the active magnetic field effect on structural and micro-structural properties of the thixotropic fluid and confirms that larger values of magnetic forces results into more modification of zero-shear viscosity of the fluid, however the variations of this property during timeline is highly damped an so more stable bubble oscillation is obtained.

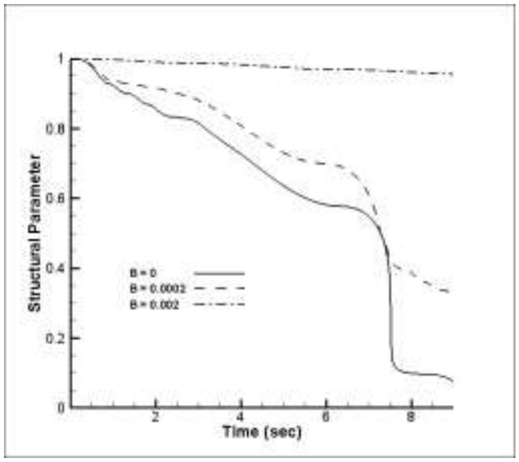

Fig. 4: Structural parameter variations due to acoustic motivations for different magnitudes of magnetic fields and$\alpha = 2. k_1/k_2 = 1000$.

Figures 5 and 6 depict the bubble dimensionless radius variations and structural parameter variations with time for the case with stronger acoustic motivations ($\alpha = 4$). In this case $k_1/k_2 = 1$ and other constant properties are the same as previous cases. The dissipative effect of externally imposed magnetic field on bubble size variations is more obvious it this case. Also Fig.5 reveals that at higher acoustic pressure excitations, vigorous bubble oscillations cause stronger structures destruction (higher rate of break-down) and as result the structural parameter (Fig.6) reduces dramatically from unity. Another important observation from Fig.6 and other figures that are explaining the structural parameter evolution through time is the chaotic behaviour of this parameter. Figure 6 can reconfirm the significance of magnetic field effect on thixotropic fluid rheological attitude in stronger acoustically motivated bubbles.

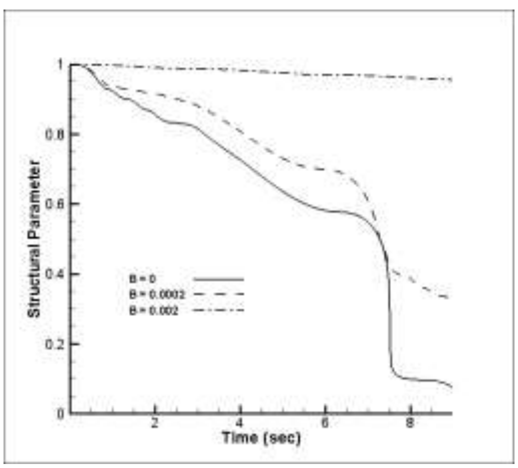

Fig. 5: Bubble size variations due to acoustic motivations for different magnitudes of magnetic fields and$\alpha = 4. k_1/k_2 = 1$.

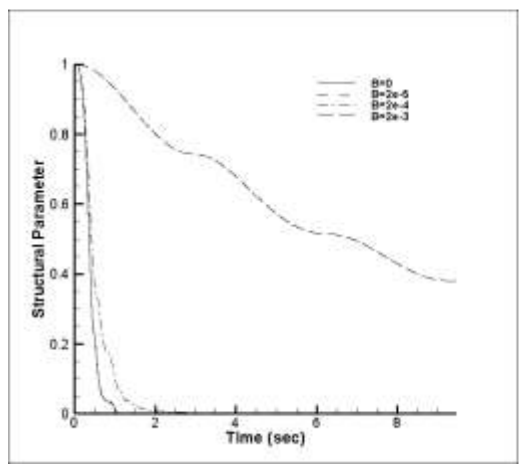

Fig. 6: Structural parameter variations due to acoustic motivations for different magnitudes of magnetic fields and$\alpha = 4. k_1/k_2 = 1$.

4. Conclusion

The effect of magnetic field on nonlinear oscillations of a spherical, acoustically forced gas bubble in thixotropic media has been studied. The effect of a constant magnetic field on non-dimensionalized bubble radius, and structural parameter of fluid has been investigated for three cases. During this comparison for these three cases it is found that it could be realistic to control the gas bubble dynamics in different industrial processes via externally imposed magnetic field. The more important target of the current study was to understand the magnetic field effect on bubble dynamics in thixotropic fluids as far as the MHD in thixotropic extensional flows has never been studied before regarding to our knowledge and these kinds of flow are possible in some bio-medical or industrial applications so far.

In current study a modified Rayleigh-Plesset equation for MHD flows which have been developed by the same authors of this study in their previous work is used considering this assumption that the gas bubbles are surrounded by a thixotrpic fluid which is obeying the Moore's model.

The results show the dissipative effect of external magnetic field on bubble oscillations which was reconfirm the previous work, but the important things that are noticeable from current work are the effect of radically imposed magnetic field on time phase shift and frequency change of oscillations and more interesting was the effect of this magnetic field and its variations on structural break-down and build-up (rheological) behavior of the thixotropic fluid. It was observed that the structural evolutions of a thixotorpic fluid could be controlled by externally imposed magnetic field. So this rheological behaviour of thixotropic fluids affects the gas bubble oscillations and maybe any other usual process in this domain. The results for bubble dimensionless radius variations versus time for different values of magnetic field makes it possible to guess that if the direction of magnetic field is being changed from outflow to inflow, the oscillations should be magnified instead of present attenuation.

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