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Controller Implementation of a Balancing Robot through a Dynamic Model with Acceleration Control Input

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Abstract **–** In this paper, we propose a new dynamic model of the balancing robot, and present the implementation method and results of the controller based on this model. The model equation of the system is derived through the Lagrangian approach. The model for tracking control of balancing robot is proposed, of which tracking reference is added as state, and implemented through LQR controller. The dynamic model proposed in this paper has the acceleration of robot body as the control input, unlike the conventional model which uses the torque or voltage as the control input. The acceleration from the LQR controller is transformed to the velocity reference of a motor and this value is applied to the real system through motor velocity controller. The motor velocity controller could be separated from the LQR controller making control reference. Therefore, the entire control could be easily implemented by changing only the motor velocity controller depending on the motor used in a balancing robot. The performance of the entire controller can be improved by controlling velocity accurately. To verify usefulness and reliability of this proposed model, two balancing robots, one of which uses step motors and the other of which uses DC motors, are implemented and results for balancing and tracking control are presented.

Keywords**:** balancing robot, Lagrange equation, LQR control, motor velocity control

1. Introduction

Balancing robots have drawn much attention and been undergoing development rapidly. Especially, it is applied to real life because it can move in a narrow and complicated urban environment and is eco-friendly. The 'Segway [1]' and 'PUMA [2]' are good examples of this application. In addition, balancing wheelchairs with better maneuver than conventional wheelchairs can play an important role in the field of personal transportation. Since such balancing robot is a nonlinear system with nonholonomic constraints and inherent unstability, several control methods have been proposed for effective control. Among them, the control of a balancing robot through PID control is most basic and general control method [4], [5], [6]. However, many trials and errors and much time are needed in the process of tuning gains, and whenever the system is changed, new gains should be found. This drawback can be removed through control based on the dynamic model. The appropriated model equation helps a controller of a robot to be implemented easily. So, many other control methods based on dynamic model have been proposed [7], [8], [9].

The dynamic model of the balancing robot proposed so far uses torque or voltage as the control input [10], [11], [12]. To implement the controller having torque reference, motor's electrical parameters and other sensors as well as encoders are required. Although the model having torque as control input could be transformed to other model using voltage control input, motor's electrical parameters are also required and the model has to be corrected for the controller to be implemented. The dynamic model proposed in this paper is derived by transforming the dynamic model having torque as control input derived using Lagrangian approach. It has a feature that the control input is robot body's acceleration. As the dynamic model consists of only mechanical parameters, the uncertainty of the model due to electrical parameters can be reduced. Since robot body's acceleration as input is transformed to motor's velocity reference, the system is controlled by motor velocity controller. It means that the controller of the robot can be configured by changing only the motor's velocity controller depending on the motor of the robot. It is easy to use various motors and even step motors that are difficult to control by voltage or torque.

In this paper, we introduce the control algorithm that is easy to apply to various motors, and the experimental results that are controlled by the controller based on this model. This paper is organized as follows: In Section. 2, kinematic model and dynamic model are derived. In Section. 3, the control algorithm based on the dynamic model is described. In Section. 4, the results of controlling the balancing robots with step motors and DC motors are presented. Finally, conclusions are presented in Section. 5.

2. Modeling of balancing robot

Two-wheeled balancing robot is mechanically unstable and has nonholonomic constraint. In this paper, considering this characteristic of balancing robot, kinematic constraint that the balancing robot should satisfy is presented and the dynamic model is derived through Lagrangian equation.

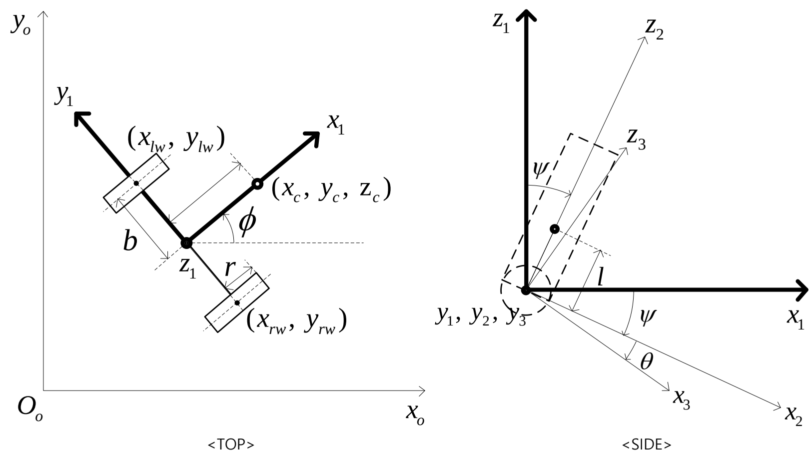

Fig. 1: The parameters and coordinates of a balancing robot.

The coordinate system used to derive the kinematics model and dynamic model is shown in Fig. 1. The frame $o_o x_o y_o z_o$ is an inertial reference frame. The frame $o_1 x_1 y_1 z_1$ is robot heading frame. The frame $o_2 x_2 y_2 z_2$ is robot body frame. The frame $o_3 x_3 y_3 z_3$ is wheel frame. The parameters are described in Table 1.

Table 1: The parameters of balancing robot.

ϕ	Robot heading angle	ψ	Tilt angle	θ	Wheel rotation angle
l	Distance between the center of gravity of the body	r	Radius of the wheel	b	Half-distance between wheels

2.1. Kinematics model

The kinematics obtained under the condition of zero lateral velocity and rolling without slip is

$$-\dot{x}\sin\phi + \dot{y}\cos\phi = 0, \tag{1}$$

$$\dot{x}\cos\phi + \dot{y}\sin\phi = \frac{r(\dot{\theta}_R + \dot{\theta}_L)}{2} + r\dot{\psi}, \tag{2}$$

$$\dot{\phi} = \frac{r(\dot{\theta}_R - \dot{\theta}_L)}{2b}. \tag{3}$$

The velocity of the center point of the axis connecting the two wheels is arranged by these equations with respect to the robot body frame. Among them, Eqs. (1) - (2) is the nonholonomic constraint and Eqs. (3) is the holonomic constraint. This equation is modified to obtain

$$\dot{q} = B(q)v = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \\ \dot{\psi} \\ \dot{\theta}_R \\ \dot{\theta}_L \end{bmatrix} = \begin{bmatrix} r\cos\phi & \frac{r\cos\phi}{2} & \frac{r\cos\phi}{2} \\ r\sin\phi & \frac{r\sin\phi}{2} & \frac{r\sin\phi}{2} \\ 0 & \frac{r}{2b} & -\frac{r}{2b} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\psi} \\ \dot{\theta}_R \\ \dot{\theta}_L \end{bmatrix}, \tag{4}$$

which takes $\dot{\psi}$, $\dot{\theta}_R$, $\dot{\theta}_L$ as inputs. The kinematics is used to transform the torque input of the dynamic model into an acceleration input, and to convert the acceleration reference to a motor velocity reference.

2.2. Dynamic model

In this paper, the dynamic model with the torque as the control input is obtained through the Lagrange equation and through this model, the dynamic model with the acceleration as the control input is derived. To obtain the Lagrange equation, the L (Lagrange function) is obtained as follows:

$$\begin{aligned} L &= K - P \\ &= K_{body,trans} + K_{wheel,trans} + K_{body,rot} + K_{wheel,rot} - P \\ &= \frac{1}{2}(m_c + 2m_w)(x^2 + y^2) + \frac{1}{2}(m_c l^2 \sin^2\psi + I_x \sin^2\psi + I_z \cos^2\psi + 2I_m + 2m_w b^2)\dot{\phi}^2 \ , \\ &+ \frac{1}{2}(m_c l^2 + I_y + 2I_w)\dot{\psi}^2 + \frac{1}{2}I_w(\dot{\theta}_R + \dot{\psi})^2 + \frac{1}{2}I_w(\dot{\theta}_L + \dot{\psi})^2 \\ &= \frac{1}{2}\dot{q}^T D(q)\dot{q} - m_c g(r + l\cos\psi). \end{aligned} \tag{5}$$

Now, the Lagrange equation is given by $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = \tau$ as

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + M^T(q)\lambda = E(q)\tau. \tag{6}$$

After substituting Eqs. (5) in Eqs. (7), and rearranging of equations, we obtain

$$\overline{D}\begin{bmatrix} \ddot{\psi} \\ \ddot{\theta}_R \\ \ddot{\theta}_L \end{bmatrix} + \begin{bmatrix} -m_c g l \psi \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -(\tau_R + \tau_L) \\ \tau_R \\ \tau_L \end{bmatrix}, \tag{7}$$

where $\overline{D}$ is given by

$$\overline{D}=\begin{bmatrix} mr^2+2rm_c l+I_\psi & \frac{mr^2}{2}+\frac{rm_c l}{2}+I_w & \frac{mr^2}{2}+\frac{rm_c l}{2}+I_w \\ \frac{mr^2}{2}+\frac{rm_c l}{2}+I_w & \frac{r^2(mb^2+I_z+2I_m+2m_w b^2)}{4b^2}+I_w & \frac{r^2(mb^2-I_z-2I_m-2m_w b^2)}{4b^2} \\ \frac{mr^2}{2}+\frac{rm_c l}{2}+I_w & \frac{r^2(mb^2-I_z-2I_m-2m_w b^2)}{4b^2} & \frac{r^2(mb^2+I_z+2I_m+2m_w b^2)}{4b^2}+I_w \end{bmatrix}. \tag{8}$$

For the derivation of a linearized model, we assumed $\psi\approx 0,\ \dot{\psi}\approx 0,\ \dot{\phi}\approx 0$, which means $\cos\psi\approx 1,\ \sin\psi\approx\psi$. Employing Eqs. (2) - (3), equations including the terms, $\ddot{\psi},\ \ddot{\theta}_R,\ \ddot{\theta}_L$, are now transformed into equations with terms, $\ddot{\psi},\ \dot{v},\ \dot{w}$. The rearranged equation is

$$\ddot{\psi}+\alpha_1\psi=\beta_1(\tau_R+\tau_L), \tag{9}$$

$$\dot{v}+\alpha_2\psi=\beta_2(\tau_R+\tau_L), \tag{10}$$

$$\dot{w}=\beta_3(\tau_R-\tau_L). \tag{11}$$

where:

$$\alpha_1=\frac{m_c gl(mr^2+2I_w)}{\gamma},\ \alpha_2=-\frac{(m_c rl)^2 g}{\gamma},\ \beta_1=\frac{2mr^2+4I_w+rm_c l}{\gamma},\ \beta_2=-\frac{r(2rm_c l+I_\psi-2I_w)}{\gamma},$$

$$\beta_3=\frac{rb}{r^2(I_z+2I_m+2m_w b^2)+2I_w b^2},\ \gamma=r^2({m_c}^2 l^2-I_\psi m-2I_\psi I_w+2I_w m)+4{I_w}^2$$

This equation is the dynamic model of balancing robot having torque input, which reveals the relationship between the torque of the left motor and the right motor and the other mechanical parameters. Through Eqs. (9) – (11), the dynamic model using robot body’s acceleration as control input is derived. By calculating $(9)\times\beta_2-(10)\times\beta_1$, we obtain

$$\ddot{\psi}+\frac{(\beta_2\alpha_1-\beta_1\alpha_2)}{\beta_2}\psi-\frac{\beta_1}{\beta_2}\dot{v}=0. \tag{12}$$

Then, state equation of Eqs. (12) is

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0\\ -\frac{(\beta_2\alpha_1-\beta_1\alpha_2)}{\beta_2} & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}+\begin{bmatrix}0\\ \frac{\beta_1}{\beta_2}\\ 0\\ 1\end{bmatrix}u, \tag{13}$$

where states denote $x_1 = \psi,\ x_2 = \dot{\psi},\ x_3 = s,\ x_4 = \dot{s} = v,\ \dot{v} = u$. Eqs. (12) is controllable dynamic model with acceleration as control input. The values of $\alpha_1,\ \alpha_2,\ \beta_1,\ \beta_2,\ \beta_3,\ \gamma$ used in Eqs. (9) - (11) consist of mechanical parameters such as moment of inertia, mass, and length. By following above process, the motor torque is removed and Eqs. (13) is the dynamic model expressed without an electrical parameters.

3. Implementation of balancing robot

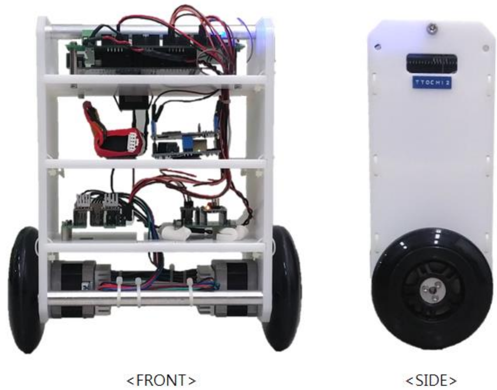

Fig. 2: Lab-built balancing robot.

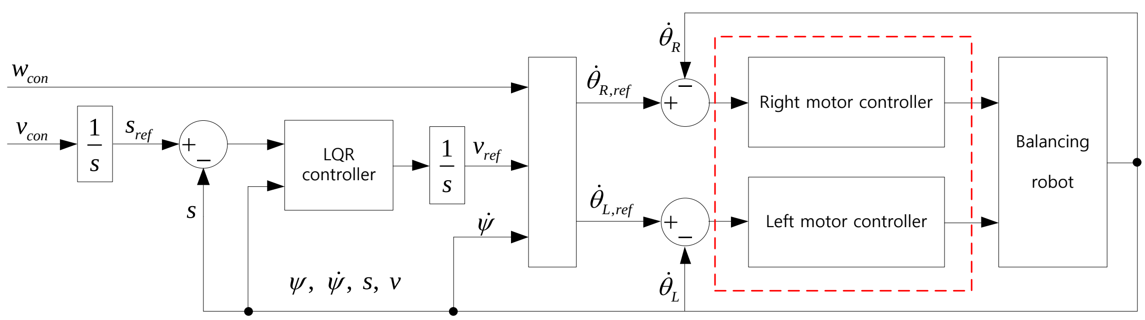

Fig. 3: Overall control algorithm.

The balancing robot designed to implement the controller based on dynamic model is shown in Fig. 2. Fig. 3 depicts the proposed control algorithm through a block diagram. The steering reference and tracking reference are the input of different blocks. For tracking control, the augmented system is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -\dfrac{(\beta_2\alpha_1 - \beta_1\alpha_2)}{\beta_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{\beta_1}{\beta_2} \\ 0 \\ 1 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} s_{ref}, \tag{14}$$

where s_{ref} is considered to be additional state, x_5 , in Eqs. (13). The acceleration reference obtained through LQR controller is $u = \dot{v} = k_1x_1 + k_2x_2 + k_3x_3 + k_4x_4 + k_5\int(s_{ref} - x_3)$. The cost function

$$J = \frac{1}{2}\int_0^\infty \left[e^T(t)Qe(t) + u^T(t)Ru(t) \right] dt \qquad (15)$$

is used for obtaining control gain, K [13]. Q and R in Eqs. (15) are symmetric positive definite matrices. The weighting matrices in Eqs. (15) are chosen to be $Q = \mathrm{daug}\left(0.02, \quad 0.0001, \quad 0.000001, \quad 0.0001, \quad 1\right)$ and $R = 0.001$. The control gain is given by $K = \left[22.7395 \quad 3.8311 \quad 32.9824 \quad 16.6589 \quad -33.2719\right]$. The motor's velocity reference is calculated with acceleration reference and steering reference as follows:

$$\dot{\theta}_{R,ref} = -\dot{\psi} + \frac{1}{r} v_{ref} + \frac{b}{r} w_{con}, \qquad (16)$$

$$\dot{\theta}_{L,ref} = -\dot{\psi} + \frac{1}{r} v_{ref} - \frac{b}{r} w_{con}, \qquad (17)$$

which are derived by Eqs. (2) - (3). Consequently, the control of balancing robot is implemented by velocity controller of motors used in the robot. The balancing robot can be implemented without motor's electrical parameters and other sensors except encoders, since dynamic model applied to LQR controller has only mechanical parameters and the system is controlled by motor velocity controllers. Additionally, motor controllers shown in dashed lines can be changed depending on the types of motors, such as DC motors, BLDC motors, PMSMs, and even step motors. It is known that step motors are difficult to control by changing torques or voltages. In this paper, two robots using DC motors and step motors are implemented respectively. The PI velocity controller is used in robot with DC motors and the robot with step motors is controlled by driving step depending on velocity reference.

4. Experiments and results

The balancing robot in this paper is controlled by LQR controller based on the dynamic model using acceleration as control input. To verify usefulness and reliability, the robots with step motors and DC motors are implemented and the results of balancing and tracking control are presented.

4.1. Balancing control

The acceleration control input is transformed to each motor's velocity reference in the controller of this paper. Virtually, since the robot is controlled through the internal motor velocity control, the performance of the velocity controller affects overall control results. These results are presented in experiments by driving different microstep resolution in step motor drivers and encoder resolution of DC motors. The displacements of ψ and the phase portrait are shown in Fig. 4, Fig. 5, and Fig. 6. Using step motors in the robot, the data of full-step, 1/2-step, 1/4-step, 1/8-step and 1/16-step is compared. In a full-step step motor, 200 steps rotate a step motor through $2\pi\ rad$. The more smaller microstep leads to better resolution. In experimental results, using full step, the range of ψ is from -1 to 1[deg], and the range is reduced to -0.2 to 0.2[deg] in using 1/16-step.

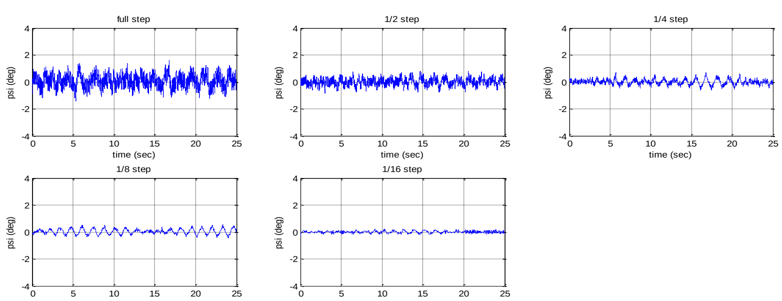

Fig. 4: ψ of the step motor robot.

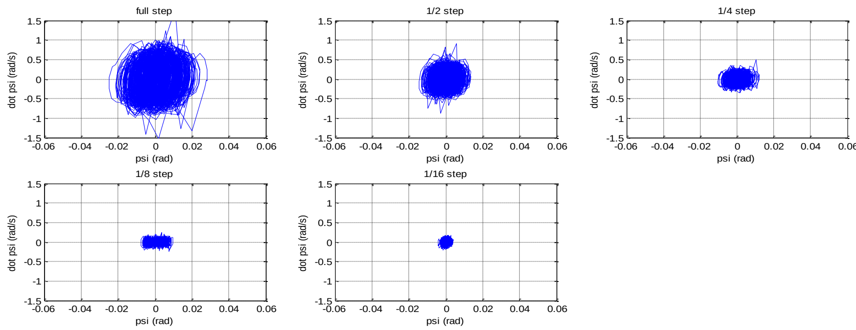

Fig. 5: The phase portrait of the step motor robot.

The range of ψ is from -2 to 2[deg] using DC motors with 64 PPR encoders, and from -1 to 1[deg] using 4096 PPR encoders. Since the DC motor have a gear box of 30:1, backlash nonlinearity has an impact on control performance. This nonlinearity causes the results with 4096PPR encoder to be same with the results using step motors of full step. Moreover, this backlash effect is seen clearly in a limit cycle of the phase portrait. The range of ψ and the size of the limit cycle get smaller as the resolution increases in both robots with step motors and DC motors. Consequently, this experiments present that it is applicable to both robots with step motors and DC motors although the control performance is different depending on the characteristic and resolution of motors.

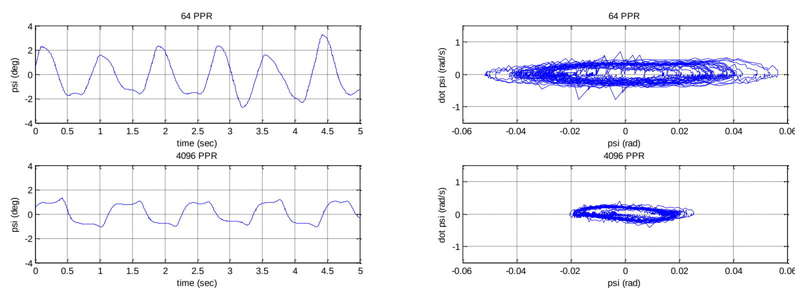

Fig. 6: Experimental data of a DC motor robot.

4.2. Tracking control

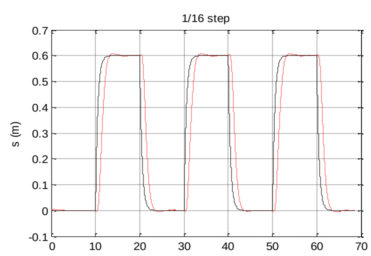

Fig. 7: tracking result.

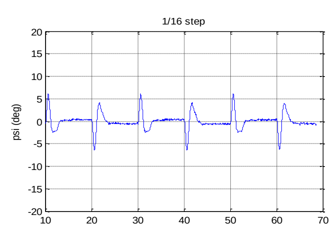

Fig. 8: ψ in tracking experiment.

The result of tracking control is presented in Fig. 7 and Fig. 8. The value of s, traveling distance of robot, depending on the value of s_{ref} is shown in Fig. 7. The value of s_{ref} is changed and the balancing robot moves forward and backward at intervals of 0.6 [m]. The value of s has undershoots caused by starting movement in steady state and overshoots caused by pausing movement, which causes about one second delay. The reason for this result is because the balancing robot's body is inclined forward or backward when movement started in upright position. This status is represented in Fig. 8 showing the range of ψ of the robot. In Fig. 7 and Fig. 8, it can be confirmed that the robot reaches the target point clearly and the value of ψ maintains 0[deg].

5. Conclusion

This paper proposed the new dynamic model of a balancing robot with acceleration input, and control method designed by this model. Since the robot body's acceleration obtained through LQR controller is transformed to the motor's velocity reference, the proposed method has a feature that the balancing robot is controlled through the velocity control of motor. The entire controller has two-part form, the LQR controller and the motor's velocity controller. The LQR controller based on the dynamic model consists of only mechanical parameters and motor's electrical parameters are also unnecessary in motor velocity controller. Since the model and controller do not depend on the motor used in the robot, motor velocity controller can be changed depending on the type of the motor. It means that more flexibility is provided in the selection and application of the motor and since in this paper, the control is implemented based on model, time and effort could be reduced compared with using PID controller. To verify reliability of this model and the controller, robots using step motors and DC motors were implemented respectively. The step motor could implement an accurate velocity by applying steps, so feedback loop was unnecessary in step motor's velocity control. The DC motor was controlled by a PI velocity controller. In the robot with step motors of 1/16-step, the range of ψ was from -0.2 to 0.2[deg], and in the robot using DC motors with 4096 PPR encoder, the range was from -1 to 1[deg]. Consequently, the result of balancing and tracking control for the balancing robot was verified although the experimental results were different depending on the characteristic and resolution of motors.

Acknowledgements

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