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# Pulsating Heat Pipe: Operation in Nonlinear Regime

**Alok Kumar[1], Nadeem Ahmed[1], Suneet Singh[1]**
[1]Department of Energy Science and Engineering,
Indian Institute of Technology Bombay, Mumbai-400076, India
17i170013@iitb.ac.in; suneet.singh@iitb.ac.in

***Abstract*** **-** The global attention toward miniaturization technologies puts compact electronic devices at the core of thermal management research. The high compact density electronic devices are very high heat-dissipating systems. The removal of this high heat flux is imperative to maintain the reliability and durability of these devices. Among many heat removal systems, Pulsating Heat Pipes (PHP) have shown both their standalone and hybrid utility. Their miniature size with wickless structure and performance in different operating conditions put them as a promising heat removing agent in small area applications, be it terrestrial or space. Despite their simple geometry, PHPs exhibit complex dynamical characteristics due to the multiphysics processes involved during the transient operation. The complex dynamics is not easy to explain while analyzing the system by the mathematical models, used to obtain operating characteristics. The interplay between the multiphysics interactions makes the system highly nonlinear, which is very sensitive to the initial conditions. Therefore, the fundamental problem lies in the understanding of the dynamics in the nonlinear regime. Nonlinear stability analysis has been carried out on a state-of-the-art model. The oscillatory behavior of the liquid slug is modeled similarly to the spring-mass system. The parameters related to several thermodynamic processes have been varied to capture the change in the dynamics using codimension one and two analyses. The system of non-linear differential equations, containing the conservation equations for the liquid slug and vapor plug separately, has been solved numerically using MATCONT. Bifurcation analysis shows the sudden changes in the dynamics while varying the parameters and the start-up operating conditions also match with the literature.

***Keywords*****:** Pulsating heat pipe, thermal management, energy conservation, nonlinear stability analysis

## 1. Introduction

Many recent technological developments, such as the miniaturization of electronic devices, compact packing, and intense data storage requirements, stressed the need for very high heat transfer technologies to ensure the operation of electronic devices without any premature thermal burnout [1]. Since the heat dissipation requirements have risen by thousand times at the chip level from a few $W/cm^2$ to thousands of $W/cm^2$. Conventional cooling systems like copper rods are unable to dissipate this excess heat and thus resulting in the deterioration of the surrounding components [2-3]. Therefore, both the reliability and the efficiency can be enhanced, if there is a technology that can dissipate this excess amount of heat efficiently. Since modern devices have greater compact densities, they are more prone to encounter such premature accidents and hence subjected to having a better thermal management mechanism installed. These heat-dissipating methods are called Cooling Technologies. Many such cooling technologies have been developed e.g. conduction, convection, radiant cooling, thermoelectric cooling, and liquid-cooled cold plates, etc. All these technologies have their own specified merits and demerits. They constitute better performances on specific applications [4]. On an individual level, each of this technology has its advantage for a specific application. These technologies can be used on a stand-alone basis or on a hybrid basis, where two or more such technologies are coupled with each other [5]. The literature shows that hybrid cooling technologies are more efficient than stand-alone [6]. One among such cooling technologies is Heat Pipe Technology (HPT), which has provided very good cooling solutions for the devices.

In due course of time, there has developed a hierarchy of heat pipes from thermosyphon to pulsating heat pipes (PHP) depending on the changing industrial needs. PHP is a meandering tube with U-shape turns and is partially filled with a working fluid. It can be closed-looped or open-looped based on the dissipating requirements. It has been proved that closed-looped PHPs are more efficient as compared to open-looped [7]. PHP, being a promising candidate for both terrestrial and space electronic applications, shows a complex dynamical phenomenon. Despite their simple geometry and relatively fewer design constraints, the physics involved in the operating mechanisms is quite complex which further limits its use in real-life applications. A focus has always been on the research related to the exploration of the real operating condition in order to use it with the least possible precision of error. However, major attention has been toward the design, start-up process, and working fluid selection. Therefore, there is a significant need to understand the operating conditions using nonlinear models.

## 2. Mathematical Modeling

The system has been modeled by considering a control volume consisting of a vapor plug squeezed between two consecutive liquid slugs. The conservation equations for this system have been derived and converted into simplified ordinary differential equations.

### 2.1. Assumptions

The following assumptions have been incorporated into the mathematical modeling.

1. Liquid and gas properties have been taken as constant throughout the process.
2. Vapour has been considered an ideal gas.
3. The heat transfer coefficient for the liquid to vapor and wall to liquid heat transfer is constant.
4. The temperature of the wall is considered constant.
5.

### 2.2. Energy Balance

Simultaneous exposure of the two ends of a PHP to a high and a low-temperature field leads to the occurrence of a chaotic or pulsating flow regime in the flow field. It makes the flow desirable to pulsations and creates instabilities in the flow field, which further enhances the heat transfer capability of the system [8]. Several mechanical problems related to these nonlinear pulsations may severely affect the overall performance of the PHPs. Therefore, the tradeoff between the nonlinear effects and mechanical problems should be thoroughly reviewed using some mathematical formulation. To do so, a mathematical model has been analyzed using non-linear analysis.

The system has been modeled using the conservation equations for the liquid slug and the vapor plug. While formulating the mathematical model, the processes like capillary and surface tension have been considered. This consideration of the mentioned processes makes the model nonlinear. Usually, the nonlinear models are solved using the linearization technique where all the nonlinear terms are either truncated or are linearised using Taylor series expansion. The present modeling is based on [1] and is very sensitive to the initial conditions because it has been solved without linearisation. The conservation equations have been written based on the schematic shown in Figure 1. The schematic shows that a vapor bubble is squeezed by two liquid slugs and enclosed by a thin liquid boundary of thickness δ. The heat flux has been provided on the walls and is transferred from the wall to liquid slug and liquid film, and from liquid slug and liquid film to vapor bubble.

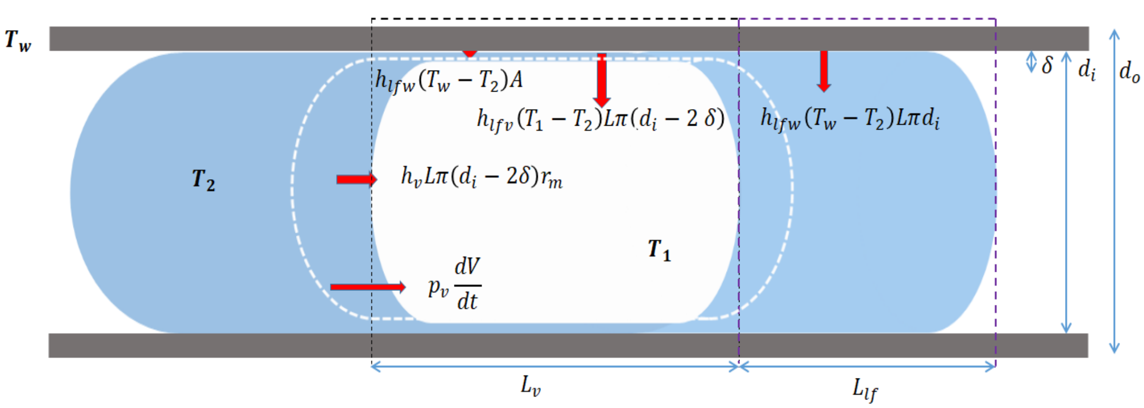

Figure 1: Schematic diagram of the system

The different modes of heat transfer have been shown by red arrows aligned with the direction of heat transfer (e.g., from wall to liquid film, liquid to vapor, etc.). As assumed, the wall temperature has been kept constant, $T_w$.

### 2.3. Time Evolution of the Vapor Plug and Liquid Slug Temperature

The temperature variation of the vapor bubble is dependent on three heat transfers, (1) from the liquid film to the vapor bubble, (2) energy transfer due to mass transfer, and (3) work done by the vapor bubble to expand. The energy conservation equation represented by Eq. (1) is a balance of the energy components. The expansion of the liquid film is negligible in comparison to that of the vapor bubble. Therefore, it has not been presented in the heat balance equation. **Error! Reference source not found.**, It has been shown that liquid film temperature is affected by heat transfer (1) from wall to liquid film, (2) from a liquid film to vapor bubble, and (3) mass transfer due to evaporation. The energy balance equation for the liquid film is given in Eq. (2).

$$m_v C_{vv} \frac{dT_1}{dt} = -h_{lfv}(T_1 - T_2)L\pi(d_i - 2\delta) - h_v L\pi(d_i - 2\delta)r_m - p_v \frac{dV}{dt} \tag{1}$$

$$m_f C_{vl} \frac{dT_2}{dt} = h_{lfw}(T_w - T_2)L\pi d_i + h_{lfv}(T_1 - T_2)L\pi\left(d_i - 2\,\delta\right) + h_v L\,\pi(d_i - 2\delta)r_m \tag{2}$$

### 2.4. Mass Conservation

The phase change due to the boiling of the liquid creates a mass transfer from liquid to vapor and vice versa in the case of condensation. Thus, the vapor flow rate depends on the interfacial mass transfer between both phases. A mass balance for the vapor bubble is represented by Eq. 3.

$$\frac{dm_v}{dt} = -\pi\left(d - 2\,\delta\right)r_m, \quad \text{where, } m_v = \rho_v V \tag{3}$$

$$\frac{dm_V}{dt} = \rho_v \frac{dV}{dt} + V\frac{d\rho_v}{dt} \quad \text{or,} \qquad \rho_v \frac{dV}{dt} \;=\; -\pi(d_i - 2\delta)r_m - r_v \pi d_i L_v \tag{4}$$

### 2.5. Oscillatory Position of the Liquid Slug

As the process of startup initiates in the core f the PHP, the liquid slugs start pulsating due to the pressure gradient which has been generated across both ends. A liquid slug experiences three force components as shown in the schematic, (1) gravitational, (2) shear stress, and (3) pressure drop. The mathematical terms for each of them are given below.

#### 2.5.1 Gravitational force

The gravitational force on a plug is equivalent to its weight force. In varying or microgravity also the same expression is valid with different values of the gravitational constant.

$$F_g = m_P g \tag{5}$$

#### 2.5.2 Wall shear stress

$$s_w = \frac{1}{2} C_f\, \rho_l\, v_p^2, \quad C_f = \{\frac{16}{Re}, \;\; Re < 1180 \tag{6}$$

Where, $C_f$ is the friction factor, and has been correlated using the Moody diagram.

#### 2.5.3 Pressure drop across the plug

The force exerted on the plug due to the pressure difference is simply the product of the surface area and the pressure drop.

$$F_p = \frac{\pi}{4} di^2(P_{v1} - P_{v2}) \tag{7}$$

The momentum conservation equation is presented in Eq. 8.

$$m_p \frac{d^2 x_p}{dt^2} = \frac{\pi}{4} di^2 (P_{v1} - P_{v2}) - \pi d_i L_P s_w + m_P g \tag{8}$$

Where $(P_{v1} - P_{v2})$ is the pressure difference between the trail and the front of a vapor bubble. The mass of the liquid slug can be written as: $m_p = \frac{\pi}{4}(d_i - 2\delta)^2 (L_o - x_p)\rho_l$, where $L_o \approx 25\, d_i$ and $\rho_l = 1000\; kg\; m^{-3}$.
The above system of equations can be represented by

$$\frac{d\vec{X}(t)}{dt} = f(\vec{X}(t), \vec{r}) \tag{9}$$

Where $\vec{X}(t)$ and $\vec{r}$ are the vectors of variables and the design parameters respectively, and can be represented as follows,

$$\overrightarrow{X}(t) = (T_1, T_2, m_v, x_p, x_p')$$
$$\vec{r} = (h_{lfv}, L, L_p, r_m, \delta, d_i) \tag{10}$$

Although the vector $\vec{r}$ represents all parameters including design and operation, only a few have been considered for the present study. The Jacobean matrix for the final system of the equation has been used to find the eigenvalues of the system by finding the roots of the characteristic equation on the given value of the fixed point. An algorithm for the same process is as follows:

| | |
|---|---|
| calculate $J = \boldsymbol{D_X}[f(\vec{X}(t), \vec{r})]$ | Step 1 |
| calculate $\vec{X}_0$ by putting $f(\vec{X}(t), \vec{r}) = 0$ for the given value of $\vec{r}$ | Step 2 |
| find **det**$\mid J - \lambda I \mid$ at the calculated $\vec{X}_0$ | Step 3 |
| Solve **det**$\mid J - \lambda I \mid$ for $\lambda$ | Step 4 |
| Repeat Step 2 to Step 4 for varying values of $\vec{r}$ | Step 5 |

where $\boldsymbol{D_X}$ represents the component-wise differentiation of the system. $\lambda$ is an unknown constant and $I$ is the $5 \times 5$ identity matrix.

## 3 Results and Discussion

The final system of equations consists of Eq. (1), (2), (3), and Eq. (8). The force balance, Eq. (8) has been converted into two first-order ODEs to solve the complete system of the equation of five ODEs. There have been several attempts to solve the mathematical model using different numerical methods. Parametric optimization using the firefly algorithm shows a range of the parametric value with some degree of variation, which matches that of the analytical values obtained from the simulations [5]. A similar model has been employed with the dynamic liquid film thickness with the brute force simulation to identify the temporal characteristics [9]. In the final system of equations, the momentum balance equation consists of a nonlinear term with negative power ($m_P^{-1}$ consists of $x_p^{-1}$). The negative nonlinearity often leads to asymptotic failure of the system. As value of $x_p$ is approaches close to $L_o$, acceleration of the slug tends to infinity, which can be regarded as a limitation of the model. The nonlinear systems with negative powers are highly adaptive to the dependence on the initial conditions. Thus, finding exact equilibrium is crucial for any solver at varying values of the parameters.

### 3.1 Initial Conditions

The initial conditions are given as:
Initial temperatures $T_1 = T_2 = 20°C$, Wall temperature $T_w = 40°C$,

the vapor flow rate is calculated using the ideal gas equations $m_v = 3.95 \times 10^{-5}\ kg\ s^{-1}$, the initial displacement of the liquid slug $x_p = 0.001\ m$ and initial velocity $v_p = 0\ ms^{-1}, R = 8.31,\ C_{vv} = 1800 \frac{J}{kg} C,\ C_{vl} = 1900 \frac{J}{kg} C,$ $L = 0.18\ m,\ d_i = 3.3 \times 10^{-3}\ m,\ \delta = 2.5 \times 10^{-5}\ m, L_p = 0.01\ m,\ h_{lfw} = 1000\ W/m^2C,\ h_v = 10\ W/m^2.$

### 3.2 Temporal Evolution

The eigenvalue analysis shows that the pair of complex eigenvalues does not diminish for different ICs by varying the liquid plug temperature. On the other hand, if the vapor temperature is varied within the feasible range, the pair of the complex eigenvalues disappears, which shows that once the liquid plug reaches the condenser side after getting vaporized, it starts oscillating with the process of condensation. In other words, the vapor bubble on the condenser side starts oscillating while getting condensed. These sustained oscillations are desired in such heat transfer devices to achieve greater heat transfer capacity. At a certain set of parameter values, these systems may behave in chaotic nature also. The study of chaotic dynamics has been planned as future work using different methods.

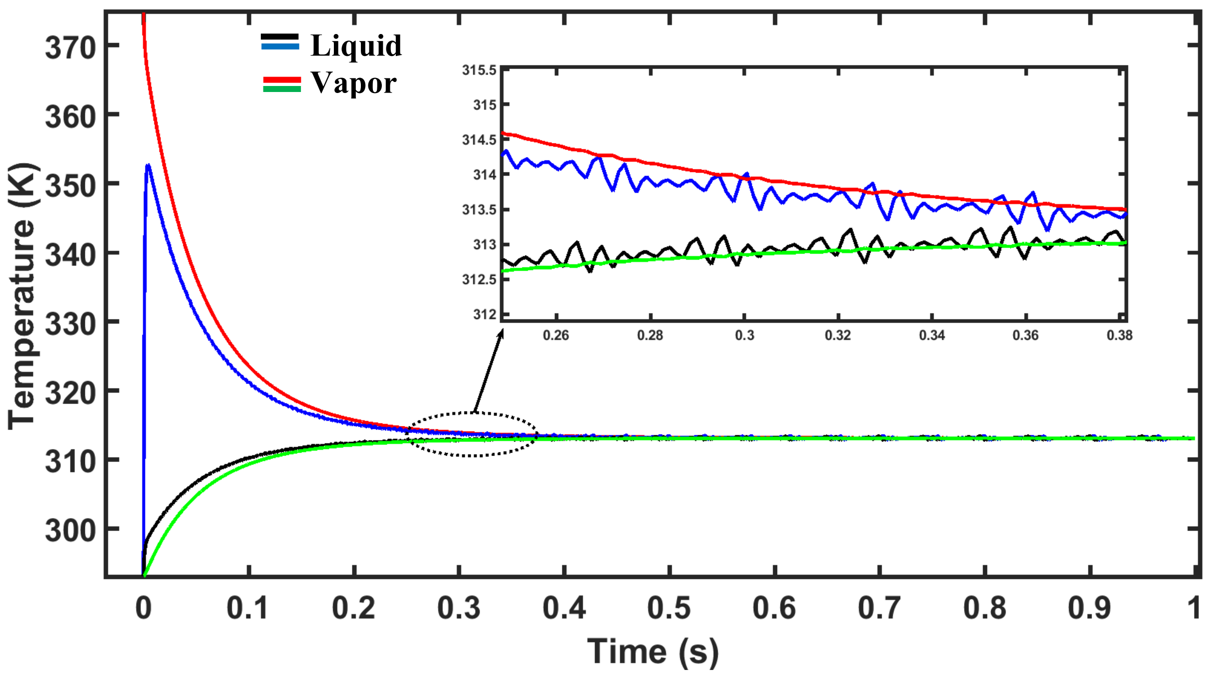

Figure 2: Temperature evolution with time at different initial conditions

The liquid slugs and vapor bubbles, which filled the volume of the adiabatic section of the heat pipe, pulsate around their mean position after moving away from their respective positions in the evaporative section. Both the temperatures of the slug and that of the plug, converge to the wall temperature. Therefore, The wall temperature ($T_w = 40\ ^oC$) is a stable attractor for the thermal profiles. The strength of the attraction can be analyzed from the size of the perturbation. The system is perturbed from the first IC ($T_1 = 293.15\ K$, $T_2 = 293.15\ K$) to the other ($T_1 = 373.15\ K$, $T_2 = 293.15\ K$). In the second IC, vapor temperature is very high as compared to that of the liquid. The heat transfer from the plug to the slug is initially very high from the vapor to liquid; therefore a spike in the slug temperature can be seen in Figure 2. It can be observed that in both the ICs, the oscillations are corresponding to the liquid slugs only. The shape of liquid temperature profiles may not look like an oscillatory behavior in Figure 2. Therefore, a phase portrait has been plotted for the system.

### 3.3 Phase-portrait

The phase portrait between the velocity and position in an oscillatory motion is always of elliptic shape. Based on the characteristics i.e., amplitude, phase, and frequency of the oscillations, the shape and size of the major and minor axis of the ellipse vary. The phase portrait shown in Figure 3 displays the oscillatory behavior of the liquid slug at a low heat transfer coefficient (HTC) ($h_{lfv} = 200\, W/m^2K$). Once the slug reaches the condenser section, its velocity drops drastically and subsequently it goes down to negative which indicates its departure once it gets condensed towards the evaporator section. This cycle continues throughout the normal operating conditions. The steep gradient at x = 0.0825 $m$ shows the highest frequency of the oscillations at the condensation site. Also, a parametric study, where the HTC is being varied up to $2500\, W/m^2K$, has been done by [10] and it has been noticed that the shape of the ellipse gets regular for a high HTC value, denoting a proper oscillatory pattern of the position.

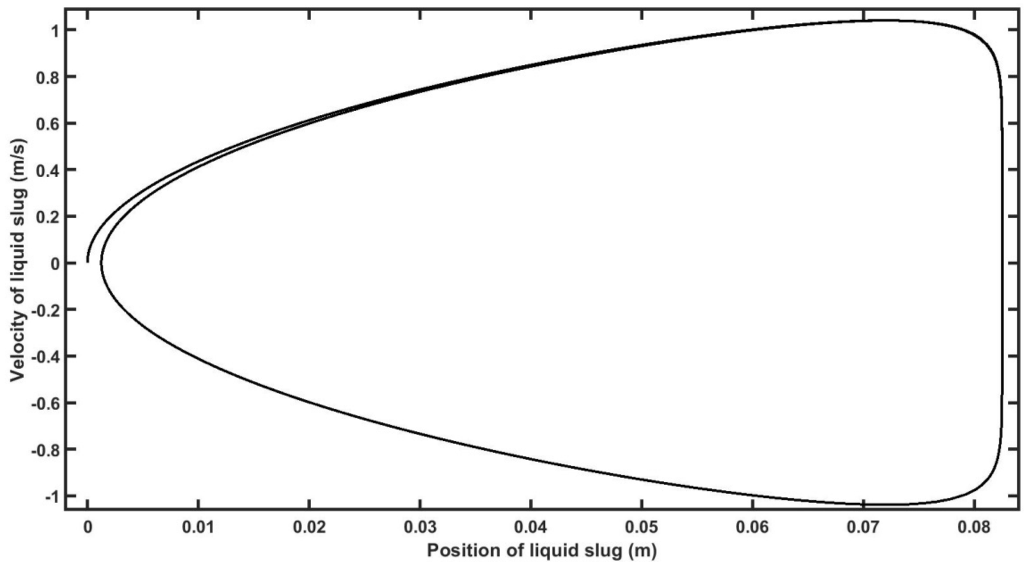


Figure 3: Phase-portrait of position and velocity of a liquid slug at $h_{lfv} = 200\ \mathrm{W/m^2K}$

### 3.4 Mass flow rate

Similarly, mass flow rate (m) decreases with time for the same values of the parameters as shown in Figure 4. Due to strong dependence on the initial conditions, choosing a feasible fixed operating point is a cumbersome task. For two

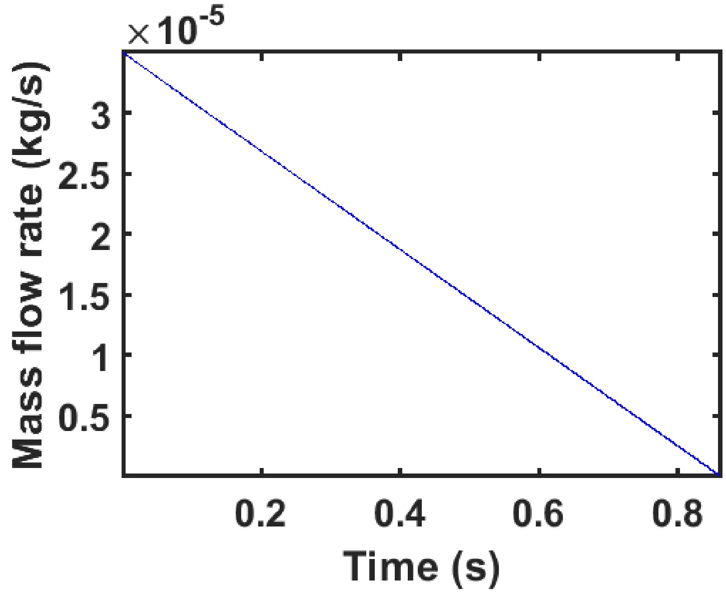


Figure 4: Time evolution of mass flow rate

slightly different initial conditions, the mass flow rate converges to two completely different flow rates. This indicates that the startup condition of the system does not depend only on the applied heat flux but also on the flow rate which is decided by the value of the filled ratio. In the present case, the initial condition has been chosen from the feasible set of the parameter values because a start-up analysis of the present model has been done by [11]. Thus, the decrease in the flow rate shows that the rate of phase change is also going to decrease with time, which further shows a reduction in the thermal gradient between the liquid and the vapor.

### 3.5 Thermal Equilibria

Furthermore, the presence of sustained oscillations has been confirmed by [12]. But, thermal equilibrium is established in the present system has been shown in Figure 5, which contradicts the previous argument. Therefore, there is a need to consider the mathematical terms for other multiphysics phenomena like variable thickness of the liquid film and actual wall temperature profile in consideration while modeling the system. The thermal equilibrium shown in Figure 5 indicates that the system becomes thermally inactive. The liquid and vapor temperature is approaching the wall temperature,

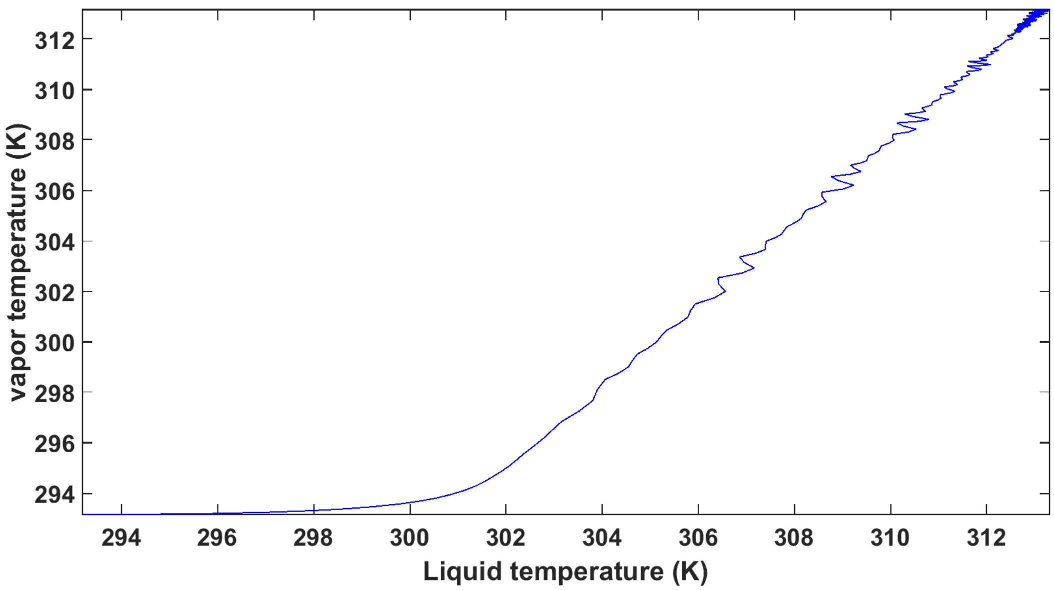


Figure 5: Phase portrait of the liquid slug temperature and the vapor plug temperature showing the thermal equilibria

meaning that any further mode of heat transfer is blocked between the three. It reduces the overall heat transfer capacity of the system. Therefore, a model which consists of the mentioned phenomenological attributes may capture the real phenomena.

## 4 Conclusion

A preliminary study of a state-of-the-art model for PHP has been done with the perspective of operation in the nonlinear regime. It has been observed that the current model captures well-established thermal equilibria in the flow field which further motivates one to consider the system without major assumptions. Both the temporal evolution and the phase-portrait shows that system operates with oscillatory behavior. The liquid temperature also approaches the wall temperature in an oscillatory pattern. The modeling equivalent to the spring-mass system provides a gist of the contraction and expansion of the vapor bubbles.

In the future, the study can be extended to higher-order nonlinear analysis. Codimension-1 and codimension-2 bifurcation analysis can be performed once the convergence of the equilibrium point is established. The interaction between the bifurcations obtained from codimension-1 and codimension-2 analysis will provide a gist of the actual operating dynamics. A mathematical model with less relaxed assumptions can be taken for analysis which will further revamp the quality of the conclusions.

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