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Super Element Method of Oil Reservoir Simulation

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Abstract **-** The paper describes the theoretical framework and results of using an rapid three-dimensional super element model of oil field development. The model makes it possible to speed up calculation of two-phase filtration in an oil reservoir by hundreds of times owing to the use of large computational cells – super elements, the number of which corresponds to the number of wells in the field. Accuracy of the numerical solution is ensured by a formulation of the problem in terms of the smooth mean fields of pressure and saturation with a selective refinement of the solution in the near of the wells on independent detailed nested grids.

Keywords**:** Oil reservoir simulation, Two-phase flow in porous media, Coarse unstructured grids, Super element method.

1. Introduction

Simulation of oil field development is traditionally performed using full-scale filtration models (Roxar Tempest More, Schlumberger Eclipse, etc.) on computational grids with a cell size of approximately several tens of meters horizontally and tens of centimeters vertically. Such models require specification of an excessive number of parameters, and in case of large fields they contain millions of cells. This complicates their adaptation and makes it virtually impossible to use them for multivariant prediction calculations.

To overcome these problems when optimizing the system of oil field development, it is proposed to use a super element modeling method (Mazo et al., 2011; Mazo et al., 2013). Compared to the traditional models, the super element model makes it possible to speed up calculation of two-phase filtration in an oil reservoir by hundreds of times owing to the use of large computational cells – super elements, the number of which corresponds to the number of wells in the field. Satisfactory accuracy of the solution on a coarse grid is maintained due to the problem formulation relative to the smooth mean fields of pressure and saturation as well as due to the preliminary upscaling of the reservoir's hydraulic and capacity properties and the use of independent detailed nested computational grids in cases of local refinement of the solution.

This paper presents the basic principles and some of the results of constructing a super element model of oil field development.

2. Computational Grid of Super Elements

For constructing a grid of super elements in a horizontal plane, we use the algorithms of centered PEBI-segmentation (Palagi et al., 1991), which make it possible to build a predominantly hexagonal grid covering of the computational domain, in which case the centers of the computational blocks coincide with the initial control network (Du et al., 1999; Nikitin, 2010). The role of triangulation centers is performed by active vertical wells as well as additional centers that ensure a uniform coverage of the field area (Fig. 1). Unstructured computational grids simplify the description of reservoir geometry and the determination of boundary conditions. They also reduce the impact of grid orientation on the numerical solution (Heinemann et al., 1991). The division of the reservoir into super elements in the vertical direction is performed along the boundaries of the geological packs containing several permeable layers.

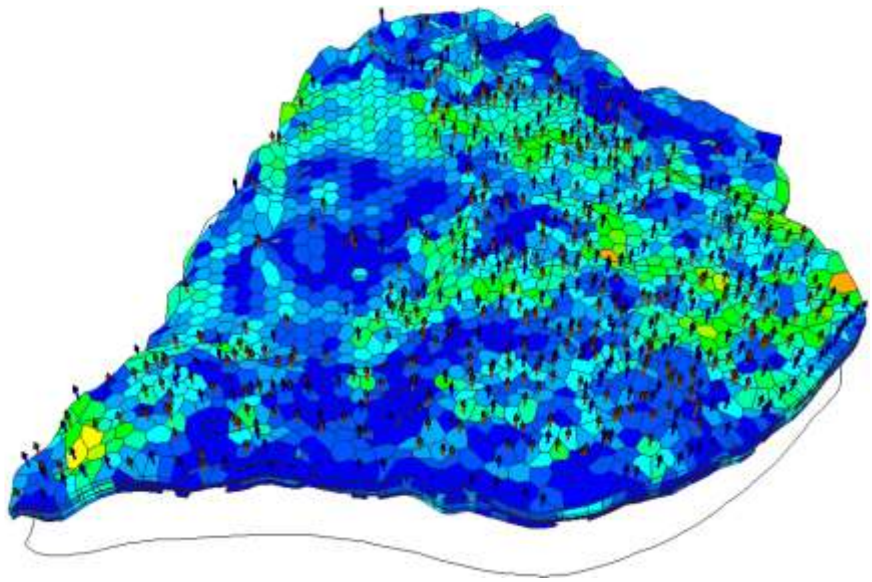

Fig. 1. Example of the coverage of an oil field by super element computational grid.

3. Description of the Super Element Model

The super element model is based on the known two-phase flow in porous media equations written without regard to capillary and gravity forces:

$$\begin{aligned}&\beta\frac{\partial p}{\partial t}+\operatorname{div}\mathbf{U}=0, \qquad \mathbf{U}=-\sigma(s)k(x,y,z)\operatorname{grad}p, \qquad \sigma(s)=\frac{k_w(s)+K_\mu k_o(s)}{\mu_w},\\ &m\frac{\partial s}{\partial t}+\operatorname{div}\big(f(s)\mathbf{U}\big)=0, \quad f(s)=\frac{k_w(s)}{k_w(s)+K_\mu k_o(s)}, \qquad K_\mu=\frac{\mu_w}{\mu_o}.\end{aligned} \tag{1}$$

Here t is the time; x, y, z are the Cartesian coordinates; p is the pressure in the reservoir fluid; s is the water saturation; m, k are the porosity and absolute permeability of the reservoir; $\mathbf{U}$ is the total flow rate; β is the compressibility; f is the fraction of water in the flow (Buckley–Leverett function); μ_w, μ_o are the viscosities of water and oil; k_w, k_o are the relative phase permeabilities, which are approximated by the power-law dependences of the form

$$k_w=S^a, \quad k_o=(1-S)^b, \quad S=\frac{s-s_*}{s^*-s_*}, \tag{2}$$

where s^*, s_* are the boundaries of phase mobility; $n=1\div 3$.

The three-dimensional equations (1) describe two-phase flow in the region D representing the reservoir tapped by the system of input and output wells. The wells can be horizontal or vertical; generally, they are directional and operate either in the mode of the given pressures (p) or in the mode of the given flow rates (q). The boundary Γ of the region D consists of the reservoir outer boundary (the impermeable apex Γ_T and the bottom Γ_B and the lateral boundary Γ_S with the given hydrostatic pressure p and the saturation s) and the inner boundaries γ_k of the reservoir, formed by the well surfaces, on which the pressure and total flow are set:

$$(x,y,z)\in\gamma_k: \quad \begin{cases} p = p_k^{\gamma}, \\ \int_{\gamma_k} U_n\, \mathrm{d}\gamma = Q_k. \end{cases} \tag{3}$$

Approximation of the equations (1)–(3) on the super element grid is constructed similarly as by the finite-volume method (Lipnikov et al., 2007), which ensures the persistence of the grid scheme. Super elements are used as finite volumes. In the solution process at each time moment t for each super element with the volume $|V|$, we find the average pressure and water saturation

$$\bar{p} = \frac{1}{|V|}\int_V p dV, \quad \bar{s} = \frac{1}{|V|}\int_V s dV .$$

The general structure of the averaged equations is preserved:

$$|V|\beta\frac{\partial \bar{p}}{\partial t} + Q = q, \qquad Q = \int_{\Gamma_V} U_n\, \mathrm{d}\Gamma, \qquad \bar{U}_n \approx -\bar{\sigma}\mathbf{K}\cdot\nabla\bar{p}\cdot\mathbf{n} = -\mathbf{k}\,\bar{\sigma}\nabla\bar{p},$$

$$|V|m\frac{\partial \bar{s}}{\partial t} + Q_w = q_w, \qquad Q_w = \int_{\Gamma_V} f(s)U_n\, \mathrm{d}\Gamma, \qquad Q_w \approx \bar{f}\,\bar{U}_n|\Gamma_V|.$$

Here Γ_V is the set of the edges of the super element V; q and q_w are the flow rates of a two-phase fluid and water according to the wells in the super element; U_n is the projection of the flow rate on the external normal to the super element boundary; $\mathbf{K}$ is the absolute permeability tensor, describing internal geological inhomogeneity of the super element and calculated by the upscaling of a three-dimensional field of absolute permeability; $\bar{\sigma} = 2\Sigma_1\Sigma_2/(\Sigma_1+\Sigma_2)$ is the average water permeability on the edge; the function $\Sigma(\bar{s})$ is the analogue of the function $\sigma(\bar{s})$ on the scale of the super element and, as the averaged Buckley–Leverett function $F(\bar{s}) = \overline{f(s)}$, can be determined by upscaling of relative permeability curves.

A low-dissipative approximation of the hyperbolic equation of saturation transport is constructed using a TVD method in its algebraic form (Kuzmin et al., 2003) in combination with the original approach to the correction of saturation movement, based on the analytic theory (Mazo et al., 2013).

Problems of absolute and relative permeability upscaling are solved on detail computational grid using multigrid method and parallel algorithm for NVIDIA CUDA platform (Demidov et al., 2010). For problems of hydraulic fractures and tectonic faults method of direct QR-factorization is used (Davis, 2011).

It appears sufficient to use a coarse super element grid for cumulative adaptation of the model. The high accuracy of the numerical solution near the computational domain’s geological and technological features (tectonic faults, high permeable lens inclusions, paleochannels, vertical and horizontal wells, and hydraulic fractures) is achieved by solving the auxiliary problems on fine nested grids (Fig. 2).

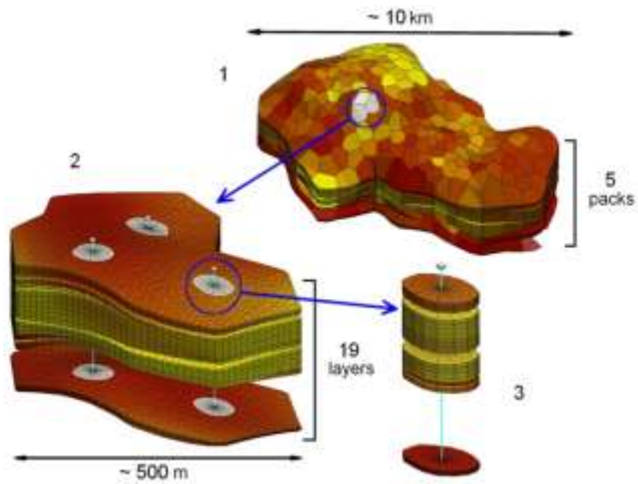

Fig. 2. Scheme of building local refinement (2) of a super element grid (1) in a domain requiring improved calculations in the vicinity of the wells (3).

4. Simulation Results

Fig. 3 demonstrates reconstruction of the production history of a reservoir by the super element method. It shows the reservoir flooding dynamics in the form of current oil saturation maps and presents the graphs of real and calculated cumulative indicators of reservoir development, i.e. current and summary oil extraction. The oil field contained 623 wells; the two-dimensional segmentation included 1873 cells with an average planned size of 500 meters. In four interlayers, 7 573 super elements were allocated (with an additional separation along the surface of the oil-water contact). The reconstruction time for the reservoir's 55-year history of development made up only 80 seconds on personal computer with processor AMD Athlon™ 64 X2, DualCore, 1.79 GHz.

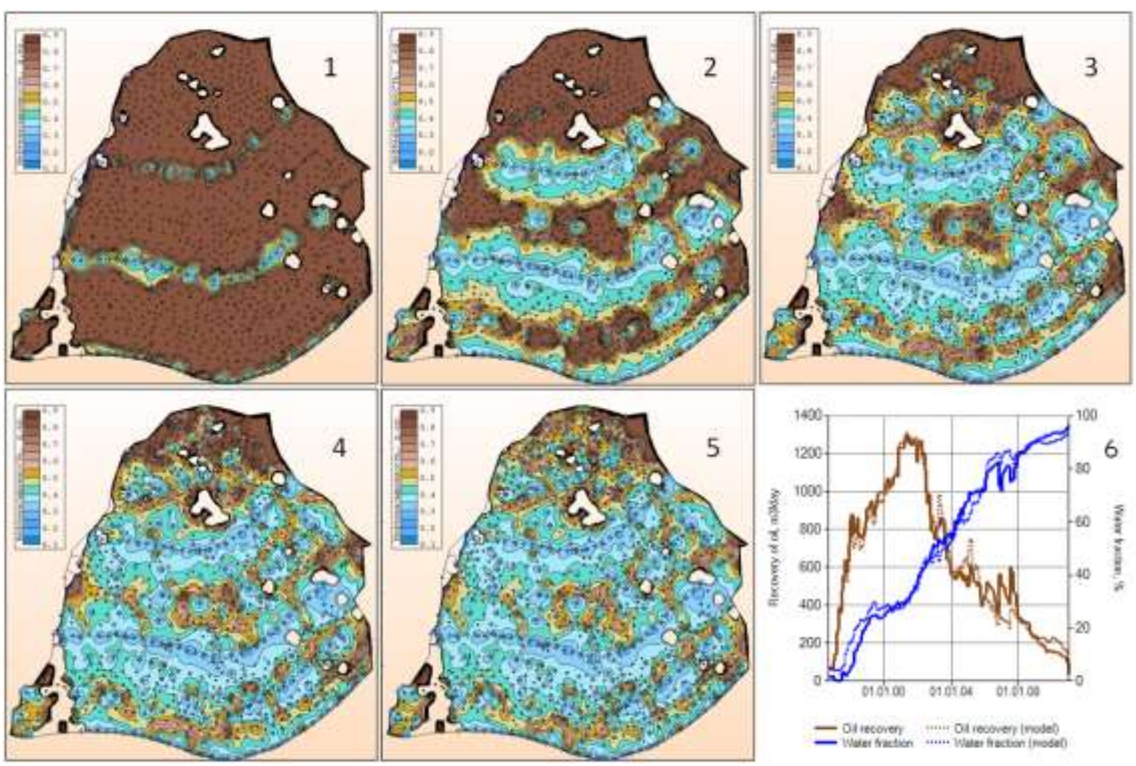

Fig. 3. Computational field of oil saturation (1-5) after each 10 years of development, and the graph of the calculated and actual oil recovery and water fraction (3)

5. Conclusion

Testing of the model on the real oil fields and comparison between the results from numerical simulation and the corresponding results obtained using the traditional models on small grids confirm wide opportunities and prospects of the super element model for rapid calculations. It is possible to speed up simulation of oil reservoir development by hundreds of times using the super element method without significant loss of accuracy.

The super element model is able to describe arbitrarily oriented wellbores, hydraulic fractures, tectonic faults and geological bodies. In each individual case, this requires using auxiliary fine grids and solving the corresponding mathematical problems.

The model is implemented by an ongoing software system for maintenance and monitoring of oil field development, which helped build the super element models for the development of a number of fields in Kazakhstan and Russia.

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References

Davis T.A. (2011). Algorithm 915: SuiteSparseQR: Multifrontal multithreaded rank-revealing sparse QR factorization. ACM Transactions on Mathematical Software, 38 (1).

Du Q., Faber V., Gunzburger M. (1999). Centroidal voronoi tessellations: Applications and algorithms. SIAM Review, 41, 637-676.

Heinemann Z.E., Brand C., Munka M., Chen Y.M. (1991). Modeling reservoir geometry with irregular grids. Soc. of Petroleum Engineers Reservoir Engineering, 6 (2), 225-232.

Kuzmin D., Moller M., Turek S. (2003). High-resolution FEM-FCT schemes for multidimensional conservation laws. Int. J. Numer. Meth. Fluids, 42, 265-295.

Palagi C.L., Aziz K. (1991). Use of Voronoi grid in reservoir simulation. Proceedings - SPE Annual Technical Conference and Exhibition, Sigma, 77-92.

Demidov D.E., Egorov A.G., Nuriev A.N. (2010). Application of NVIDIA CUDA technology for numerical solution of hydrodynamic problems. Uchenye Zapiski Kazanskogo Universiteta. Seriya Fiziko-Matematicheskie Nauki, 152 (1), 142-154. (In Russian)

Mazo A.B. (2011). Super elements. A new approach to simulation of oil field development. Neft. Gaz. Novatsii, 11, 6-8. (In Russian)

Mazo A.B. (2013). Simulation of oil field development by the super element method. Matematicheskoe Modelirovanie, 8, 83-87. (In Russian)

Lipnikov K., Svyatskiy D., Shashkov M., Vassilevski Yu. (2007). Monotone finite volume schemes for diffusion equations on unstructured triangular and shape-regular polygonal meshes. J. Comp. Phys., 227, 492-512.