

# Numerical Simulation of Rock Thermal Properties

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**Abstract** – New approach for numerical simulation of rock thermal properties and its validation are presented in this paper. Values of thermal conductivity and thermal diffusivity of porous media are determined both numerically and experimentally. The digital rock model size of the porous sample coincides with the same real samples size used in experiments. In addition the distribution of real mineral composition in digital rock model is constructed. Influence of cement bridges between mineral grains of rock on the thermal properties are also taken into account. The unsteady heat equation in 3D case for the numerical study of the problem is solved. The initial distribution of phases in the saturated sample is created by using a numerical code based on the density functional method. The both properties thermal conductivity and thermal diffusivity were experimentally studied for numerical simulation validation. Thermal properties were measured at dry and water saturated states with a new non-contact and non-destructive optical scanning instrument. We compared the numerical solutions and experimental data of these thermal characteristics.

**Keywords:** Thermal conductivity, Thermal diffusivity, Digital rock, Saturated porous medium.

## 1. Introduction

Thermal properties of porous media are an important part of overall problems of studying geophysical properties of rocks: interpretation of temperature logging data, theoretical modelling heat and mass transfer in formations, determination of heat flow density and its distribution along wells and interpretation of its vertical variations, prediction of other formation's physical properties from the correlations found between thermal and other physical properties (Popov et al., 2010a). In geothermal energy industry the thermal properties are especially important since economic aspects. Increasing necessity in thermal property data stimulated the development of new effective approaches and equipment to provide more reliable and detailed information about rock's thermal properties. Along with the traditional experimental methods for studying this problem, currently rapidly developing line of research associated with numerical methods for studying digital models of the core. The availability of modern  $\mu$ CT scanners, as well as special approaches for image reconstruction makes completely solvable the task of creating digital rock models. Development of computing power and the creation of modern approaches to study of multiphase multicomponent flows in porous media channels (see Dinariev, 1995, Demianov et al., 2009, Demianov et al., 2011) enable to obtain both the hydrodynamic characteristics of the porous sample and the initial phase distribution fields. These fields are treated as the initial conditions for further analysis of the porous medium properties, such as thermal properties, electromagnetic properties, elastic properties, etc. A comparison of the numerical results and experimental data on the similar samples enables to validate the computational and theoretical methods for their further implementation in the oil industry.

## 2. Mathematical Statement of the Problem

We consider the unsteady problem of heat conduction for digital sample saturated rock in 3D case. Moreover, its thermal characteristics (mass density  $\rho$ , specific heat capacity  $c$ , thermal conductivity  $\lambda$ ) will be non-uniform in space. In this case, the governing equation can be written as follows:

$$\rho c \frac{\partial T}{\partial t} = \partial_x \lambda \partial_x T + \partial_y \lambda \partial_y T + \partial_z \lambda \partial_z T \quad (1)$$

Here:  $T$  – absolute temperature;  $t$  – time;  $\partial_a = \partial / \partial a$  – partial derivative with respect to the Cartesian coordinates  $a = x, y, z$ . Solution to the problem is sought in a rectangular area with linear sizes  $L_x, L_y, L_z$ . The initial conditions in the sample is given a constant value of temperature  $T(t = 0, a) = T_0 = const$ . Boundary conditions are specified as follows: on one of the area faces (for example in the  $a$  direction) the temperature value is maintained  $T(t, a = 0) = T_1 = const$ , on the opposite face the temperature value is maintained  $T(t, a = L_a) = T_0$ , ( $T_1 > T_0$ ); zero heat fluxes are defined on all other borders. If a cement bridge is between two neighbour cells with different minerals  $i, j$  on uniform numerical mesh in the  $a$  direction, the additional thermal resistance  $r_c$  (associated with a cement bridge) is included in the formula for the thermal resistance  $R_{ij}^a$  (Patankar, 1980):

$$R_{ij}^a = \frac{h_a}{2} \left( \frac{1}{\lambda_i} + \frac{1}{\lambda_j} \right) + r_c \quad (2)$$

Here:  $h_a$  – cell size of numerical mesh in  $a$  direction;  $\lambda_i, \lambda_j$  – coefficients of thermal conductivity for neighbour cells  $i, j$  respectively. This model takes into account the presence of cement between cells of the computational grid (sub-grid level). Equation (1) is solved numerically using an implicit finite-difference Douglas-Gunn splitting scheme (Douglas and Gunn, 1964). As a result, we can find a stationary temperature field. Thereafter the mean value of the thermal conductivity in  $a$  direction  $\bar{\lambda}_a$  is reduced to the following formula:

$$\bar{\lambda}_a = \frac{L_a}{V(T_1 - T_0)} \iiint q_a dV \quad (3)$$

Here  $V = L_x L_y L_z$  – volume of the area,  $q_a = -\lambda \partial_a T$  – heat flux in  $a$  direction. After calculating  $\bar{\lambda}_a$  the sample average value of thermal diffusivity  $\bar{\chi}_a = \bar{\lambda}_a / (\bar{\rho} \bar{c})$  can be determined. Average values of density  $\bar{\rho}$  and specific heat capacity  $\bar{c}$  are simply computed.

## 2. 1. Use of Analytical Solutions

However, there is a more accurate method for calculating thermal diffusivity (call this value as  $\chi_a^u$ ). For its presentation we use the analytical solution property of the linear heat equation (Carslaw and Jaeger, 1959). This solution for our problem has the following form:

$$T(t, a) = T_1 + (T_0 - T_1) \frac{2}{\sqrt{\pi}} \int_0^\eta e^{-\eta^2} d\eta \quad (4)$$

Here:  $\eta = \frac{a}{2\sqrt{\chi_a^u t}}$  similar variable. Solution (4) is used to the time until starting to affect the boundary  $a = L_a$ . Due to the inhomogeneous distribution of thermal parameters in the sample  $\bar{\chi}_a \neq \chi_a^u$  ( $\bar{\chi}_a < \chi_a^u$ )

## 3. Experimental Study

Optical scanning method is principally new way in the measurements of thermal properties (thermal conductivity and diffusivity) of solids. Determining these parameters and combining them with rock sample density can define the rock's specific heat capacity. Optical scanning method is based on the heating the explored rock samples by optical radiation, concentrated in the little spot and then moved along a plane or cylindrical surface of the rock samples (core, for example) with a constant speed. Laser

or other optical sources are used for heating. Sample heating level and sample initial temperature level are registered by infrared detectors, which fields of view move along the same rock surface with the same speed that the heat spot moves. Two standard samples with known thermal conductivity and thermal diffusivity are processed in the same series as the studied samples. Thermal properties are determined by comparing heat levels of the rock samples with heat levels of standard samples, using the original theoretical base (Popov et al., 1999, Popov et al., 2010 a, b). As a result, the optical scanning method provides the precision level of thermal properties measurements (in accordance with metrological tests for a confidence probability of 0.95: for thermal conductivity – precision (1.5%) and accuracy (1.5%) within the range of 0.1...70.0 W/(m·K), for thermal diffusivity precision (2%) and accuracy (2%) within the range of 0.1...5.0·10<sup>-6</sup> m<sup>2</sup>/s that rates with the well-known standard divided bar method.

#### 4. Results

Berea sandstone (see Fig.1) selected as an example of a digital model with the following volumetric mineral composition: quartz – 80%, feldspar – 15%, cement – 5%. Sample porosity equals 17%. The numbers of numerical cells in calculated area equals 200 \* 200 \* 200, its size are  $L_x = L_y = L_z = 0.008\text{m}$ , temperature gradient is given in the  $x$  direction,  $T_0 = 300\text{ K}$ ,  $T_1 = 302\text{ K}$ . Sample wetting is hydrophilic, it is saturated with water and oil phases and water saturation value is  $S_w = 50\%$ . All the necessary thermal characteristics rock and fluids set of handbook (Schoen, 1996)

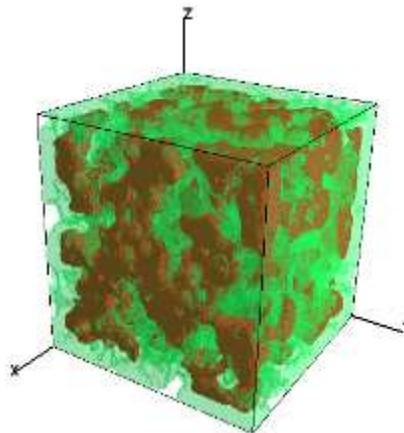


Fig. 1. Digital rock model of Berea sandstone: quartz – green color, feldspar – brown color.

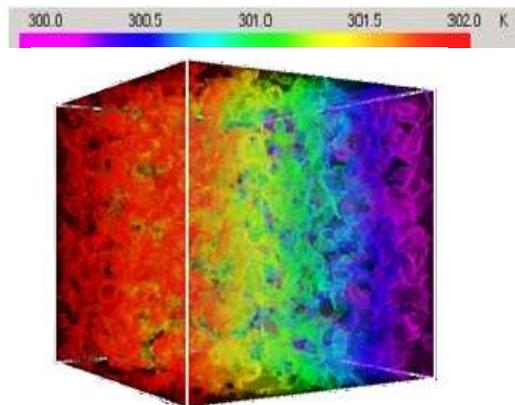


Fig. 2. Stationary temperature field of hydrophilic porous media (Berea sandstone) with water saturation  $S_w = 50\%$ .

Thermal properties were measured on a conventional core plug with diameter 30 mm. Core preparation before measurements was performed in accordance with conventional core analysis practice API RP-40: coreplugs were dried at 105<sup>0</sup>C till the stable weight before measurement at dry state and fully

saturated under vacuum before measurements at water and kerosene saturated states. Experimental data are presented in Table 1.

Table 1. Comparison of experimental and numerical results on the thermal conductivity and thermal diffusivity.

	Numerical $\bar{\lambda}_z$ $W/(m \cdot K)$	Numerical $\bar{\chi}_a$ $10^{-6}m^2/s$	Numerical $\chi_a^u$ $10^{-6}m^2/s$	Experimental $\bar{\lambda}_z$ $W/(m \cdot K)$	Experimental $\bar{\chi}_a$ $10^{-6}m^2/s$
Dry core	2.51	1.50	1.58	2.35	1.49
Kerosene sat.	3.30	1.78	1.88	3.21	1.69
Water sat.	4.11	1.79	1.9	4.33	1.88

At the moment we are working on the validation of the method on more representative samples.

## 5. Conclusion

Experimental and calculation method for determining the thermal properties of saturated rocks are presented. We observed a good coincidence between numerical and experimental data therefore discrepancy can be caused by rock heterogeneity.

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## References

- Carslaw H.S., Jaeger J.S. (1959) "Conduction of heat in solids". Second edition. Oxford.
- Demianov A.Yu., Dinariev O.Yu., Evseev N.V. (2009) "Basics of the density functional theory in Hydrodynamics". Fizmatlit, Moscow, [In Russian]
- Demianov A., Dinariev O., Evseev N. (2011) Density functional modeling in multiphase compositional. Hydrodynamics. Can. J. Chem. Eng., V.89, pp. 206-226.
- Dinariev O.Y. (1995) A Hydrodynamic Description of a Multicomponent Multiphase Mixture in Narrow Pores and Thin Layers. J. Appl. Math. Mech., V. 59(5), pp. 745-752.
- Douglas J., Gunn J.E. (1964) A general formulation of alternating direction methods. Part I. Parabolic and hyperbolic problems. Numerische Math., V. 6, pp. 428-453.
- Patankar S. (1980) "Numerical heat transfer and fluid flow". Hemisphere Publishing Corp., New York.
- Popov Yu., Miklashevskiy D., Romushkevich R., Safonov S., Novikov S. (2010a) Advanced Technique for Reservoir Thermal Properties Determination and Pore Space Characterization. Proceedings of World Geothermal Congress 2010, Bali, Indonesia, 25-29 April 2010.
- Popov Yu., Korobkov D., Miklashevskiy D., Safonov S., Novikov S. (2010b) New Experimental and Theoretical Base for Reservoir Thermal Properties Determination and Pore Space Characterization. Proceedings of the International Symposium of the Society of Core Analysts Halifax, Canada. SCA2010-10.
- Popov Yu. A., Pribnow D., Sass J., Williams C., and Burkhardt H., 1999. "Characterization of rock thermal conductivity by high-resolution optical scanning". Geothermics, 28, pp.253-276.
- Schoen J. H. 1996. Physical properties of rocks: Fundamentals and principles of petrophysics, 1st ed. Oxford: Pergamon Press. 583 p.