Addressing Preferential Flow in Landfills by Finite Difference and Marker-in-cell Method

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Abstract: The leachate emission to groundwater is considered to be largest longterm impacts related to landfilling. Therefore it is important to study landfill hydrology for quantifying the emission potential of landfill. Municipal solid waste landfills are heterogeneous, unsaturated and highly porous. We believe this nature of landfills leads to the emergence of preferential pathways for the leachate produced. In this research we explore the origin of preferential flow in a porous media in a two dimensional deterministic model. In this model the water transport is represented by Richards equation and the non-sorbing, single component, solute transport by advection dispersion equation. We discretized the Richard’s Equation by fully implicit finite difference method and the advection dispersion equation by characteristics based on marker-in-cell scheme. We simulate three systematic heterogeneous dry soil domains with defined hydraulic properties and different infiltration rates, which are compared with the homogeneous soil domain with similar infiltration conditions. We investigate the occurrence of channeled flow and solute transport in heterogeneous soil domain explained as emerging non-equilibrium. Upscaled drainage relationship of cumulative mass break through curves with number of pore volumes addresses this non-equilibrium effect and hence helps to notify the preserved preferential phenomenon. In our unsaturated two dimensional model we consider different spatial heterogeneities and different infiltration scenarios as controlling factors affecting equilibrium of flow and solute transport leading for preferential pathways as could be analogous to preferential pathways in a full scale landfill.

Keywords: Landfill, Preferential flow, Non-equilibrium, Finite difference method, Marker-in-cell.

1. Introduction

Landfilling is always a final storage solution for handling municipal solid wastes (MSW) from our society. Modern landfills are advanced technological installations separating the waste from the environment, while providing means for capturing and handling adverse emissions such as landfill gas and leachate. After the landfill is completely filled, a landfill cover is installed limiting the amount of water infiltrating in to the landfill, thus preventing leachate production and emission to the environment. This approach, requires an eternal aftercare of the landfill cover, as the landfill may contain a significant amount of contaminants that potentially could migrate to the environment.

Research on landfills carried out in the last couple of decades has shown that waste in a landfill is subject to a range of natural processes which tend to reduce the emission potential. This research has led to the development of approaches based on recirculation of leachate or aeration (Charles et al. (2009); El-Fadel (2010); Hudgins et al. (2002); McCreanor and Reinhart (1999); Pohland and Alyousfi (1994); Warith and Takata (2004)) which aim to reduce the emission potential within a relatively short period of time. Both recirculation of leachate and aeration of the landfill body stimulate the naturally occurring processes in the
landfill body, leading to an enhanced degradation of organic matter and an increased fixation of inorganic contaminants. The idea is that a landfill in this fashion will require much less aftercare. In order for regulators and landfill operators to agree on the required level of after care, a quantitative estimation of remaining long-term emission potential is required. Emission potential can be defined a quantity of remaining amount of pollutants present inside landfill in aftercare process. To determine this emission potential, study of landfill hydrology is very important.

Field measurements have shown that there is huge uncertainty of moisture content inside the landfills ranging between saturated conditions to complete dryness (Blight et al. (1992)). This is because the hydraulic properties of the refuse ranges from highly water absorbent (e.g. sponge) to water repellent (e.g. metal, plastic, glass) and from impermeable to readily permeable (Powerie and Beaven (1999)). Other construction elements like gas wells, daily cover layers and areas with low and high mechanical compaction add further additional heterogeneities which makes flow in the landfills non uniform. This flow follows a complex pattern which can be explained as preferential pathways as shown in Fig 1. Preferential flow is defined as rapid movement of water and solute through the porous media, characterized by enhanced flux regions participating in most of the flow through small fractional channels (Hendrickx and Flury (2001); Nimmo (2005)). Physical non-equilibrium in flow and solute in an unsaturated heterogeneous porous medium is a possible cause of preferential flow (Jarvis (1998)). Non-equilibrium effect in landfills could be defined as an infiltration phenomenon of leachate flowing faster in fracture or fissure than the surrounding matrix. Non-equilibrium is the most important feature of the preferential flow in which the infiltrating water does not have sufficient time to equilibrate with slowly moving resident water in bulk matrix (Jarvis (1998); Skopp (1981)).

In this research we explore the origin of preferential flow by introducing spatial heterogeneity with various infiltration scenarios in a homogeneous two dimensional (2D) domain. Our deterministic model consists of Richard’s Equation (RE) discretized by finite difference method (FDM) and advection dispersion equation (ADE) by marker-in-cell (MIC) governing coupled water and solute transport respectively. The required algorithms for solving the coupled flow and solute transport model is formulated as a MATLAB toolbox. In order to show how physical heterogeneity can lead to the emergence of non-equilibrium flow, we simulate for three different spatially heterogeneous domains and compare it with the homogeneous one. We have considered different types of soils with known van Genuchten parameters (van Genuchten (1980)), located systematically in 1.0 m × 1.0 m square domain. The considered soil profiles are relatively dry which are infiltrated with water in continuous and square wave pattern. Simulating coupled model for different infiltrations scenarios, we observe flow channeling in three heterogeneous domains as compared equilibrium flow in homogeneous domain. Later on we described our various 2D numerical results in zero dimensional upscaled concentration mass break through curves (BTCs) against number of pore volumes flushed during drainage. The BTCs helps to understand the term emission potential and they also helps to notify the non-equilibrium effect observed in heterogeneous domains which preserves the observed preferential phenomenon. We observed that both infiltration scenarios and heterogeneity are significant factors affecting equilibrium of flow and solute transport in our simple 2D model with inbuilt heterogeneities leading for preferential pathways as could be analogous for those occurring in a full scale landfill.

2. Theory
2.1. Water Transport Model

The hydraulic model governing water flow in variably-saturated, single phase porous medium is described by mixed based form of RE with picard’s iteration (Celia et al. (1990)).
\[
\frac{\theta^{a+1} - \theta^a}{\Delta t} + \frac{C_m(\psi^{a+1,b}) + S_w S_z \delta^{b+1} + \nabla \cdot q}{\Delta t} = 0
\] (1)

\[
q = -k_r(\psi)K_{\text{sat}}[\nabla (\psi + z)]
\] (2)

where \( \psi \) is the pressure head [\( L \)], \( t \) is the time [\( T \)], \( \theta \) is the moisture content and function of \( \psi \), \( C_m \) is the specific moisture capacity \([1/L]\). Picard’s iteration process is used for solving the nonlinearity in RE, it continues until the difference between pressure heads at each node of two successive iteration levels becomes less than the predefined tolerance \( \delta \), see Eq.(3), where \( a \) is the time level and \( b \) is the iteration stage.

\[
\delta^{b+1} = (\psi^{a+1,b+1} - \psi^{a+1,b})
\] (3)

\( S_w \) is storativity coefficient and \( S_z \) is fluid compressibility coefficient. \( q \) is the Darcy velocity \([L/T]\), \( k_r \) is relative hydraulic permeability function and \( K_{\text{sat}} \) is the saturated hydraulic conductivity tensor \([L/T]\), written as \( \begin{bmatrix} K_{xx} & 0 \\ 0 & K_{zz} \end{bmatrix} \) for where \( K_{xx} \) and \( K_{zz} \) are the saturated hydraulic conductivity in \( x \) and \( z \) direction, \( x \) is horizontal direction and \( z \) is the vertical dimension assumed positive upwards.

The (van Genuchten (1980)) functions which are needed to solve RE, assuming no hysteresis, can be expressed as.

\[
K(\psi) = k_r K_{\text{sat}}
\] (4)

\[
S_{\text{eff}} = [1 + \alpha |\psi|]^{-m}
\] (5)

\[
\theta(\psi) = \theta_r + S_{\text{eff}}(\theta_s - \theta_r)
\] (6)
\[ k_r = S_{eff}^{\frac{1}{2}} \left[ 1 - \left( 1 - S_{eff}^{\frac{1}{2}} \right)^m \right]^2 \] (7)

\[ S_w = S_{eff} + \theta_r \] (8)

\[ C_m = \frac{\alpha m}{1 - m} (\theta_s - \theta_r) S_{eff}^{\frac{1}{2}} \left( 1 - S_{eff}^{\frac{1}{2}} \right)^m \] (9)

\( \theta_r \) is residual water content \([L^3/L^3]\), \( \theta_s \) is saturated water content \([L^3/L^3]\), \( S_{eff} \) is effective saturation, \( \alpha \) is air entry value \((1/L)\), \( n \) and \( m \) are van Genuchten parameters for unsaturated flow.

### 2.2. Solute Transport Model

The solute transport is represented by classical ADE (Fetter (1993); Bear (1988)).

\[ \frac{\partial \theta c}{\partial t} + \nabla \cdot u = 0 \] (10)

\[ u = -\theta D \nabla c + q c \] (11)

where \( c \) is the concentration \([M/L]\), \( t \) is the time \([T]\), \( u \) is concentration flux, \( D \) is hydrodynamic dispersion tensor coefficient \([L^2/T]\) can be written as \( D = \begin{bmatrix} D_{xx} & 0 \\ 0 & D_{zz} \end{bmatrix} \), where \( D_{xx} \) and \( D_{zz} \) are the hydraulic dispersion in \( x \) and \( z \) direction. The hydrodynamic dispersion \( D [L^2/T] \) is the tensor, whose matrix elements are given by

\[ D_{\alpha\beta} = \alpha_T |v| \delta_{\alpha\beta} + (\alpha_L - \alpha_T) v_{\alpha} v_{\beta} / |v| + D_m \delta_{\alpha\beta} \] (12)

\[ v = \frac{q}{\theta} \] (13)

where \( D_{\alpha\beta} \) is the dispersion coefficient in respective directions, \( D_m \) is the molecular diffusion \([L^2/T] \), \( v \) is the average pore water velocity \([L/T] \), \( \alpha_L \) and \( \alpha_T \) are the longitudinal and transverse dispersivities \([L]\), respectively. The subscripts \( \alpha \) and \( \beta \) represent the \( x \) and \( z \) coordinate directions. Substitution of \( x \) and \( z \) for \( \alpha \) and \( \beta \) yields four values for the dispersion tensor. Terms \( q \) and \( \theta \) are coupling variables mathematically presented in Eq. 2 and Eq. 6.

The conventional finite difference formulation for solving ADE produces numerical diffusion caused by erroneous evaluation of the spatial derivative of concentration i.e the advection term \( \nabla c \) (See Eq. 11) (Al-Lawatia (2012); El-Amrani and Seaid (2012)). These strong oscillations can be removed by higher order numerical schemes like MIC techniques (Gerya and Yuen (2003)). MIC method uses Lagrangian and Eulerian-Lagrangian advection algorithm which involves combining the use of Lagrangian advecting points (markers) with an immobile, Eulerian grid. In this research we have used MIC method for reducing numerical diffusion.
Fig. 2. Different spatial scenarios, namely FM (a); SCP (b); SP (c) and H (d).

Table 1. van Genuchten parameters for different soil profiles

<table>
<thead>
<tr>
<th>Hydraulic parameters</th>
<th>coarse sand</th>
<th>fine clay</th>
<th>medium sand</th>
<th>clay silt</th>
<th>clay loam</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$ [m$^{-1}$]</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$n$</td>
<td>1.5</td>
<td>2.5</td>
<td>1.378</td>
<td>1.242</td>
<td>1.6</td>
</tr>
<tr>
<td>$\theta_s$</td>
<td>0.4</td>
<td>0.45</td>
<td>0.41</td>
<td>0.39</td>
<td>0.41</td>
</tr>
<tr>
<td>$\theta_r$</td>
<td>0.04</td>
<td>0.08</td>
<td>0.041</td>
<td>0.075</td>
<td>0.02</td>
</tr>
<tr>
<td>$K_{sat}$ [m s$^{-1}$]</td>
<td>$5.0 \times 10^{-2}$</td>
<td>$5.0 \times 10^{-5}$</td>
<td>$4.2 \times 10^{-5}$</td>
<td>$2.0 \times 10^{-6}$</td>
<td>$2.3 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

3. Materials and Methods

To determine the effect of heterogeneities in porous media, we have considered different spatial scenarios. We have named them as Five Materials (FM); Sand, Clay and Plastic (SCP); Sand and Plastic (SP) and their simulation results are compared with a Homogeneous (H) scenario. Detailed diagrams of these scenarios is given in Fig 2.

In all the spatial scenarios, the initial condition for water transport model was $\psi(x,z,0) = (z-z_{ref})$, where $z_{ref} = -2.0$ m is phreatic surface and $z$ is vertical dimensional coordinates of domain. For solute transport model, $c(x,z,0) = c_{ini}$, where $c_{ini} = 1.0$ kg m$^{-3}$ is the initial concentration of domain.

In water transport model all two types of boundary conditions namely Neumann and Robbin were used. Neumann boundary condition was applied at the top horizontal edge $q(x,0,t) = q_{top}$ for continuous and intermittent square wave pattern water inflow. Where $q_{top}$ magnitude is ,, , and 7.0 m s$^{-1}$ . The continuous and square wave infiltration were carried out for 1.0h. At the bottom horizontal edge Robbins boundary condition $q(x,-1,t) = K_{surf} (\psi_{amb} - \psi)$ is applied for all infiltration and spatial scenarios. Where $K_{surf} = 5.0$ s$^{-1}$ is the surface pressure coefficient and $\psi_{amb} = -1$ m is ambient pressure head.

In solute transport model, Dirichlet and Robbins type of boundary conditions are applied on top and bottom horizontal edges of domain respectively. At the top edge there is no inlet concentration only drainage of initial concentration i.e $c(x,0,t) = c_{top}$, where $c_{top} = 0.0$ kg m$^{-3}$. At bottom horizontal edge Robbins boundary condition is applied of zero concentration gradient substituting $\nabla c = 0$ in Eq.(11) i.e. $u(x,-1,t) = q_c$. The left and right vertical edges are considered to be no flux boundaries.

The van Genuchten parameters considered for different soil profiles as shown in Fig.(2). are indicated in Table 1. For impermeable plastics in some of the spatial scenarios, we assume the $K_{zz} = 0.0$ m s$^{-1}$. We have considered the compressibility of water as $S_s = $. The molecular diffusion i.e. $D_m = $ and the dispersivity coefficients $\alpha_L = 0.1$ m and $\alpha_T = 0.01$ m for all scenarios. Over all there are five continuous inflow infiltration scenarios and five square wave inflow infiltration scenarios for four different spatial scenarios. So we have simulated our coupled model for 40 different scenarios.
4. Results and Discussions

The main emphasis in this paper is to analyze different effects of heterogeneities and type of infiltration and its magnitude on the equilibrium of flow and solute transport in a porous media. A detailed investigation for every aspect of flow and solute transport for all scenarios is not practical option for a single paper, though it is likely to affect the equilibrium in different manner. The results presented in this paper could correspond to lab scale samples and are general enough as could be extended to the processes occurring at a large full scale landfill.

4.1. Effect of Spatial Heterogeneity on Pressure Head, Moisture and Solute Distribution

In Figs 3 simulation results for pressure head, moisture content and solute content at the end of 1 hour for square wave pattern infiltration are seen. The anisotropic nature of hydraulic conductivity in heterogeneous scenarios leads for a development of horizontal gradient of water pressure at different intersecting edges of soil profiles. This leads to development of local ponding at the upper intersecting interstices of different soil profiles and/or plastics. In Fig 3(a, b and c) positive pressure head denotes local ponding. Because of this local ponding of water pressure, the flow get channelized. In Fig 3(e, f and g) the moisture content profiles shows the channeled flow transport observed in between foreground and background soil profiles. The flow lines seemed to be converging into newly developed flow paths for FM, SCP and SP scenarios unlike for the H scenario (Fig 3(d, h and l)), where the flow lines are directed in vertically downward directions. These converging channeled flow paths are relatively dryer may have faster flow rates, whereas the regions with low hydraulic conductivity are relatively wetter and releases water at slower rate. The coupled solute transport results shown in Fig 3(i and j), for FM and SCP shows that there is a slow release of concentrations in finner foreground soil profiles. Whereas in Fig 3(k and l) the solute concentration is already released to 0.0 kg m$^{-3}$, from the soil domain before the end of simulation period.

4.2. Upscaled Drainage Relationship of Cumulative Concentration Mass and Number of Pore Volumes

In close landfills the internal heterogeneity is not visible. Thus the location of different waste material is not known. Also there is huge uncertainty in the hydraulic properties and thus chemical composition of wastes inside the landfills. These reasons make deterministic modeling for landfills very unrealistic (Bun et al. (2013)). In full scale landfills where the outlet parameters like electrical conductivity and drainage rates of leachate are only known measurements, non equilibrium models like dual porosity or dual permeability models are configured by reverse modeling (Abbaspour et al. (2004); Fellner and Brunner (2010)). In similar way we have plotted the outlet cumulative mass with the number of pore volumes flushed from the domain during simulation time period.

In Fig 4((a) and (b)) upscale drainage results of cumulative mass concentration and number of pore volumes shows that for all spatial scenarios for continuous infiltration the cumulative mass of concentration released is more for lower inflow rates than for higher inflow rates. Similarly in Fig 4((c) and (d)) for intermittent square inflow infiltration, the cumulative mass of concentration released is more for lower inflow rates as compared to higher inflow rates. This is because of slow release of concentrations during zero inflow condition in intermittent square wave pattern infiltration.

In Fig 4(c) it can be seen a lag in release of number pore volumes for different spatial scenarios which is because of decreasing heterogeneity. The number of pore volumes flushed are more for higher flow rates indicating flushing or water with slow release of mass. This indicates non-equilibrium in flow and solute transport. It can also be seen that in the mass release in homogeneous scenario is rather fast than the heterogeneous scenarios. This is because of occurrence of local ponding in heterogeneous scenarios (FM, SCP and SP) which causes slow release of mass from denser medium to the background less denser
Fig. 3. Pressure head, moisture and concentration contents for different spatial scenarios for intermittent square wave inflow infiltration with rate of \( \), at the end of 1.0 h.

material. This process of migration of solute concentration in channeled paths can be coined as preferential phenomenon.

In Fig 5 we have plotted the results of FM scenario with square wave pattern inflow infiltration, which indicates that lower inflow rates can remove more concentration mass with less pore volumes flushed. It means the emission potential of a landfill can be reduced if recirculation of water irrigation can be carried out at lower rates.

5. Conclusions

In this research we have showed through our results that introducing spatial heterogeneities can lead to emergence of non equilibrium in flow and therefore non equilibrium in coupled solute transport. There is a high possibility that the pressure, moisture and solute pattern observed in all the considered heterogeneous spatial scenarios along with infiltration scenarios would mimic some locations existing in MSW landfills.

This non equilibrium is caused by local ponding observed in heterogeneous scenarios causes channeled flow paths. In between these newly emerged channeled flow paths during intermittent inflow infiltration, solute mass migrates preferentially. The method of relating upscaled discharge concentration mass against
Fig. 4. Upscale discharge concentration mass against number of pore volumes for continuous ((a) and (b)) and intermittent square wave inflow infiltration((c) and (d)) at , and . Graph lines red (FM), magenta(SCP), green(SP), blue (H) indicate the spatial scenarios.

Fig. 5. Increase in magnitude of intermittent inflow infiltration decreases the release of concentration mass.
number of pore volumes help to quantify the term emission potential and notifies the preserved preferential pathways. Further research regarding relation of outlet concentration and outlet discharge can be validated using upscaled transfer functions in black box models.

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