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Low-cost Housing Using Sustainable Materials and Catenary Shapes

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Abstract **-** The United Nations estimates that approximately 12% of the world's population live in slums, which is about 860 million people. The use of the word "slums" infers that its occupants live in informal housing, or housing that does not comply with building codes and regulations, are inferior in construction and often present health hazards. Although every nation is inherently responsible to ensure that this basic need is met, usually finances and social prioritization are the inhibiting factors. In fact, the rate of increase in slums is exceeding population growth. The world is digressing. This paper is therefore an attempt to provide an alternative solution to the housing problem. A housing solution should be affordable, include sustainable building materials and optimize the structural shape. A prototype house is presented, which incorporates cement-stabilized earth blocks (CSEB) and blocks made from recycled building material. A typical pitched roof is also replaced with a catenary brick vault.

Keywords**:** Cement stabilized earth blocks, Sustainable building materials, Dry-stack construction, Low-cost housing, Catenary vaults, Shell structures.

1. Introduction

UN Habitat estimates that nearly one billion people will live in slums in the next 20 years. South Africa is a developing country, and is faced with housing shortages that appear to be insurmountable. Approximately 2.2 million people live in substandard homes, which are inferior in construction, lack basic sanitation, clean water and are unsafe for human habitation. The country has been engaged in housing projects for the past 20 years, but the demand far outstrips what is supplied. The inhibiting factor is a lack of finance, by the homeowner (who are often unemployed or underemployed) and government (who heavily subsidize the projects). The need for a cost effective house is apparent.

We often gravitate to conventional homes, which are linear and box-like, with a pitched roof. Although these types of structures are functional and pleasing, they may not be structurally optimal, nor incorporate sustainable materials.

2. Cement Stabilized Earth Blocks

Although earth blocks have been used in construction for thousands of years, cement stabilized earth blocks (CSEB) is a recent development. Most guidelines on earth bricks generally adhere to a few basic requirements [1]:

1. The soil should be well graded and have a good distribution of gravel, sand, silt and clay. Ideally, the distribution should be to achieve the highest density. The most important issue is the quantity of clay, which should be between 15% to 20% of the mix. Clay is necessary for cohesion of the earth particles, and assists in waterproofing the blocks. However, clay is potentially unstable—expansion or contraction is possible when saturated with water, or dried.
2. The soil material is sieved to break up clumps into smaller particles. The breaking up of particles is necessary to ensure that the cement is evenly distributed, and able to bind the soil particles.
3. Polymer waterproofing agents may be added to the mix to prevent capillary action, due to exposure to rainwater. These admixtures substantially increase the durability of the blocks. There are several examples in South Africa of CSEB structures that were constructed more than 20 years ago, that do not show any signs of deterioration, erosion or cracking in the blocks.
4. The amount of cement added to the mix typically ranges from 8% to 12%. Usually, most guidelines require a minimum of 10% cement.
5. The cement and soil are mixed thoroughly, and molded into a block by applying compression. Usually, a pressure

of 20 MPa is applied to the mix to achieve an average bulk density of 1900 kg/m^3.

6. The blocks are then cured for several weeks, to gain strength and minimize shrinkage cracking.

Earth blocks typically have a compressive strength between 7 – 10 MPa. A compressive strength of 10 MPa is common, with adherence to the above guidelines. This strength is more than adequate to support residential structures.

Why use earth blocks? Earth blocks have the advantage of being manufactured on-site, using the surrounding soil. Material and transportation costs of the blocks are therefore minimized. Furthermore, the amount of cement is reduced (compared to a cement brick), culminating in an overall saving. Habona [2] has found that CSEB is approximately 18% cheaper than conventional bricks.

Bricks may also be produced from building rubble. The brick and concrete from demolished buildings are crushed, particle size assessed, cement added and moulded into a block by applying a compression. The manufacturing process is similar to CSEB, but the main constituent is the crushed recycled building rubble.

3. Wall Construction

The walls of the ground floor are dry-stacked, and laid in a stretcher pattern. This construction method requires that the bricks are stacked one upon another without mortar. The wall profile is illustrated in Fig. 1.

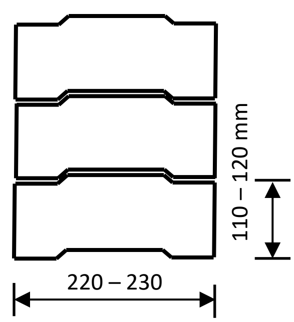

Fig. 1: Dry-stacked brick walls.

The blocks are moulded with keys and indentations so that the bricks interlock, similar to a Lego ® set. Furthermore, the bricks are stacked without mortar, to permit rapid and economical construction (no mortar). The ground floor was constructed of 220 mm wide bricks (equivalent to a double brick wall).

CSEB is truly a sustainable material, and production is known to consume less energy and cause less CO_2 emissions. Waziri [3] reported that CSEB produces 9 times less CO_2 emissions than clay fired blocks and nearly 7 times less emissions than concrete blocks.

4. Roof Structure

The roof structure makes up a substantial part of the cost of a home. If a shell vault replaces the roof, additional living area is added to the space created by the vaults. The vault becomes the walls and the roof of the second storey. A typical low-cost house in South Africa is about 50 m^2. A second floor will double the living space. The extra space is sorely needed, since the average occupancy of a low-cost home in South Africa is about eight people.

To accommodate internal fittings, furniture and cabinets, it was decided to mix the layout of the house plan—a square ground floor layout and shell structure on the upper floor. The plan dimensions of the prototype house 7 m x 7 m, giving a ground floor area of 49 m^2. The roof was replaced with a catenary shell vault. However, to minimize unusable space due to sloping walls, the height of the vault would have to be about 7.3 m. A single vault is therefore not practical. For this reason, the design includes two vaults (see Fig. 2). The high sloping walls are also necessary to minimize the horizontal thrust, at the base of the vault.

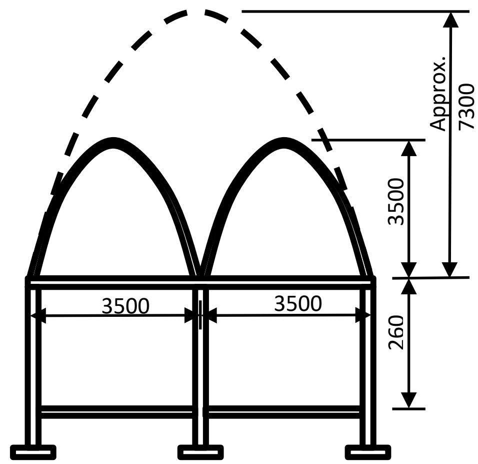

Fig. 2: Elevation layout of the low-cost prototype house

The catenary vault was, by far, the most technically challenging part of the design. The vaults are constructed of unreinforced masonry. Unreinforced masonry only works in compression. Tensions in masonry lead to cracking, which is unsightly, accelerates deterioration, causes leaks and may present stability/safety problems. Only one vault shape would satisfy these requirements—the catenary vault.

The word catenary, meaning chain, is a shape of a unique type of arch, in which the stresses are carried in pure compression, and without bending and shears. Bending and shear forces are highly uneconomical, and increases the stress in structures substantially, ranging from 12 to 24 times greater than axial stresses [4]. The efficiency of shell structures is well-documented [5]. The most efficient structures are those in which the stress flows along the axis, referred to in shell theory and as membrane stresses.

A chain that is suspended between two points and allowed to drape will form a catenary curve, which is an extremely important shape in structural applications (especially in masonry). A hanging chain is in pure tension, and incapable of resisting bending and shear forces. If the chain links are locked and the shape is flipped upright, the forces are reversed and in pure compression. This form is regularly applied to arches and domes.

Referring to Fig. 3, the catenary arch equation is given by Eqn. (1).

$$y = H - a \cosh\left(\frac{x}{a}\right) + a \qquad (1)$$

The value "*a*" is usually determined beforehand by knowing the basic dimensions *H* and *L*. "a" is solved by iterating Eqn. (2).

$$H = a \cosh\left(\frac{L}{a}\right) - a \qquad (2)$$

The length of the arch (*S*) is determined by integrating the curve of Fig. 3 from 0 to *x*, and multiplying by 2.

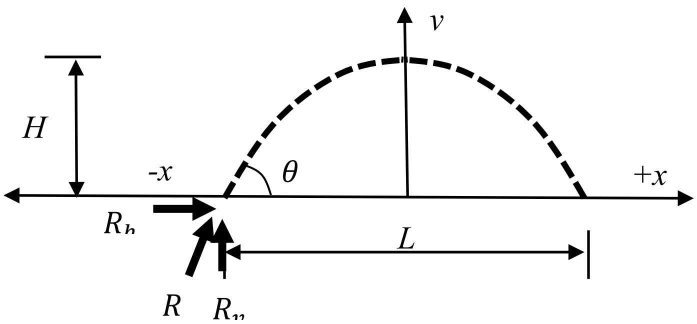

Fig. 3: Geometry of a catenary arch.

$$S = 2a\ sinh\left(\frac{L}{2a}\right) \tag{3}$$

The total weight of the arch (W) is given by Eqn. (4), where γ_m is unit weight of the arch material, h is the thickness and b is the width (or unit width) of the arch.

$$W = \ \gamma_m\ Shb \tag{4}$$

The vertical reaction at the base is therefore,

$$R_v = \ \frac{W}{2} \tag{5}$$

The slope at the base of the shell is determined by taking the derivative of the catenary equation,

$$\theta = tan^{-1}\left[\sinh\left(\frac{L}{2a}\right)\right] \tag{6}$$

Solving for the reaction that is tangent with the base of the catenary curve,

$$R = \ \frac{R_v}{sin\ \theta} \tag{7}$$

Therefore, the horizontal reaction at the base is equal to:

$$R_h = R\ cos\ \theta = \ \frac{W\cos\theta}{2\sin\theta} \tag{8}$$

In catenary arches, the horizontal reaction at any point in the arch is constant from apex to base. The stress at the apex of the arch ($y = H$) is expressed as,

$$N = \ \frac{R_h}{hb} = \ \frac{W\cos\theta}{2hb\sin\theta} \tag{9}$$

And at the base (y = 0),

$$N = \frac{R_h}{hb\sin\theta} = \frac{W}{2hb\sin\theta} \tag{10}$$

Assume a linear distribution of stress, and express the axial stress in terms of W.

$$N = \frac{W}{2hb\sin\theta}\left[\left(\frac{\cos\theta - 1}{H}\right)y + 1\right] \tag{11}$$

Eqn. 11 is the equation to determine the stress at any point in a catenary arch. The thrust-line, which is the resultant of the stress distribution, will be coincident with the axis of the catenary arch. This characteristic of the catenary arch is the reason why catenaries are free of bending moments and out-of-plane shear forces.

The profile of the arch is given in Fig. 4. The dashed line in the center of the profile is both the centroidal axis and the location of the thrust-line. The self-weight of the arch is usually the dominant force. Other forces, such as wind and temperature, will cause a deviation of the thrust-line profile. As long as the thrust-lines of the other loadings are kept within the middle third of the arch, the arch will remain in compression. However, the location of the other thrust-line patterns are determined from other analytical methods, such as the segmental equilibrium method [4].

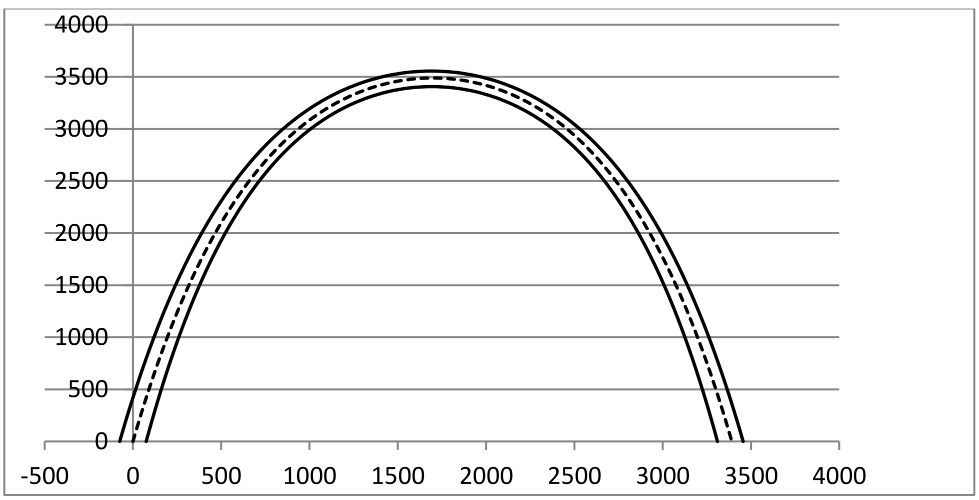

Fig. 4: Thrust-line and geometry of the catenary arch.

5. Construction of the Prototype House

The brickwork on the ground floor is dry-stacked. This system reduces the cost of the structure, since no mortar is used between the joints the construction time is reduced, as well as the number of labourers. Construction of the ground floor of the prototype home was rapid-the walls were built in under two days. Once the walls were completed, a rib and block slab was place on top of the ground floor walls. Construction consists of first laying the precast prestressed ribs at regular centres, and then placing blocks between the ribs. In-situ concrete is then placed on top of the ribs and blocks to form the slab. The blocks act as void formers, and do not add to the structural strength of the slab. The ribs and blocks covered the entire floor area, and therefore formwork is not required. The final step is to construct the vaults. It is critical that the vaults are constructed with a high degree of accuracy. The thickness of the vault walls is only 150 mm. All the thrust-lines must fall within a narrow band of 50 mm (middle third). To achieve this accuracy, two 1.5 m wide sections of formwork were fabricated from a steel frame and steel sheeting riveted to the outside. The formwork was designed for re-use, ease of handling and transportation. The formwork was propped into place and bricks were laid flat against the formwork, creating the vault.

As each section of the vault was completed, the formwork was dropped and slid into a new position. A cement slurry was then applied to the outside to smooth the surface and then painted with an acrylic waterproofing. The completed house is given in Fig. 5.

Fig. 5: Completed low-cost house using sustainable materials.

6. Conclusion

Catenary vaults are highly efficient structures. These shapes are the only forms that are void of bending and shear forces. By eliminating bending and shear, the stresses in the arch are drastically reduced (potentially by 90%). Since the quantity of materials is a function of the level of stress, it makes perfect sense to incorporate shells into low-cost housing designs—less construction material is required and an increased strength is achieved by lowering the stress levels.

The use of sustainable (CSEB) and recycled (building rubble) materials, and dry-stacked construction makes the proposed house a viable solution for low-cost housing.

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