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Numerical Resolution for Fin Profiles Optimization

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***Abstract*-**In the present work a numerical method is proposed in order to optimize the thermal performance of finned surfaces. The bidimensional temperature distribution on the longitudinal section of the fin is calculated by restoring to the finite volumes method. The heat flux dissipated by a generic profile fin is compared with the heat flux removed by the rectangular profile fin with the same length and volume. In this study it is shown that a finite volume method for quadrilaterals unstructured mesh is developed to predict the two dimensional steady-state solutions of conduction equation, in order to determine the sinusoidal parameter values which optimize the fin effectiveness. In this scheme, based on the integration around the polygonal control volume, the derivatives of conduction equation must be converted into closed line integrals using same formulation of the Stokes theorem. The numerical results show good agreement with analytical results. To demonstrate the accuracy of the method, the absolute and root-mean square errors versus the grid size are examined quantitatively.

***Keywords*:** Stokes theorem, Unstructured grid, Heat transfer, Complex geometry.

1. Introduction

In many engineering sectors, where high thermal fluxes must be transferred, the finned surface power removers are today an usual tool. Since finned surfaces allow evident improvements in heat transfer effectiveness, the heat exchangers field is one of the most interested in their applications. Moreover new industrial sectors present an increasing interest in the introduction of extended surfaces for heat flux removal. In particular, the electronics industry has promoted a new interest in developing heat removers, aimed at transferring heat from electronic components to the environment, in order to reduce the working temperature and to improve the characteristics and the reliability (A. Bar-Cohen and A. D. Kraus), (C. W. Leung, S. D. Probert),(G. Fabbri).

This study is motivated by the need for a numerical approach that is not only capable of performing accurate computations but that also provides an easier way to implement these computations. Our objectives is to develop a simple and accurate procedure to deal with curved boundaries, which capable of achieving second order accuracy with relative economy, for heat transfer and flow problem, employing unstructured mesh. For this objective, we develop a method based on same formulation of the Stokes theorem. For testing our method, we consider the computation of passive scalar in fin profile, the numerical method calculates the heat flux dissipated by the sinusoidal profile heat remover on the basis of the bidimensional temperature distribution on its longitudinal section, which is obtained with the help of our method.

2. Fin Model

In the orthogonal coordinate system we will refer to a heat remover with longitudinal section symmetrical with respect to the *x* axis and with a rectangular profile, as shown in Fig. 1, then with the proposed model that described by the sinusoidal function *y(x),* as shown in Fig. 2. The fin width and length L, is immersed in a fluid with a constant bulk temperature T_F. Moreover, the fin base temperature T_0 is known.

In order to calculate the heat flux removed by such a fin it is necessary to determine the temperature distribution in the longitudinal section (plane *xy*). This distribution must satisfy the Laplace's equation:

$$\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) = 0 \tag{1}$$

With the boundary conditions:

$$T(0, y) = T_0 \tag{2}$$

$$\left[\frac{\partial\ \ T}{\partial x}\right]_{L,y} = -\frac{h_f}{k}\left[T(L, y) - T_F\right] \tag{3}$$

$$\left[\frac{\partial\ \ T}{\partial x}\right]_{x,0} = 0 \tag{4}$$

$$\left[\frac{\partial\ \ T}{\partial x}\right]_{x,f(x)} - \left[\frac{d\ \ f}{dx}\frac{\partial\ \ T}{\partial x}\right]_{x,f(x)} = -\frac{h\sqrt{\left(\frac{d\ \ f}{dx}\right)^2_{\ x}}}{k}\left[T(x, f(x)) - T_F\right. \tag{5}$$

$h$andh_f being the convective heat transfer coefficients for the longitudinal fin surface and for the final surface and for the final transversal one, respectively, k being the thermal conductivity of the fin. Due to the complexity of the problem it is convenient to determine the temperature distribution numerically using for example our method.

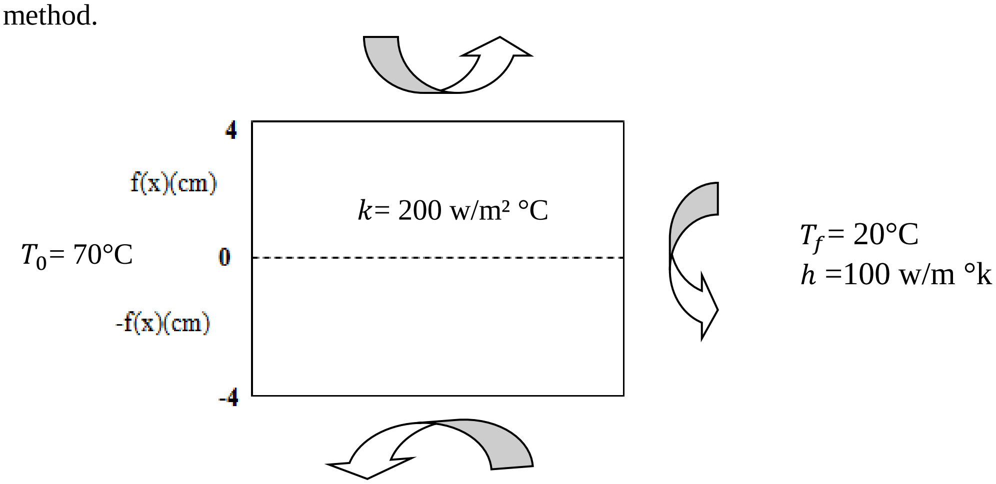

Fig.1. Longitudinal section of a symmetrical profile for an rectangular profile.

3. Effectiveness of the Fin

The fin performance can be evaluated on the basis of the compared effectiveness, i.e. the ratio between the heat flux (Q_d) dissipated by the heat remover with a generic profile and the heat flux (Q_r) removed by a fin of the same volume and length and with rectangular profile:

$$E = \frac{Q_d}{Q_r} \tag{6}$$

Let us then consider a rectangular fin of width $2\bar{f}$,$\bar{f}$ being the average value of *f(x):*

$$\bar{f} = \frac{1}{L}\int_0^L f(x)dx \tag{7}$$

The temperature distribution on the longitudinal section of such a fin must satisfy equation (1), the boundary conditions (2)-(4) and the following:

$$\left[\frac{\partial \ T}{\partial x}\right]_{x,f(x)} = -\frac{h}{k}\left[T(x,f(x)) - \ T_F\right. \quad (8)$$

Since both longitudinal and final transversal surfaces are plane we can assume h equal toh_f. By integrating equation (1) with the above boundary conditions the following solutions is obtained (H. S. Carslaw and J. C. Jaeger):

$$T(x,y) = T_F + \frac{2h(T_0-T_F)}{k}\sum_{n=1}^{\infty}\left[\frac{1}{\left(\alpha_n^2+\frac{h^2}{k^2}\right)\bar{f}+\frac{h}{k}}\frac{\cos(\alpha_n y)}{\cos(\alpha_n \bar{f})}\times\frac{\alpha_n \cosh[(\alpha_n(L-x)]+\frac{h}{k}\sinh[\alpha_n(L-x)]}{\alpha_n\cosh(\alpha_n L)+\frac{h}{k}\sinh(\alpha_n L)}\right] \quad (9)$$

beingα_n the solution of the equation:

$$\alpha\tan\left(\alpha\bar{f}\right) = \frac{h}{k} \quad (10)$$

The heat flux dissipated for unit of length is then:

$$Q_r = 4h(T_0 - T_F)\sum_{n=1}^{\infty}\left[\frac{\tan\ (\alpha_n\bar{f})}{\left(\alpha_n^2+\frac{h^2}{k^2}\right)\bar{f}+\frac{h}{k}}\frac{\alpha_n\sinh\ (\alpha_n L)+\frac{h}{k}\cosh(\alpha_n L)}{\alpha_n\cosh\ (\alpha_n L)+\frac{h}{k}\sinh\ (\alpha_n L)}\right] \quad (11)$$

We can calculate the heat flux dissipated by the remover for unit of width in the following way:

$$Q_d = 2(\sum_i g_{oi}(T_0 - T_i) + g_{h0}(T_0 - T_F)) \quad (12)$$

g_{oi}being the thermal conductance between the fin base and the *ith* element, where it is zero for all the elements which are not adjacent to the fin base. While g_{h0} being the thermal conductance between the fin base and the coolant fluid.

3. Numerical Procedure

We now propose the numerical method which is able to determine the values of the fin profile describing parameters which allow the highest compared effectiveness. We will consider heat removers for which the profile function $f(x)$ has a sinusoidal form and we have 2 cases:

$$f_1(x) = w - a_0\sin\left(\frac{2\pi x}{\lambda_0}\right) \quad (13)$$

$$f_2(x) = w + a_0\sin\left(\frac{2\pi x}{\lambda_0}\right) \quad (14)$$

a_0, λ_0being the amplitude and length of the sinusoid respectively.

By increasing the number of the sinusoid more and more articulate fin profiles will be taken into account.

The Laplace's equation is integrated in space using a finite volume method that is developed for an unstructured grid made up of quadrilaterals (G.K. Despotis, and S. Tsangaris), (S. Boivin, F. Cayré, and J. M. Hérard), (N. Piskounov).

For the integration around finite volume, the derivations of the flow equation must be converted into closed line integrals using same formulation of the Stokes theorem, which is described by the following equation:

$$\oint_E \vec{T}\ .\overrightarrow{dr} = \iint_s\ rot\,\vec{T}\ .\vec{n}dS \tag{15}$$

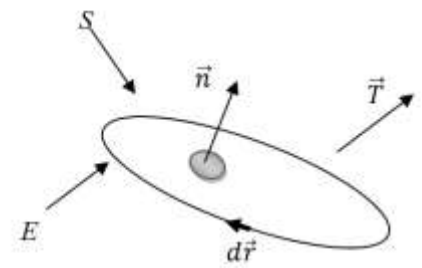

Fig. 2.Formulation of Stokes's theorem.

Where $\overrightarrow{dr}$ is the elementary arc, dS is the elemntary surface and is the normal vector to this surface. The computational domain is discretized on a quadrilateral unstructured grid where each node is the centre of polygonal cell constituted of four elements; all computed variables are stored at the centres of the polygonal as:

3. 1. Approximation of the First Derives

The convective terms are calculated at the node P (fig.2). The nodal finite volume descritization scheme is used for the discretization of the convective terms that appear in the governing equation. The first differences are calculated as:

$$\left(\frac{\partial T}{\partial x}\right)_c = \frac{1}{A_C}\int_{Sc} T.dy = \frac{1}{A_C}\sum_{i=1}^{NC}\frac{T_{i+1}+T_i}{2}(y_{i+1}-y_i) \tag{16}$$

$$\left(\frac{\partial T}{\partial y}\right)_c = \frac{1}{A_C}\int_{Sc} T.dx = \frac{1}{A_C}\sum_{i=1}^{NC}\frac{T_{i+1}+T_i}{2}(x_{i+1}-x_i) \tag{17}$$

Where $A_C$is the area of the polygonal control volume (1,2,3,…NE), T the temperature and x,$y$are the coordinate of the polygonal vertices, and I refers to the vertices number of external polygonal control volume.

3. 2. Approximation of the First Derives

This terms must be calculated at the node P and this achieved by computing the second order derivatives at the same point. The required second differences may be computed as:

$$\left(\frac{\partial^2 T}{\partial x^2}\right)_c = \left[\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right)\right]_C = \frac{1}{A_{CI}}\int_{SCI} T.dy = \frac{1}{A_{CI}}\sum_{i=1}^{NC}\left(\frac{\partial T}{\partial x}\right)_E (y_{i+1}-y_i) \tag{18}$$

$$\left(\frac{\partial^2 T}{\partial y^2}\right)_c = \left[\frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right)\right]_C = \frac{1}{A_{CI}}\int_{SCI} T.dx = \frac{1}{A_{CI}}\sum_{i=1}^{NC}\left(\frac{\partial T}{\partial y}\right)_E (x_{i+1}-x_i) \tag{19}$$

$A_{CI}$is the area of polygonal control volume (2,4,…NE) (fig.2) and I refer to the vertices number of internal polygonal control volume. Where, the first differences at the middle of the edge are defined as:

$$\left(\frac{\partial T}{\partial x}\right)_E = \frac{1}{A_E}\int_{SE} T.dy = \frac{1}{A_E}\sum_{i=1}^{4}\frac{T_{i+1}+T_i}{2}(y_{i+1}-y_i) \tag{20}$$

$$\left(\frac{\partial T}{\partial y}\right)_E = -\frac{1}{A_E}\int_{SE} T.dx = -\frac{1}{A_E}\sum_{i=1}^{4}\frac{T_{i+1}+T_i}{2}(x_{i+1}-x_i) \tag{21}$$

A_E is the area of the quadrilateral control volume ((1),(2),(3),(4))(Fig.2.) and the four vertices of quadrilateral control volume.

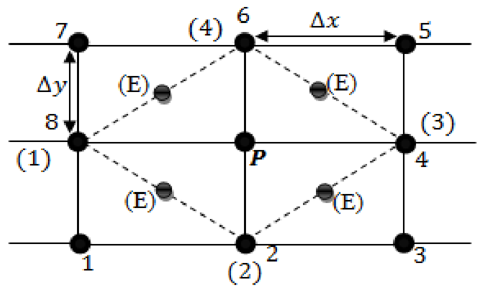

Fig. 3.The computational control volume structure.

4. Results

We take the results of our numerical method for the problem considered:

4. 1. Errors and Convergence

We define two different errors to examine the numerical solution:

• Absolute Errors:

$$\varepsilon = \left| \frac{T_{analytique} - T_{numérique}}{T_{analytique}} \right| \tag{22}$$

This gives an average absolute error:

$$rms = \sqrt{\frac{\sum_{i=1}^{n} (T_{analytique} - T_{numérique})^2}{n}} \tag{23}$$

To quantitatively examine the convergence of the mean error and mean square error are not presented according to the mesh grid in the Fig. 4 for no meshes studied, we note that the errors increases lightly refining the mesh.

In Fig.5 below we clearly see the temperature distribution in different grid size and influence of the mesh refining on the distribution where we approach to the exact solution.

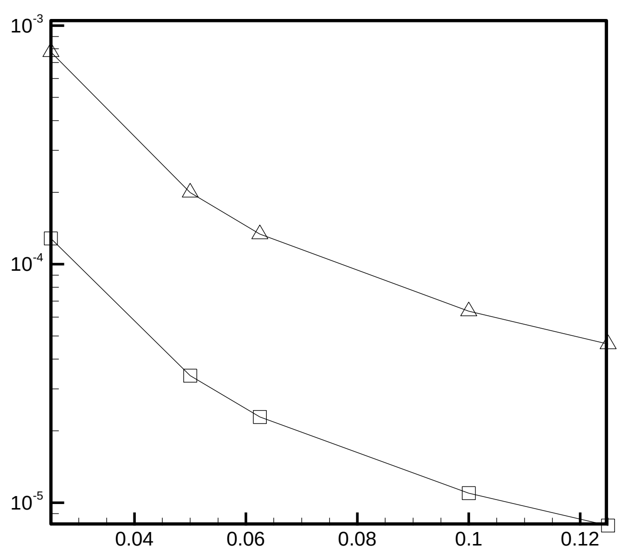

Fig .4.Theaverageandrmserroraccording tothe grid size.

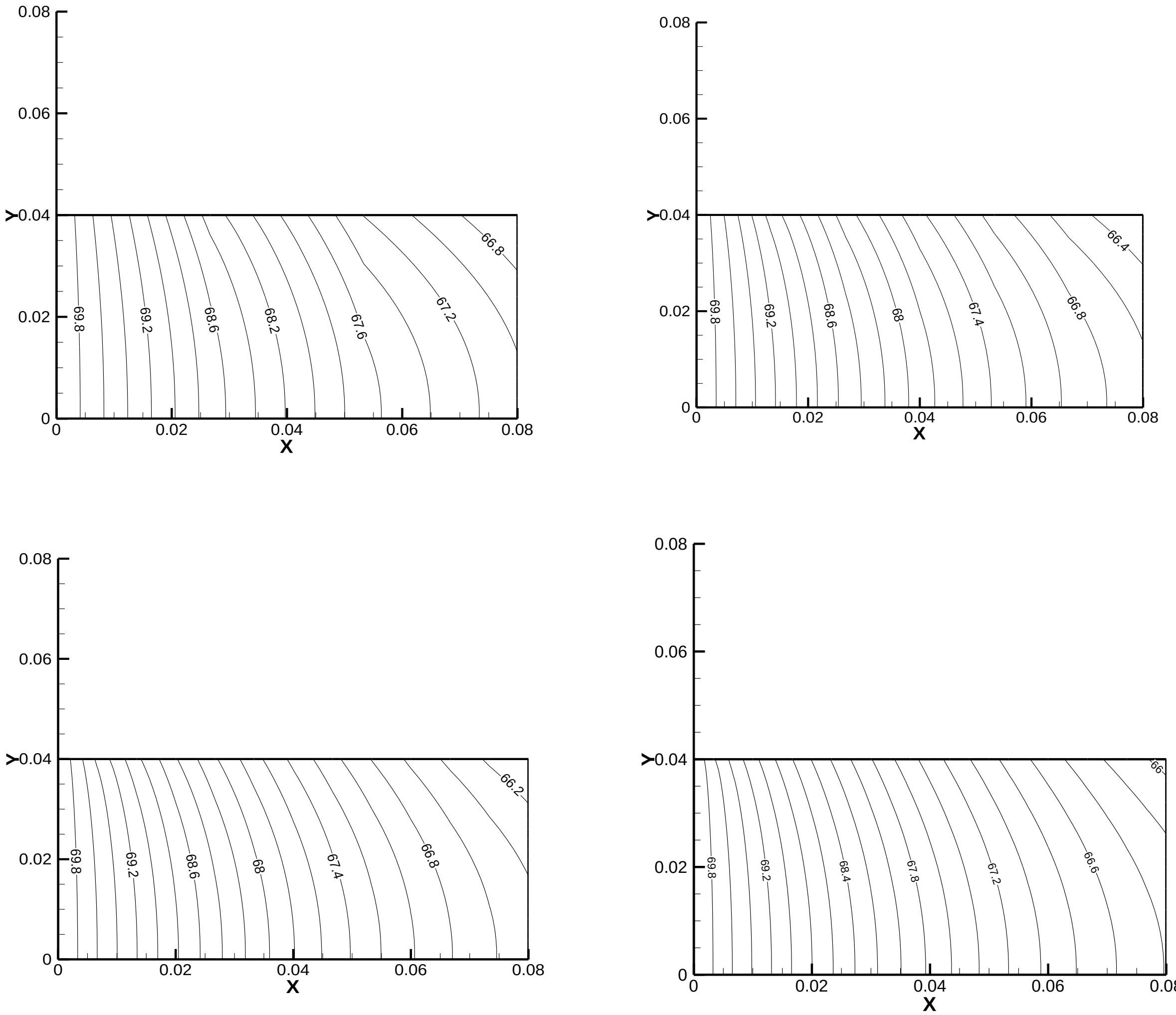

Fig .5 . The temperature distribution in different grid size.

The proposed numerical method has been utilized in order to optimize the sinusoidal profile of aluminium fins. For the finite volumes model parameters, the values are reported in table 1 have been assumed.

The coefficient *h* has been assumed constant and equal to 100 w /m²°k.

The numerical method was utilized by choosing first of all the beginning of the function *f(x)*, once from the top and in the second time from the lower (Fig.6), then we compared the effectiveness of the sinusoidal profile with the rectangular.

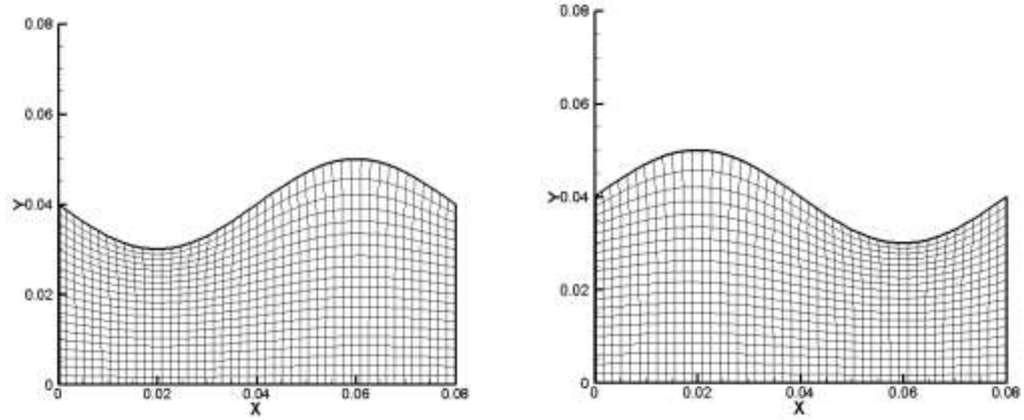

Fig. 6. The proposed geometries to compared the effectiveness according to the function f(x).

The compare effectiveness, in fact, always grows with the fin who started from the lower (first undulation down then sublimate)(Fig. 7).

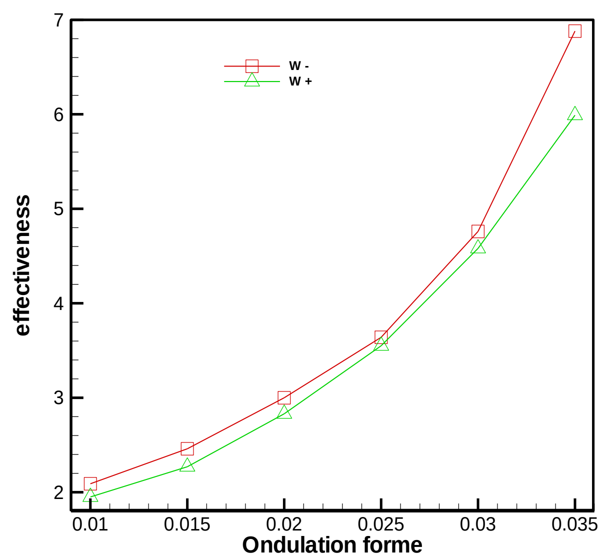

Fig. 7.The compared effectiveness between the two configurations.

5. Conclusion

The proposed numerical method seems able to solve various problems of heat transfer especially when we have complex geometries and even the problems of optimization of the longitudinal profile of a fin, to improve its performance compared to a rectangular longitudinal section of the same volume and length.

And for the efficiency of the method of the discretization error can be reduced by way of the mesh grid is refined and the order of convergence is defined by the mesh refinement that these errors may improve.

The developed method can be easily used to treat:

- The various problems of conduction and with various boundary conditions.
- The stationary problem of conduction.

Finally, this method can be extended to treat:

- The advection-diffusion problem or more of a velocity field temperature is taken into account.
- The flow problem (Navier-Stokes) with complex geometric boundaries, but requires more sophisticated that the problem of conduction or advection-diffusion numerical considerations.

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