Proceedings of theInternational Conference on Heat Transfer and Fluid Flow
Prague, Czech Republic, August 11-12, 2014
Paper No. 67

Effect of Magnetic Field on the Heat and Mass Transfer in a Rotating Horizontal Annulus

Sofiane Aberkane, Mourad Mouderes
Université de M'hamed Bougara
Avenue de l'independence 35000, Boumerdes, Algérie
aberkane.sofian@gmail.com; mouradw002@yahoo.fr

Malika Ihdene
Université de Yahia Farès, Médéa Algérie
mihdene@yahoo.fr

Abderahmane Ghezal
Université des Sciences et de la Technologie de Houari Boumediene,
Laboratoire de Mécanique des Fluides Théorique et Appliquée,
Bab Ezzouar, Alger-16111, Algérie
abd.ghezal@gmail.com

***Abstract* -** This paper presents the effect of an axial magnetic field imposed on incompressible flow of electrically conductive fluid between two horizontal coaxial cylinders. The imposed magnetic field is assumed uniform and constant, we also take into account the effect of heat generation due to viscous dissipation for some cases. The inner and outer cylinders are maintained at different and uniform temperatures and concentrations. The movement of the fluid is due to the rotation of the cylinders with a constant speed. An exact solution of the governing equations for momentum and energy are obtained in the form of Bessel functions. A finite difference implicit scheme was used in the numerical solution to solve the governing equations of convection flow and mass transfer. The concentration and temperature distributions were obtained with and without the magnetic field. The results show that for different values of the Hartmann number, the concentration between the two cylinders decreases as the Hartmann number increases. Also, it is found that by increasing the Hartmann number, the local Nusselt and Sherwood numbers decreases.

***Keywords*:** Rotating cylinders, Heat transfer, Mass transfer, Magnetic field, Bessel function, Finite difference.

1. Introduction

The study of flow of electrically conductive fluids, called magnetohydrodynamic (MHD) has attracted much attention due to its various applications. In astrophysics and geophysics, it is applied to the study of stellar structures, terrestrial cores and solar plasma. In industrial processes, it finds its application in MHD pumps, nuclear reactors, the extraction of geothermal energy, metallurgical and crystal growth in the field of semiconductors, the control of the behavior of fluid flow and heat and mass transfer and the stability of convective flows. Several studies have been conducted to evaluate the effect of magnetic field on the convective flows for different conditions. M. Molki et al (1990) applied the naphthalene sublimation technique to an annulus with a rotating inner cylinder in order to study heat transfer in the entrance region to obtain heat transfer data for laminar flows and compare them with results of mass transfer. H. Ben Hadid, and D. Henry (1996) investigated numerically the effect of a constant magnetic field on a three-dimensional buoyancy-induced flow in a cylindrical cavity, they put in light the structural changes of the flow induced by the magnetic field for each field orientation. Singh S. K. et al (1997) presented exact solutions for fully developed natural convection in open-ended vertical concentric annuli under a radial magnetic field. Kefeng Shi, Wen-Qiang Lu (2006) simulated numerically the characteristics of transient double-diffusive convection in a vertical cylinder using a finite element method. Mohamed A. Teamah (2009) carried out. a numerical study of double-diffusive laminar mixed convection within a two-dimensional, horizontal annulus rotating cylinders. The results for both average Nusselt and Sherwood numbers were correlated in terms of Lewis number, thermal Rayleigh number and buoyancy ratio. Bessaih R., et al (2009) studied the MHD stability of an axisymmetric rotating flow in a cylindrical enclosure containing liquid metal (Pr = 0.015), with an aspect ratio equal to 2, and subjected to a vertical temperature gradient and an axial magnetic field. W. Wrobel et al (2010) presented an experimental and numerical analysis of a thermo-magnetic convective flow of paramagnetic fluid in an annular enclosure with a round rod core and a cylindrical outer wall under gravitational and magnetic environments. Venkatachalappa M., et al (2011) carried out numerical computations to investigate the effect of axial or radial magnetic field on the double-diffusive natural convection in a vertical cylindrical annular cavity. R.H Mozayyeni and A.B Rahimi (2012) investigated numerically the problem of mixed convection of a fluid in the fully developed region between two horizontally concentric cylinders with infinite lengths, in the presence of a constant magnetic field with a radial MHD force direction, considering the effects of viscous heat dissipation in the fluid in both steady and unsteady states. Seth G.S. and Singh J.K. (2013) presented a study of the unsteady MHD Couette flow of class-II in a rotating system with Hall effects in the presence of a uniform transverse magnetic field.

Although the exact solutions for the Hartmann flow and the MHD Couette flow have been achieved for more than seventy years, the solutions for a heat transfer in flow between concentric rotating cylinders, also known as Taylore Couette flows, under external magnetic field have been restricted to high Hartmann numbers.

The aim of the present study is to examine analytically and numerically the effects of an external axial magnetic field applied to the forced convection flow of an electrically conducting fluid between two horizontal concentric cylinders, considering the effects of viscous heat dissipation in the fluid. Also we investigated numerically the effects of the magnetic field on the mass transfer in the annular cavity.

2. Formulation of the Problem

Consider a laminar flow of a viscous incompressible electrically conductive fluid between two coaxial cylinders. The inner cylinder of radius r_1 is rotated at a constant speed Ω_1 and the outer cylinder of radius r_2 is fixed. The inner and outer walls are maintained at a constant and different temperatures and concentrations, but their values for the inner are higher than the outer, while the top and bottom walls are insulated and impermeable. The two cylinders are electrically isolated. The flow is subjected to a magnetic field B_0 of constant magnitude, uniform and axially oriented. We assume that the magnetic Reynolds number is neglected. When the magnetic field is uniform and externally applied, its time variations can be neglected and the set of flow equations further simplified to involve only the Navier-Stokes equations and the conservation of the electric current. Also we assume that the electric field is zero. In this study the viscous dissipation term in the energy equation is considered.

3. Analytical Study

The flow is assumed to be steady, laminar and unidirectional, therefore the radial and axial components of the velocity and the derivatives of the velocity with respect to θ and z are zero. Under these assumptions and in cylindrical coordinates, the non-dimensional equations governing the flow together with the boundary conditions in the azimuthal direction can be written as follows (where the stars are dropped for convenience):

$$\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} - \left(\frac{Ha^2}{(1-\eta)^2} + \frac{1}{r^2}\right) v = 0 \tag{1}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right)=-Ec\,Pr\left(\frac{\partial v}{\partial r}-\frac{v}{r}\right)^2 \tag{2}$$

$$r=\eta: v(r)=1, \theta=1 \tag{3}$$

$$r=1: v(r)=0, \theta=0 \tag{4}$$

Where:

$$r^*=\frac{r}{r_2}, v^*=\frac{v}{\Omega_1 r_1}, \eta=\frac{r_1}{r_2}, \theta=\frac{T-T_2}{T_1-T_2}, Ha=B_0 d\sqrt{\frac{\sigma}{\rho\upsilon}}, Pr=\frac{\upsilon}{a}, Ec=\frac{(\Omega_1 r_1)^2}{Cp\Delta T}:$$

are the dimensionless variables and parameters.
The velocity profile in the annular space is obtained by solving the Eq. (1) as follows:

$$v(r)=C_1 I_1(Mr)+C_2 K_1(Mr)=0 \tag{5}$$

Where $M=\frac{Ha}{1-\eta}$

Where C_1 and C_2 are the constants of integration, which are determined from the boundary conditions on the velocity.

$$C_1=\frac{K_1(M)-bK_1(\eta M)}{I_1(\eta M)K_1(M)-K_1(\eta M)I_1(M)}$$

$$C_2=\frac{bI_1(\eta M)-I_1(M)}{I_1(\eta M)K_1(M)-K_1(\eta M)I_1(M)}$$

I_1 is the modified Bessel function of the first kind of order 1, and K_1 is the modified Bessel function of the second kind of order 1.
To obtain the temperature field from Eq. (2), we performed calculations by using the expansions with three terms of the modified Bessel functions $I_1(Mr)$ and $K_1(Mr)$ used by Omid M. et al (2012), for small values of Ha.

It can be used as following:

$$I_1(Mr)\cong\frac{1}{2}Mr+\frac{(Mr)^3}{16}+\frac{(Mr)^5}{384} \tag{6}$$

$$K_1(Mr)\cong\frac{1}{Mr}+\left[\frac{1}{2}\ln(\frac{Mr}{2})-\frac{1}{4}(-2\gamma+1)\right](Mr)+\left[\frac{1}{16}\ln(\frac{Mr}{2})-\frac{1}{32}(\frac{5}{2}-2\gamma)\right](Mr)^3 \tag{7}$$

Where

γ is Euler's constant defined by: $\gamma=\lim_{x\to\infty}\left[1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+.....+\frac{1}{m}-\ln(m)\right]=0,5772156649...$

By substituting the values of $I_1(Mr)$ and $K_1(Mr)$ from the above expansions in the velocity equation, Eq. (5), and using the new velocity distribution in Eq. (2) to find the temperature field.
The temperature gradient is given then by the following equation:

$$\frac{\delta\theta}{\delta r}=\frac{C_3}{r}-\frac{Br}{r}\left[\begin{array}{l} 2C_2{}^2\ln\left(\frac{Mr}{2}\right)+\frac{2C_2{}^2}{M^2r^2}+C_5\left(Mr\right)^6\ln\left(\frac{Mr}{2}\right)+C_6\left(Mr\right)^6-\frac{1}{384}C_2{}^2\left(Mr\right)^6\left(\ln\left(\frac{Mr}{2}\right)\right)^2+C_7\left(Mr\right)^2 \\ +C_8\left(Mr\right)^4-\frac{1}{32}C_2{}^2\left(Mr\right)^4\ln\left(\frac{Mr}{2}\right)+C_9\left(Mr\right)^8-\frac{1}{3072}C_1C_2\left(Mr\right)^8\ln\left(\frac{Mr}{2}\right)-\frac{1}{92160}C_1{}^2\left(Mr\right)^{10} \end{array}\right] \tag{8}$$

Where the constants C_5 to C_9 are given in terms of C_1 and C_2 as follows:

$$C_5=\frac{11}{2304}C_1{}^2-\frac{1}{192}C_1C_2-\frac{1}{192}C_2{}^2\gamma$$

$$C_6=\frac{11}{2304}C_2{}^2\gamma+\frac{7}{2304}C_1C_2-\frac{1}{384}C_2{}^2\gamma-\frac{1}{384}C_1{}^2-\frac{125}{55296}C_2{}^2-\frac{1}{192}C_1C_2\gamma$$

$$C_7=\frac{1}{4}C_2{}^2\gamma-\frac{7}{16}C_2{}^2+\frac{1}{4}C_1C_2$$

$$C_8=\frac{1}{32}C_2{}^2-\frac{1}{32}C_2{}^2\gamma-\frac{1}{48}C_1C_2$$

$$C_9=\frac{7}{24576}C_1C_2-\frac{1}{3072}C_1C_2\gamma-\frac{1}{3072}C_1{}^2$$

The solution to the temperature profile is given by:

$$\theta=C_4+C_3\ln(r)+Br\left[\begin{array}{l} C_{10}\left(Mr\right)^6+C_{11}\left(Mr\right)^8+C_{12}\left(Mr\right)^4+\frac{1}{2}C_7\left(Mr\right)^2-\frac{1}{921600}C_1{}^2\left(Mr\right)^{10}+C_2{}^2\left(\ln\left(\frac{Mr}{2}\right)\right)^2 2C_2{}^2\ln\left(\frac{Mr}{2}\right)+\frac{C_2{}^2}{\left(Mr\right)^2}+\frac{1}{6}C_5\left(Mr\right)^6\ln\left(\frac{Mr}{2}\right) \\ +C_6\left(Mr\right)^6-\frac{1}{2304}C_2{}^2\left(Mr\right)^6\left(\ln\left(\frac{Mr}{2}\right)\right)^2+\frac{1}{6912}C_2{}^2\left(Mr\right)^6\ln\left(\frac{Mr}{2}\right)-\frac{1}{128}C_2{}^2\left(Mr\right)^4\ln\left(\frac{Mr}{2}\right)-\frac{1}{24576}C_1C_2\left(Mr\right)^8\ln\left(\frac{Mr}{2}\right) \end{array}\right] \tag{9}$$

Where: the Constants C_{10}, C_{11} and C_{12} are given as follows:

$$C_{10}=\frac{1}{36}C_5+\frac{1}{6}C_6-\frac{1}{41472}C_2{}^2$$

$$C_{11}=\frac{1}{8}C_9+\frac{1}{196608}C_1C_2$$

$$C_{12}=\frac{1}{4}C_8+\frac{1}{512}C_2{}^2$$

4. Numerical Study

In this numerical study, we consider a two-dimensional and axisymmetric unsteady flow. We opted for the numerical formulation speed - pressure due to its rapidity of prediction, its lower cost, and its ability to simulate real conditions. The finite difference scheme adopted for the resolution is very similar to that used by R.Peyret (1976), A.Ghezal et al (1992) and (2011), this is a semi implicit scheme of Crank-Nicholson type. We used the Marker And Cell (MAC) for the spatial discretization. The iterative procedure is assumed converged when the following test is verified

$$\max(|L_u|,|L_v|,|L_w|,|L_\theta|,|L_C||D|)<\varepsilon$$

where L_u, L_v, L_w, L_θ, L_C and D represents operators differences relating to system equations corresponding to the problem variables u, v, w, θ, C and Π respectively, ε is of the order of 10^{-5} depending on the considered case. We then proceeded to a study of the mesh sensitivity of the field of study. This study led us to retain a mesh of 336 nodes along the direction r and 48 nodes in the z direction.

4. 1. Mathematical Equations

Based on these dimensionless variables, the time dependent governing equations for conservation of mass, momentum, energy and species are written as follows (where the stars are dropped for convenience):

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0 \tag{10}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}-\frac{v^2}{r}+w\frac{\partial u}{\partial z}=-\frac{\delta\Pi}{\delta r}+\frac{1-\eta}{Ta}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial z^2}-\frac{u}{r^2}\right)-\frac{Ha^2 u}{(1-\eta)Ta} \tag{11}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+\frac{vu}{r}+w\frac{\partial v}{\partial z}=\frac{1-\eta}{Ta}\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}+\frac{\partial^2 v}{\partial z^2}-\frac{v}{r^2}\right)-\frac{Ha^2 v}{(1-\eta)Ta} \tag{12}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}=-\frac{\partial\Pi}{\partial z}+\frac{1-\eta}{Ta}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\right) \tag{13}$$

$$\frac{\partial\theta}{\partial t}+u\frac{\partial\theta}{\partial r}+w\frac{\partial\theta}{\partial z}=\frac{1-\eta}{Pr\,Ta}\left(\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}+\frac{\partial^2\theta}{\partial z^2}\right)+\frac{(1-\eta)Ec\Phi}{Ta} \tag{14}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial r}+w\frac{\partial C}{\partial z}=\frac{1-\eta}{ScTa}\left(\frac{\partial^2 C}{\partial r^2}+\frac{1}{r}\frac{\partial C}{\partial r}+\frac{\partial^2 C}{\partial z^2}\right) \tag{15}$$

Where:

$Ha=Bd\sqrt{\dfrac{\sigma}{\rho\upsilon}}$ is the Hartmann number,

$Ta=\dfrac{\Omega_1 r_1 d}{\nu}$ is the Taylor number,

$d=R_1-R_2$ is the width of the annular space.

$\Pr=\dfrac{\upsilon}{a}$ is the Prandtl number

$Sc=\dfrac{\upsilon}{D}$ is the Schmidt number

$\Phi=2\left[\left(\dfrac{\partial u}{\partial r}\right)^2+\left(\dfrac{u}{r}\right)^2+\left(\dfrac{\partial w}{\partial z}\right)^2\right]+\left(\dfrac{\partial u}{\partial z}+\dfrac{\partial w}{\partial r}\right)^2+\left(\dfrac{\partial v}{\partial r}-\dfrac{v}{r}\right)^2+\left(\dfrac{\partial v}{\partial z}\right)^2$ is the viscous dissipation function.

The rate of heat transfer in non – dimensional for the inner and outer cylinder is given by:

$Nu_i(z)=-\gamma\left.\dfrac{\partial\theta}{\partial r}\right|_{r=\eta}$, $Nu_e(z)=-\gamma\left.\dfrac{\partial\theta}{\partial r}\right|_{r=1}$, With: $\gamma=1-\eta$

Similarly, we can calculate both local Sherwood number as follows:

$Sh_i(z)=-\gamma\left.\dfrac{\partial C}{\partial r}\right|_{r=\eta}$, $Sh_e(z)=-\gamma\left.\dfrac{\partial C}{\partial r}\right|_{r=1}$

4. 2. Initial and Boundary Conditions

At the time t=0:

$$u(r,z,0)=v(r,z,0)=w(r,z,0)=\Pi(r,z,0)=\theta(r,z,0)=C(r,z,0)=0 \qquad (16)$$

The boundary conditions are as follows:

$$\begin{aligned}
&r=\eta \quad z\geq 0: \quad u(\eta,z)=v(\eta,z)=w(\eta,z)=0,\ \theta(\eta,z)=1, C(\eta,z)=1\\
&r=1 \quad z\geq 0: \quad u(1,z)=v(1,z)=w(1,z)=\theta(1,z)=C(1,z)=0 \qquad (17)\\
&\eta<r<1 \quad z=0: \quad u=v=w=0,\ \frac{\partial\theta}{\partial z}=\frac{\partial C}{\partial z}=0\\
&z=L: \quad u=v=w=0, \frac{\partial\theta}{\partial z}=\frac{\partial C}{\partial z}=0
\end{aligned}$$

5. Results and Discussion

In order to understand the physical situation of the problem and the effects of the Hartmann and Eckert numbers entering the problem, we have computed the numerical and analytical values of the temperature, and the numerical results of the concentration, the Nusselt number and Sherwood number. The results are presented in the figs below.

The distribution of the velocity and the temperature for numerical results reaching a steady-state at t=120.There is not much difference in velocity and temperature at t = 60 compared to t = 120.

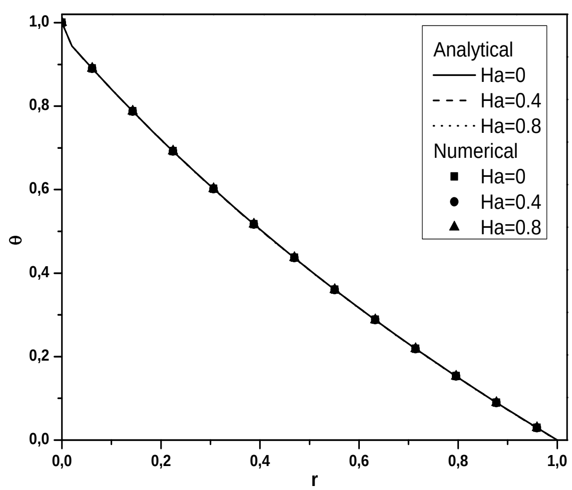

Fig. 1 . Temperature profile as a function of Hartmann number, for η = 0.5, Ta=20, Pr = 0.02, Ec=0.5, *t=120*.

The temperature is evaluated analytically and numerically for different values of Hartmann number in fig 1, comparing the numerical results with those obtained analytically, we find that the analytical and numerical results are in good agreement, it can be seen that the effect of magnetic field on the radial profile of temperature is insignificant for small values of Hartmann number (Ha<1).

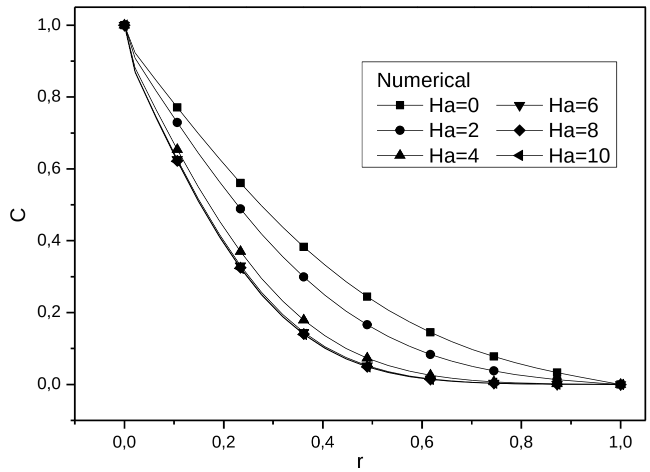

Fig. 2 . Concentration profile as a function of Hartmann number, for η = 0.5, Ta=20, Pr = 0.02, Ec=0, *t=120, z/d=7.*

Fig. 2 displays the effect of Hartmann number on the concentration at the midlength, as shown in this figure. It is observed that the concentration decreases with an increase in the values of Hartmann number in the annular cavity.

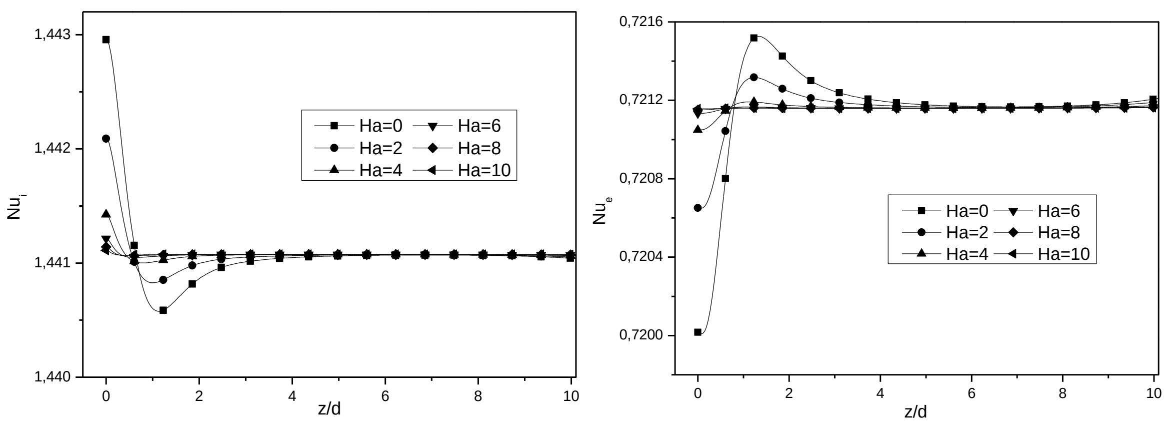

Fig. 3. Effect of Hartman number on local Nusslet number distribution on (a) inner and (b) outer cylinders, for *η = 0.5, Pr = 0.02, Ec=0, t=120* (numerical results)

The heat and mass transfer rates across the annular cavity are investigated using the computed local Nusselt and Sherwood numbers for different Hartmann number and are displayed in Figs 3 and 4.

In fig. 3 the local Nusselt number on the inner and outer surfaces are shown for different values of Hartmann number. It it can be seen that for significant increase of Hartmann number, the local Nusselt number on the inner and outer surfaces decreases, this is due to suppression of convection by the magnetic field, which results in a gradual decrease in the Nusselt number. The analysis of the variation of local Nusselt number on the inner and outer cylinder shows that it tends to a limit value, located at a value as lower than the Hartmann number is large and this is more obvious for high value of Eckert number.

In fig. 4 it can be noticed that the rate of mass transfer is, higher in the inner cylinder than in the outer cylinder. This is reasonable to expect, since the velocity and concentration gradient are higher for the inner cylinder than for the outer cylinder. The rate of mass transfer profile in the inner surface is increased with increasing the values of the magnetic field parameter, on the other hand the application of a transverse magnetic field tends to decrease the mass transfer between the outer surface and the fluid.

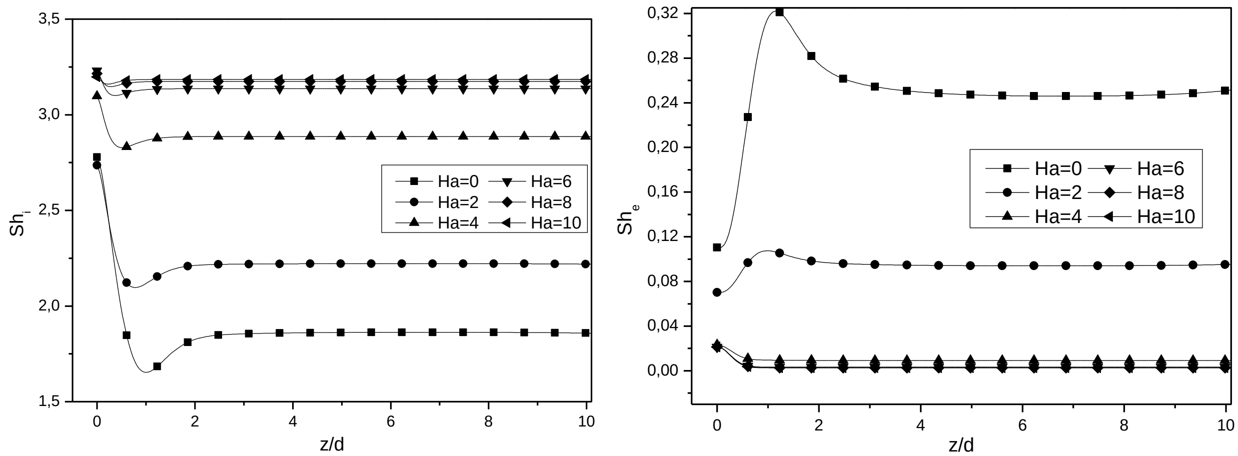

Fig. 4. Effect of Hartman number on local Sherwood number distribution on (a) inner and (b) outer cylinders, for *η = 0.5, Sc=10, Ec=0, t=120* (numerical results)

4. Conclusion

In this study, the MHD forced convection flow and mass transfer of an electrically conducting fluid between two horizontal concentric cylinders in the presence of an axial magnetic field and a temperature gradient considering the effects of viscous heat dissipation in the fluid has been investigated numerically and analytically. The velocity distribution in the annulus is obtained analytically in terms of the modified Bessel functions whose argument contains Hartmann number and radial coordinate. To obtain the temperature, the expansions of the modified Bessel functions, with three terms are used in the energy equation. It is found that the velocity and concentration decreases in the annulus with increase of Hartmann number. However an increase in Hartmann number does not affect the temperature. The results show that an increase in Hartmann number reduces the Nusselt number on both surfaces of the cylinders. The application of a magnetic field generates some interesting changes in mass transfer, an increasing in Hartmann number causes a reduction on the locale Sherwood number.

References

Ben Hadid H., and Henry D. (1996). Numerical simulation of convective three-dimensional flows in a horizontal cylinder under the action of a constant magnetic field, “J. Cryst. Growth”, 166, 436-673.

Bessaïh R., Boukhari A. and Marty Ph. (2009). Magnetohydrodynamics stability of a rotating flow with heat transfer, “International Communications in Heat and Mass Transfer”, 36, 893-901.

Ghezal A., Porterie B., Loraud J.C. (1992). Modélisation du transfert de chaleur, avec couplage conduction convection, entre un obstacle en mouvement hélicoïdal est un fluide visqueux en écoulement confiné, “ Int.J.Heat.Mass.Trans”. 329-34.

Ghezal A., Porterie B. (2011). Loraud J.C., Etude dynamique et thermique d’un écoulement pulsé en présence d’un solide chauffé en rotation, “ Mécanique & Industries”, 12, 45-65.

Kakarantzas S.C., Sarris I.E., Vlachos N.S. (2011). Natural convection of liquid metal in a vertical annulus with lateral and volumetric heating in the presence of a horizontal magnetic field, “International journal of Heat and Mass Transfer”,54, 3347-3356.

Kefeng Shi, Wen-Qiang Lu (2006). Time evolution of double-diffusive convection in a vertical cylinder with radial temperature and axial solutal gradients, “International Journal of Heat and Mass Transfer”, 49 995–1003.

Molki M., Astill K. N., Leal E. (1990). Convective heat-mass transfer in the Convective heat-mass transfer in the entrance region of a concentric annulus having a rotating inner cylinder entrance, “Int. J. Heat and Fluid Flow”, Vol. 11, No. 2.

Mozayyeni H.R., Rahimi A.B. (2012). Mixed convection in cylindrical annulus with rotating outer cylinder and constant magnetic field with an effect in the radial direction, “Scientia Iranica B”, 19 (1), 91–105.

Omid M., Shohel M., Ioan P. (2012). Analysis of first and second laws of thermodynamics between two isothermal cylinders with relative rotation in the presence of MHD flow, “International Journal of Heat and Mass Transfer”, 55, 4808–4816.

Peyrret R. (1976). Unsteady evolution of horizontal jet in a stratified fluid, “J. Fluid mechanics”, 78, part 1, 49-63.

Seth G.S. and Singh J.K. (2013). Effects of Hall current of unsteady MHD Couette flow of class-II in a rotating system, “Journal of Applied Fluid Mechanics”, Vol. 6, No. 4, pp. 473-484.

Singh S. K., Jha B. K., Singh A. K. (1997). Natural convection in vertical concentric annuli under a radial magnetic field, “Heat and Mass Transfer”, 32, 399–40.

Teamah Mohamed A. (2007). Numerical simulation of double diffusive laminar mixed convection in a horizontal annulus with hot, solutal and rotating inner cylinder, “International Journal of Thermal Sciences”, 46 637–648.

Wrobel W., Fornalik-Wajs E., Szmyd J.S. (2010), Experimental and numerical analysis of thermo-magnetic convection in a vertical annular enclosure,“International Journal of Heat and Fluid Flow”, 31,1019–1031.

Venkatachalappa M., Do Younghae, Sankar M. (2011). Effect of magnetic field on the heat and mass transfer in a vertical annulus, “International Journal of Engineering Science”, 49, 262–278.