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Non-Newtonian MHD Stagnation Point Flow with Slip

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Abstract **-**The steady two-dimensional stagnation point flow of a second-grade fluid with slip in the presence of a transverse magnetic field is examined. The fluid impinges on the wall orthogonally. Numerical solutions are obtained using a quasi-linearization technique. The effects of the viscoelastic parameter, the magnetic field and the slip condition on the flow are examined.

Keywords**:** Incompressible, Stagnation-point flow, Second-grade fluid, Slip condition, Magnetic field, Steady.

1. Introduction

Magnetohydrodymanic (MHD) stagnation point flows are relevant to many engineering applications such as petroleum engineering, chemical engineering, MHD pumps, heat exchangers and metallurgy industry. Examples of MHD flows in the metallurgy industry include the cooling of continuous strips and filaments drawn through a quiescent fluid and the purification of molten metals from non-metallic inclusions. The Hiemenz flow of a viscous fluid in the presence of a magnetic field was examined by Ariel (1994). Kumari and Nath (1999) studied the flow and heat transfer in a stagnation-point flow of a viscous fluid over a stretching surface in the presence of a magnetic field. Attia (2000) investigated the steady flow of a non-Newtonian fluid at a stagnation point with heat transfer in an external uniform magnetic field.

One class of flows which has been studied extensively in fluid mechanics is the stagnation-point flows. Hiemenz (1911) derived an exact solution of the steady flow of a Newtonian fluid impinging orthogonally on an infinite flat plate. Stuart (1959), Tamada (1979) and Dorrepaal (1986) independently investigated the solutions of a stagnation point flow when the fluid impinges obliquely on the plate. Beard and Walters (1964) used boundary-layer equations to study two-dimensional flow near a stagnation-point of a viscoelastic fluid. Rajagopal et al. (1983) have studied the Falkner-Skan flows of an incompressible second grade fluid. Dorrepaal et al. (1992) investigated the behaviour of a viscoelastic fluid impinging on a flat rigid wall at an arbitrary angle of incidence. Labropulu et al. (1993)studied the oblique flow of a second grade fluid impinging on a porous wall with suction or blowing.

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Another interesting stagnation-point flow is one with wall slip which was recently considered in a paper by Wang (2003). Wall slip can occur if the working fluid contains concentrated suspensions (Soltani and Yilmazer (1998)). Also, this problem arises in some applications where a thin film of light oil is attached to the plate or when the plate is coated with special coatings such as a thick monolayer of hydrophobic octadecylthichlorosilane (see Derek et al. (2002)).

When the molecular mean free path length of the fluid is comparable to the system's characteristic length, then rarefaction effects must be considered. The Knudsen number K_n, defined as the ratio of the molecular mean free path to the characteristic length of the system, is the parameter used to classify fluids that deviate from continuum behaviour. If $K_n > 10$, it is free molecular flow, if $0.1 < K_n < 10$ it is transition flow, if $0.01 < K_n < 0.1$ it is slip flow, and if $K_n < 0.01$ it is the viscous flow (see Wang (2003), Kogan (1969)). Flows in the slip-flow region have been modeled using the Navier-Stokes equations and the traditional non-slip condition is replaced by the slip condition

$$u_y = A_p \frac{\partial u_t}{\partial n} \tag{1}$$

where u_t is the tangential velocity component, n is normal to the plate, and A_p is a coefficient close to 2(mean free path)/$\sqrt{\pi}$ (see Sharipov and Seleznev (1998)]). This condition was first proposed by Navier (1827) nearly two hundred years ago.

In the present work, we follow Wang (2003) and investigate the behaviour of the non-Newtonian second-grade fluid impinging on a rigid wall with slip in the presence of a transverse magnetic field. The fluid impinges on the wall orthogonally. In particular, we study the effects of the slip condition, the magnetic field and the effects of the viscoelasticity of the fluid on the stagnation-point.

2. Flow Equations

The steady two dimensional flow of a viscous incompressible second grade fluid in the presence of transverse magnetic field is governed by the following equations in non-dimensional form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$\begin{aligned} u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{\partial p}{\partial x} &= \nabla^2 u + We\left\{\frac{\partial}{\partial x}\left[2u\frac{\partial^2 u}{\partial x^2} + 2v\frac{\partial^2 u}{\partial x \partial y} + 4\left(\frac{\partial u}{\partial x}\right)^2 + 2\frac{\partial v}{\partial x}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right]\right. \\ &\left. + \frac{\partial}{\partial y}\left[\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + 2\frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + 2\frac{\partial v}{\partial x}\frac{\partial v}{\partial y}\right]\right\} + \lambda\frac{\partial}{\partial x}\left[4\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right] - Mu \end{aligned} \tag{3}$$

$$\begin{aligned} u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \frac{\partial p}{\partial y} &= \nabla^2 v + We\left\{\frac{\partial}{\partial x}\left[\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + 2\frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + 2\frac{\partial v}{\partial x}\frac{\partial v}{\partial y}\right]\right. \\ &\left. + \frac{\partial}{\partial y}\left[2u\frac{\partial^2 v}{\partial x \partial y} + 4\left(\frac{\partial v}{\partial y}\right)^2 + 2v\frac{\partial^2 v}{\partial x^2} + 2\frac{\partial u}{\partial y}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right]\right\} + \lambda\frac{\partial}{\partial y}\left[4\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right] \end{aligned} \tag{4}$$

where $u = u(x, y)$, $v = v(x, y)$ are the velocity components, $p = p(x, y)$ is the pressure, $\nu = {}^{\mu}/_{\rho}$ is the kinematic viscosity where ρ is the constant fluid density and μ is the constant coefficient of viscosity, and α_1, α_2 are the normal stress moduli, $We = \frac{\alpha_1 \beta}{\rho \nu}$ is the Weissenberg number, $M = \frac{\sigma B_0}{\rho \beta}$ is the Hartmann's number, σ is the electrical conductivity and B_0 is the magnetic field, $\lambda = \frac{\alpha_2 \beta}{\rho \nu}$ and β has the units of inverse time.. It is assumed that the magnetic field $\vec{B}$ is perpendicular to the velocity field $\vec{V}$ and $\sigma B_0 \ll 1$, so it is possible to neglect the effect of the induced magnetic field. The star on a variable indicates its dimensional form. Dunn and Fosdick (1974) and Dunn and Rajagopal (1995) have shown that if the second-grade fluid is to undergo motions which are compatible with Clausius-Duhem inequality and the assumption that the free energy density of the fluid be locally at rest, then the material constants must satisfy the following restrictions:

$$\mu \geq 0, \quad a_1 \geq 0, \quad \alpha_1 + \alpha_2 = 0$$

Continuity equation (2) implies the existence of a stream-function $\psi(x, y)$ such that

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{5}$$

Substitution of (5) in equations (3) and (4) and elimination of pressure from the resulting equations using $p_{xy} = p_{yx}$ yields

$$\frac{\partial\left(\psi, \nabla^2 \psi\right)}{\partial\left(x, y\right)} - We\frac{\partial\left(\psi, \nabla^4 \psi\right)}{\partial\left(x, y\right)} + \nabla^4 \psi - M\frac{\partial^2 \psi}{\partial y^2} = 0 \tag{6}$$

Having obtained a solution of equation (6), the velocity components are given by (5) and the pressure can be found by integrating equations (3) and (4).

The shear stress component τ_{12} is given by

$$\tau_{12} = \mu\beta\left\{\frac{\partial^2 \psi}{\partial y^2} - \frac{\partial^2 \psi}{\partial x^2} + We\left[\frac{\partial \psi}{\partial y}\left(\frac{\partial^3 \psi}{\partial x \partial y^2} - \frac{\partial^3 \psi}{\partial x^3}\right) - \frac{\partial \psi}{\partial x}\left(\frac{\partial^3 \psi}{\partial y^3} - \frac{\partial^3 \psi}{\partial x^2 \partial y}\right) + 2\frac{\partial^2 \psi}{\partial x \partial y}\frac{\partial^2 \psi}{\partial y^2} + 2\frac{\partial^2 \psi}{\partial x^2}\frac{\partial^2 \psi}{\partial x \partial y}\right]\right\}_{y=0} \tag{7}$$

3. Solutions and Discussion

We assume that the infinite plate is at $y = 0$ and that the fluid occupies the entire upper half plane $y > 0$. Furthermore, we assume that the stream-function far from the wall is given by $\psi = xy$ (see Hiemenz [6]). Thus, the non-dimensional boundary conditions are given by

$$\frac{\partial \psi}{\partial x} = 0 \text{ at } y = 0, \qquad \psi(x, y) \to y \text{ as } y \to \infty \tag{8}$$

The slip condition in equation (1) becomes

$$\frac{\partial \psi}{\partial y} = \gamma \frac{\partial^2 \psi}{\partial y^2} \tag{9}$$

where $\gamma = A_p\sqrt{\beta}\,\nu$ is a parameter representing the slip to viscous effects.
Following Wang (2003), we assume that

$$\psi(x, y) = xF(y) \tag{10}$$

Substituting (10) in equations (6) and boundary conditions (8) and (9), we obtain the following ordinary differential equations after one integration

$$F''' + F F'' - F'^2 - We\left(F F^{(iv)} - 2 F F''' + F''^2\right) - MF' = -1 - M$$
$$F(0) = 0, \quad F'(0) = \gamma F''(0), \quad F'(\infty) = 1 \tag{11}$$

where prime denotes differentiation with respect to y.

System (11) with $M = 0$ and $We = 0$ has been solved for various values of γ by Wang (2003). If $M = \gamma = 0$, system (11) has been solved by Garg and Rajagopal (1990) and Ariel (2002) for various values of We.

Using the quasi-linearization technique described by Garg and Rajagopal (1990), we find that $F'(0) = 1.23259$ when $We = M = \gamma = 0$. Numerical values of $F''(0)$ for various values of We, M and γ are shown in Tables 1 and 2. These values are in good agreement with the values obtained by Wang (2003) when $We = M = 0$ and the values obtained by Garg and Rajagopal (1990) and Ariel (2002) when $M = \gamma = 0$. Figure 1 depicts the profiles of F' for $M = \gamma = 1$ and various values of We. The profiles of F' for $We = 0.2,\ \gamma = 1$ and various values of M are shown in Figure 2. The profiles of F' for $We = 0.2$, $M = 1$ and various values of γ are shown in Figure 3.

4. Conclusions

The steady two-dimensional stagnation-point flow of a second grade fluid with slip in the presence of a transverse magnetic field is examined. The fluid impinges on the wall orthogonally. The quasi-linearization technique is used and numerical results are obtained for various values of the Weissenberg number We, the Hartmann's number M and the slip parameter γ. It can be observed that as the elasticity of the fluid increases, the values of $F'(y)$ near the wall are decreasing and as the slip parameter γ is increasing the values of $F'(y)$ near the wall are also increasing. Furthermore, as the magnetic parameter M is increasing, the values of $F'(y)$ near the wall are increasing. From Tables 1 and 2, we can see that $F''(0)$ increases with the magnetic parameter M. The reason for this behaviour is that the magnetic field $\vec{B}$ induces a force along the surface which supports the motion. As a result, the velocity along the surface is increased everywhere with increasing Hartmann's number.

Table 1. Numerical values of $F''(0)$ for various values of We and γ when $M = 0$.

	$We = 0$	$We = 0.2$	$We = 1$	$We = 5$
$\gamma = 0$	1.23259	1.05818	0.75276	0.41288
$\gamma = 0.6$	0.76428	0.68939	0.53933	0.33566
$\gamma = 1$	0.37589	0.35671	0.31125	0.22981
$\gamma = 5$	0.09404	0.09280	0.08939	0.08110
$\gamma = 10$	1.23259	1.05818	0.75276	0.41288

Table 2. Numerical values of $F''(0)$ for various values of We and γ when $M = 2.25$.

	$We = 0$	$We = 0.2$	$We = 1$	$We = 5$
$\gamma = 0$	1.93895	1.69217	1.24230	0.70921
$\gamma = 0.6$	0.92516	0.85191	0.69735	0.46833
$\gamma = 1$	0.67839	0.63683	0.54329	0.38938
$\gamma = 5$	0.18309	0.17974	0.17080	0.15033
$\gamma = 10$	0.09560	0.09467	0.09210	0.08568

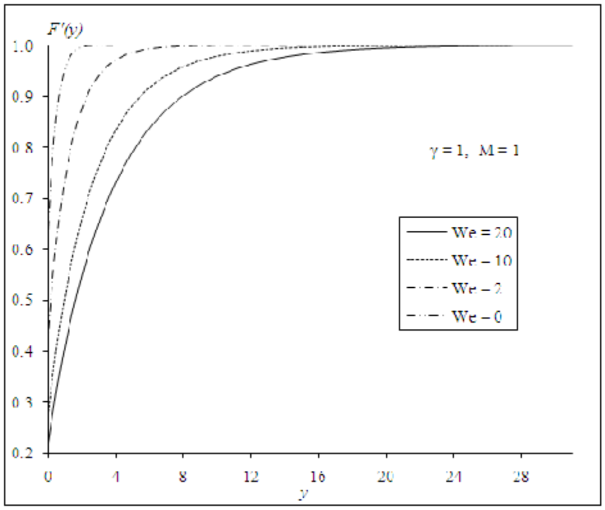

Fig. 1. Variation of $F'(y)$ for $M = 1,\ \gamma = 1$ and various values of We.

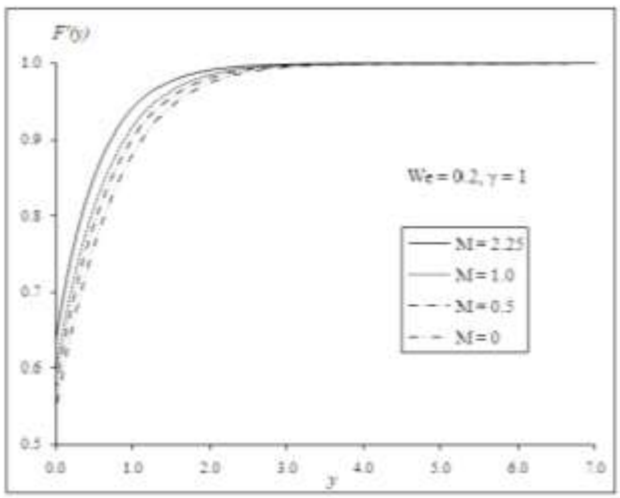

Fig. 2. Variation of $F'(y)$ for $We = 0.2,\ \gamma = 1$ and various values of M.

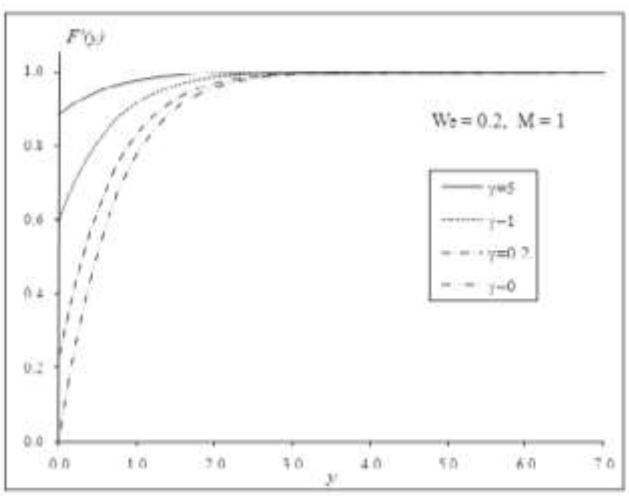

Fig. 3. Variation of $F'(y)$ for $We = 0.2,\ M = 1$ and various values of γ.

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