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Entropy Generation Analysis for Heat Exchanger Operation Subject to Step Changes in Inlet Conditions

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Abstract - In this paper, the techniques for predicting entropy generation rates in steady heat exchanger operation are extended to include cases in which step changes in inlet conditions of one of the fluids occur. An alternate set of dimensionless parameters is suggested in the formulation such that maintaining a constant rate of transfer before and after the step change occurs is more easily accounted for. Results show that contrary to the traditional advice of minimizing entropy generation by balancing the heat capacities of the two fluids, entropy generation can both increase and decrease when deviating from nominal conditions with the accompanying values of the fluids' heat capacities either converging or diverging. Nonetheless, it is argued that a heat exchanger operating at nominal conditions close to balanced heat capacities represents a reasonable benchmark for heat exchanger design.

***Keywords*:** Entropy generation, Transient Heat Exchanger Analysis

1. Introduction

As technological demands increase across a range of sectors, answering the question of how optimally to manage thermal energy is becoming increasingly important. A specific example of this is in vehicle energy management, where waste heat must be dissipated due to heating and air conditioning demands and vehicle electronics [1-2]. Waste heat removal in vehicles is typically accomplished in part through forced internal convection in heat exchangers. Optimally managing the heat exchanger's performance can be particularly difficult due to the transient nature of the on-board heat dissipation demands. Moreover, the heat exchangers themselves are components in a larger thermal management system that may integrate with other subsystems such as the propulsion system in an aircraft. Fortunately, it is well known that the second law of thermodynamics, and the notion of entropy generation minimization, can be used to quantify the best theoretical performance attainable for a range of integrated thermodynamic systems. Entropy generation minimization is particularly useful in vehicle thermal management applications where it can be leveraged as a universal metric to weigh the impact of a given design decision on various subsystems that must operate concurrently.

In the literature, design and analysis of heat exchangers is often considered only in a static sense. In most cases, researchers derive an overall heat transfer coefficient for a given heat exchanger and use steady-state assumptions to impose a desired heat transfer rate as an operational constraint. Existing works usually consider analyzing a single aspect of heat exchanger design or performance such as physical mass or footprint [3] or economic cost [4]. While this type of work is valuable for optimizing some aspects of steady-state heat exchanger performance, it does not consider the *transient* behavior which dictates performance in applications such as vehicle thermal management, nor does it consider the invaluable notion of minimizing entropy generation. Moreover, much of the research that does consider transient modeling of heat exchangers does so in the context of synthesizing feedback controllers or analyzing dynamic time constants to inform heat exchanger operation rather than looking at entropy generation [5-6].

Most work examining entropy generation effects in generic heat exchangers focuses on steady conditions, such as that described in Bejan [7]. A smaller number of works have examined transient entropy generation in heat exchangers using models of differing fidelity. In [8], Pizzolato et al. used high-fidelity models to analyze transient entropy generation in hot water storage tanks. More recently, Nash and Jain researched transient entropy generation minimization as a control metric in an aircraft thermal management system [9]. However, heat exchangers within the modeling framework were treated as a single thermal capacitance, thereby making it difficult to leverage the model to inform meaningful design decisions.

To the authors' knowledge, absent from the literature is a modeling framework that incorporates transient entropy generation to characterize the effects of step changes to key operational inlet parameters on heat exchanger performance in a physically meaningful manner. For example, maintaining desired heat removal rates during transient operation of heat exchangers involves commanding step changes in the mass flow rate of one or both heat exchanger fluids. Using such a model to make design decisions also requires that parameters in the model be physically meaningful; in other words, the model must characterize second-law performance effects using design parameters that are physically meaningful, such as the number of transfer units (*NTU*), heat exchanger effectiveness, or others where appropriate.

In this work, we present a transient modeling analysis of entropy generation for forced internal convection in heat exchangers. Our model is parameterized to characterize transients in entropy generation due to step changes in inlet conditions such as mass flow rates and can be used to inform design decisions for physical heat exchanger sizing variables to manage thermal energy more optimally for a range of applications such as vehicle heat removal.

2. Basic Equations

Figure 1 shows a schematic diagram of a counter flow heat exchanger along with the temperature distribution of the two fluids as a function of position for steady-state operation. Here it is assumed that hot and cold fluids are Fluid 1 and Fluid 2, respectively. Both fluids are considered to be incompressible with constant specific heats, and the pressure drops are assumed negligible.

With the aforementioned assumptions, conservation of energy applied to the two fluids results in

$$\dot{Q} = (\dot{m}c_p)_1\left(T_{1,in} - T_{1,out}\right) = (\dot{m}c_p)_2\left(T_{2,out} - T_{2,in}\right). \tag{1}$$

The second law of thermodynamics applied to the entire heat exchanger gives the rate of entropy generation to be

$$\dot{S}_{gen} = (\dot{m}c_p)_1 \ln\left(T_{1,out}/T_{1,in}\right) + (\dot{m}c_p)_2 \ln\left(T_{2,out}/T_{2,in}\right). \tag{2}$$

The rate of heat transfer can also be put in terms of the inlet fluid temperatures only,

$$\dot{Q} = \varepsilon(\dot{m}c_p)_{min}\left(T_{1,in} - T_{2,in}\right) \tag{3}$$

where ε is the heat exchanger effectiveness and $(\dot{m}c_p)_{min}$ is either $(\dot{m}c_p)_2$ or $(\dot{m}c_p)_1$, whichever is smaller. The effectiveness in turn is a function of the number of transfer units, $NTU=UA/(\dot{m}c_{p,min})$ and the ratio of heat capacities of the two fluids, $C = \dot{m}c_{p,min}/\dot{m}c_{p,max}$:

$$\varepsilon = f(NTU, C). \tag{4}$$

Effectiveness approaches unity in the limit of a counter flow heat exchanger with infinite area [10].

3. Entropy Generation Trends for Steady Operation

After much manipulation, Eqs. (1), (2), and (3) can be arranged to yield

$$N_S = \frac{\dot{S}_{gen}}{(\dot{m}c_p)_2} = \ln\left\{\left[1 + \varepsilon C\left(\frac{T_{2,in}}{T_{1,in}} - 1\right)\right]^{1/C}\left[1 + \varepsilon\left(\frac{T_{1,in}}{T_{2,in}} - 1\right)\right]\right\}, \tag{5}$$

where N_S is the dimensionless entropy generation for the heat exchanger. Here it has been assumed that Fluid 2 has the smaller heat capacity so that $C = \dot{m}c_{p,2}/\dot{m}c_{p,1}$. Equation (5) is of particular utility, as it shows that the entropy generation for a heat exchanger is a function of three parameters, namely, the ratio of the inlet fluid temperatures, the ratio of the fluid heat capacities, and the heat exchanger effectiveness.

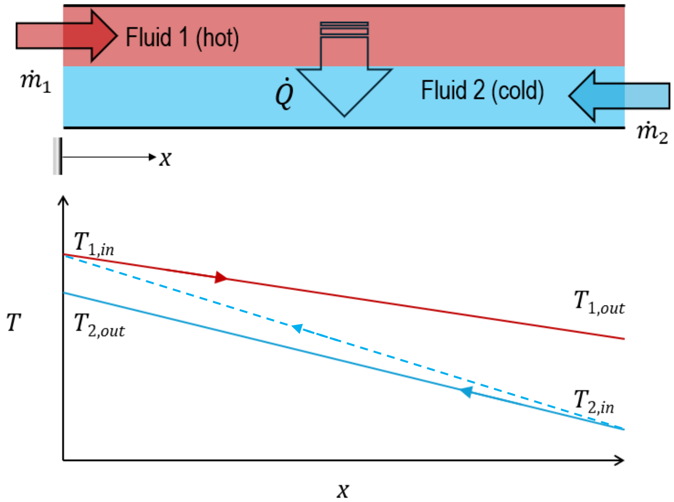

Fig. 1: A counterflow heat exchanger with temperature changes as a function of heat exchanger location. The dashed blue line corresponds to $\varepsilon = 1$ for which $T_{2,out} = T_{1,in}$ when the cold fluid has the smaller value of $\dot{m}c_p$.

In the limiting case of ε=1, Eq. (5) becomes

$$N_S = \frac{\dot{S}_{gen}}{(\dot{m}c_p)_2} = \ln\left\{\left[1 + C\left(\frac{T_{2,in}}{T_{1,in}} - 1\right)\right]^{1/C}\left[\frac{T_{1,in}}{T_{2,in}}\right]\right\}, \tag{6}$$

and the entropy generation becomes a function of only C and $T_{1,in}/T_{2,in}$. Figure 2 shows this relation.

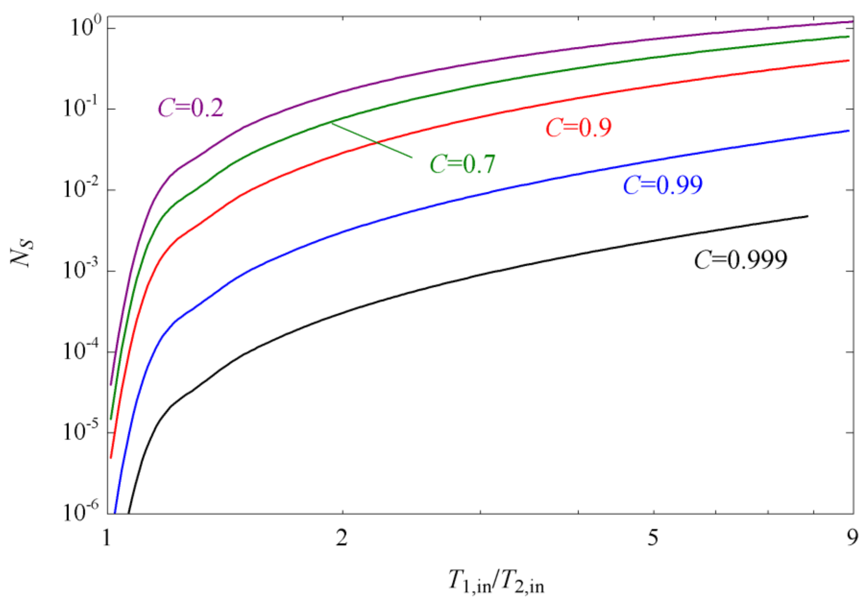

Fig. 2: Entropy generation trends for heat exchangers with ε=1.

Bejan has labelled the irreversibility due to values of C less than unity as a flow imbalance irreversibility [7]. A clear piece of advice stemming from Fig. 2 is that when the fluid inlet temperatures are fixed, the values of the heat capacities the two fluids should be made as close to one another as possible so that $C = \dot{m}c_{p,min}/\dot{m}c_{p,max} \approx 1$. This may seem somewhat counterintuitive, since higher values effectiveness result when C approaches zero [10]. Hence, the assumption of $\varepsilon = 1$ in Eq. (6) may not be a reasonable modelling assumption except in cases of very large values of overall heat transfer coefficient and/or heat exchanger area.

In light of this fact, nonideal heat exchangers can be modelled by incorporating the effectiveness-NTU relation appropriate to the heat exchanger at hand. For a double pipe counter flow heat exchanger, this is given by

$$\varepsilon = \frac{1 - \exp[1-(1-C)]}{1 - C\exp[1-(1-C)]}. \tag{7}$$

Figure 3 gives the entropy generation trends for such a heat exchanger using Eqs. (5) and (7).

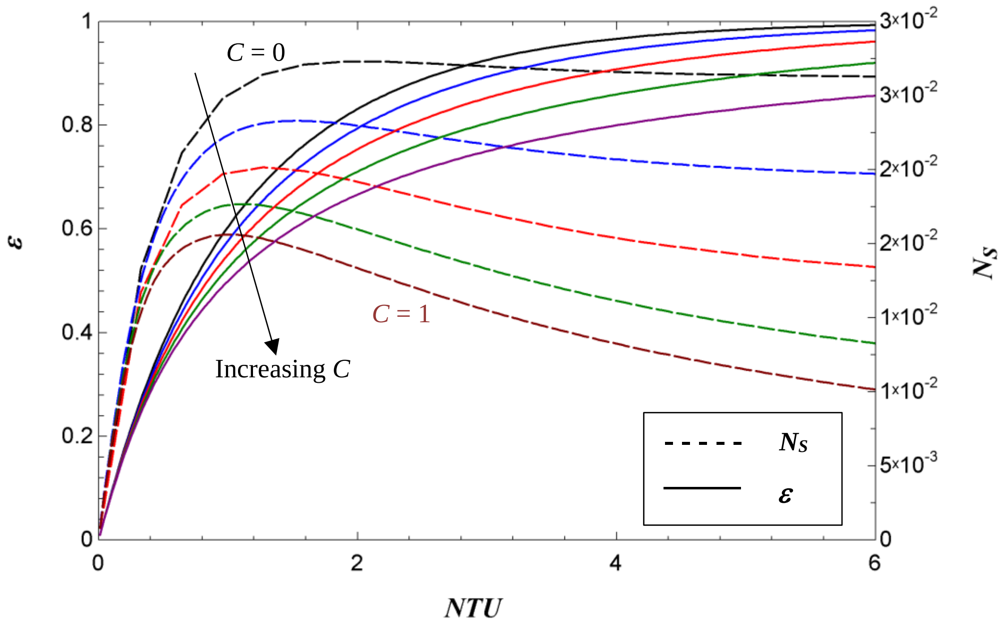

Fig. 3: Effectiveness and entropy generation for counterflow double pipe exchanger with $T_{1,in}/T_{2,in} = 3.3$.

Of note in Fig. 3 is that although effectiveness continues to rise monotonically with larger NTU, the entropy generation reaches a local maximum. (It can be shown that this maximum occurs at a value of $NTU = 1/(1+C)$, regardless of heat exchanger type [11].) Thus, general advice as to what combination of ε and C are appropriate for minimizing entropy generation is not always straightforward. Nonetheless, it does seem clear that larger values of both NTU and C result in smaller entropy generation, even if it comes at the cost of a less effective, and therefore larger heat exchanger.

4. Changes in Entropy Generation Due to Step Changes in Inlet Temperature

In many applications, a required rate of heat removal must be maintained when changes in the inlet conditions of one or both fluids occur. Assuming these changes occur in a step-like fashion, the preceding analysis can still be applied to find the new rate of entropy generation after the new steady operation has been achieved. Changing the inlet conditions of just one fluid while keeping the rate of heat transfer constant, however, will result in *all* of the dimensionless parameters changing simultaneously; that is, ε, C, and $T_{1,in}/T_{2,in}$ will all take on new values. In addition, those parameters are strongly coupled. Hence, an alternate formulation to that given in Eq. (5) is warranted.

4.1 Entropy Generation Formulation for Constant Heat Transfer

In cases where the rate of heat transfer is to be held constant, it is useful to express the rate of entropy generation with heat transfer appearing explicitly in the formulation. To that end, Eqs. (1) and (2) can be rearranged as

$$N_S = \frac{\dot{S}_{gen}}{(\dot{m}c_p)_2} = \ln\left\{\left[1 - \frac{\dot{Q}}{(\dot{m}c_p)_1 T_{1,in}}\right]^{1/\omega}\left[1 + \frac{\dot{Q}}{(\dot{m}c_p)_2 T_{2,in}}\right]\right\}, \tag{8}$$

where $\omega = \dot{m}c_{p,2}/\dot{m}c_{p,1}$. This notation is used to avoid ambiguity, since ω may be equal to either C or $1/C$, depending on which fluid has the smaller heat capacity $(\dot{m}c_p)_{min}$. Furthermore, for large enough step changes in inlet conditions it is possible for the fluid corresponding to $(\dot{m}c_p)_{min}$ to change from Fluid 1 to Fluid 2 or vice versa.

Like Eq. (5), Eq. (8) shows us that dimensionless entropy generation entails three degrees of freedom. The role of the rate of the fluid heat capacities still appears in the guise of ω, whereas ε and $T_{1,in}/T_{2,in}$, parameters more related to heat exchanger analysis, have been replaced by two similar looking dimensionless groups relating the heat transfer to the inlet conditions of the two fluids.

4.2 Case Study

The additional constraints of constant heat transfer and constant inlet conditions of one of the two fluids brings the number of degrees of freedom in Eq. (8) (or Eq. (5) for that matter) to one. Such a constraint may correspond to time varying conditions in which the inlet temperature of one fluid is a variable and its flowrate changes accordingly in order to maintain a desired heat transfer rate.

In order to explore the trends associated with such step changes, a double pipe, counterflow heat exchanger with the following attributes is explored. The configuration of the heat exchanger is shown in Fig. 4 with the specific values at nominal conditions given in Table 1. For the conditions given, the total rate of heat transfer between the hot and cold fluids is $\dot{Q}$ = 49,400 W with a dimensionless entropy generation of $N_S = 24.9\times10^{-3}$.

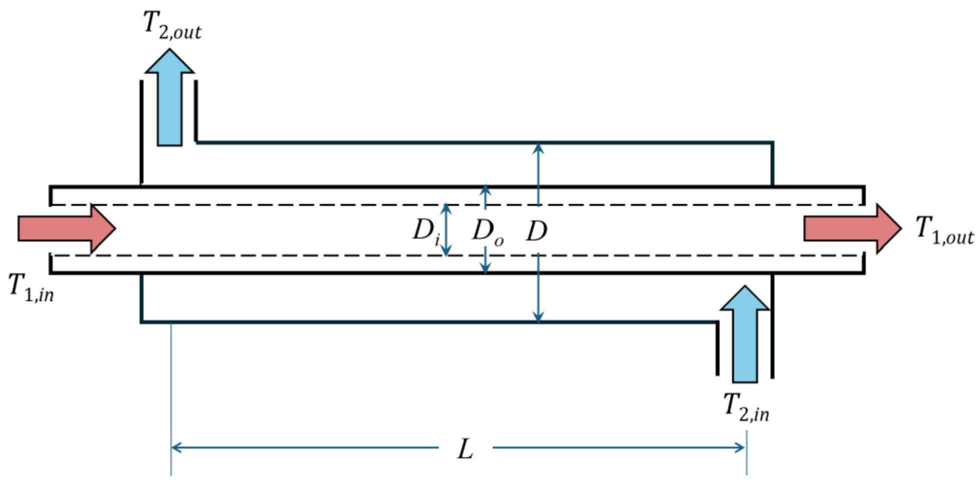

Fig. 4: Counter flow double pipe heat exchanger used in case study. For the values of the parameters in Table 1, the rate of heat transfer is $\dot{Q}$ = 49,400 W.

Table 1: Values of parameters for the counterflow heat exchanger in Fig. 4 at nominal conditions.

	T_{in} [K]	T_{out} [K]	$\dot{m}$ [kg/s]	c_p [J/kg-K]
Fluid 1, hot (C_3H_8, propane)	423	313	0.200	2247
Fluid 2, cold (Water)	283	334	0.233	4195
Geometry	D_i = 2.4 cm	D_o = 2.5 cm	D = 4.0 cm	L = 13.4 m

If the inlet temperature of the hot fluid T_1 is to change, the total heat transfer can be maintained by adjusting the flowrate $\dot{m}_1$ accordingly. The relationship between the variables is constrained not just by the conservation of energy as represented in Eq. (1), but by the physical size of the heat exchanger and the changing convective heat transfer coefficient of the hot fluid. The additional equations needed come from standard heat exchanger analysis and are given by Eqs. (9) and (10).

$$\dot{Q} = (UA)_2 \frac{(T_{1,in} - T_{2,out}) - (T_{1,out} - T_{2,in})}{\ln\left[(T_{1,in} - T_{2,out})/(T_{1,out} - T_{2,in})\right]} \tag{9}$$

$$U_2 = \left(\frac{D_o}{D_i h_1} + \frac{1}{h_2}\right)^{-1} \tag{10}$$

The convective heat transfer coefficients in Eq. (10) are found using the relation given by Dittus-Boelter [12],

$$Nu = \frac{hD}{k} = 0.023 Re^{0.8} Pr^{n}, \tag{11}$$

where *Re* is Reynolds number, *Pr* is Prandtl number, and the exponent n = 0.3 for cooling and 0.4 for heating.

Figures 5 and 6 show the entropy generation trends for inlet conditions departing from the nominal values given in Table 1. Lowering $T_{1,in}$ results the higher $\dot{m}_1$, with the lower values of $T_{1,in}$ outpacing the increased flowrate so that $\dot{Q}/(\dot{m}c_p)_1 T_{1,in}$ decreases. The entropy generation thus decreases. The opposite trend is true for increasing $T_{1,in}$.

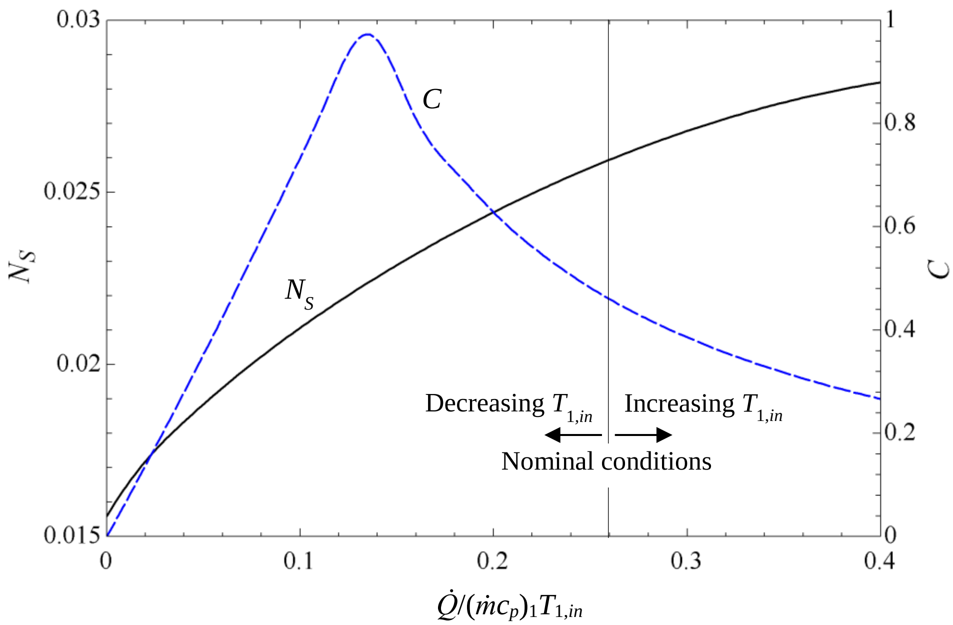

Fig. 5: Entropy generation and effectiveness trends for variable hot fluid inlet conditions. The maximum value of C is 1 occurring at $\dot{Q}/(\dot{m}c_p)_1 T_{1,in}$ = 0.132.

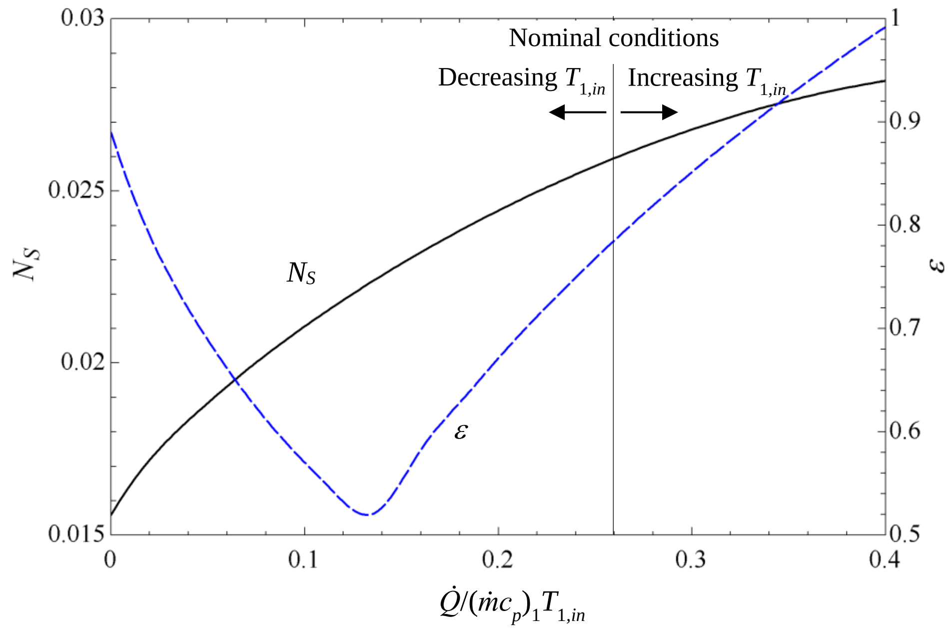

Fig. 6: Entropy generation and effectiveness trends for variable hot fluid inlet conditions. The minimum value of ε is 0.512 occurring at $\dot{Q}/(\dot{m}c_\text{p})_1 T_{1,in} = 0.132$.

Figures 5 and 6 also show the values of C and ε, respectively. The figures show that the rule of thumb that a value of C approaching unity should decrease entropy generation appears not always to hold in the case of constant heat transfer rate. Up to a critical value of $\dot{Q}/(\dot{m}c_p)_1 T_{1,in}$ of 0.132, larger C values increase entropy generation and effectiveness falls. Past this value, C decreases and entropy continues to rise as normally expected, with perhaps the condolence that effectiveness now increases.

In both cases, it is important to note that the critical value of $\dot{Q}/(\dot{m}c_p)_1 T_{1,in}$ occurs where $C = 1$ and identity of the fluid for which $\dot{m}c_p = \dot{m}c_{p,min}$ switches from the cold fluid to the hot fluid. That ε takes on its minimum value at $C = 1$ is exactly what standard heat exchanger analysis tells us, namely, that effectiveness approaches one as C goes to zero.

As far as entropy generation is concerned, however, the lack of a local minimum or maximum makes the choice of a optimal operating point less clear. One suggestion could be that designing a heat exchanger to operate under conditions for which $C \approx 1$ leads to a modicum of entropy generation (at least of the case study at hand) where deviations from that condition may lead to either larger or smaller entropy generations, but would always result in a larger effectiveness.

One should exercise caution when generalizing from this case study, of course, as the trends are specific for the conditions stated. For example, we could imagine that for a different hot fluid with a lower value of c_p decreases in $T_{1,in}$ could lead to increases in $\dot{Q}/(\dot{m}c_p)_1 T_{1,in}$ instead, in turn making the slope of the entropy generation curves in Figs. 5 and 6 negative instead of positive. Nonetheless, the tools offered here should still be valid insofar as assessing the general correlations between various operating conditions of heat exchangers subject to step changes in inlet conditions.

5. Conclusion

In this paper we have outlined a method by which entropy generation for heat exchangers subject to step changes in inlet conditions can be evaluated. In so doing, three dimensionless groups are introduced, two corresponding to rate of heat transfer and the inlet conditions of the two fluids, and the remaining group relating the heat capacities of the two fluids to one another. The use of these parameters rather than ones used in previous work allows for trends corresponding to a constant heat transfer rate to be more easily discernable.

The entropy generation trends for changing inlet conditions while maintaining a constat heat transfer do not generally follow the advice given for minimizing entropy generation in other studies, namely, that values of C approaching one should lead to smaller entropy generation. Furthermore, a clear minimum or maximum in entropy generation does not necessarily exist for varying inlet conditions. Nonetheless, a heat exchanger designed with $C \approx 1$ may offer a good starting point in that deviations from nominal conditions may lead to either smaller or larger entropy generation, but will always result in improvements in effectiveness.

References

[1] J. Doty, K. Yerkes, L. Byrd, J. Murthy, A. Alleyne, M. Wolff, S. Heister, and T. S. Fisher., “Dynamic thermal management for aerospace technology: Review and outlook,” *Journal of Thermophysics and Heat Transfer*., vol. 31, no. 1, pp. 86–98, Jan. 2017.

[2] M. Bodie, G. Russell, K. McCarthy, E. Lucas, J. Zumberge, and M. Wolff, “Thermal analysis of an integrated aircraft model,” *in Proc. 48th AIAA Aerospace Sci. Meeting Including New Horizons Forum Aerospace Exposition*, 2010, p. 288.

[3] C. Liu, W. Bu, and D. Xu, 2017. “Multi-objective shape optimization of a plate-fin heat exchanger using CFD and multi-objective genetic algorithm”. *International Journal of Heat and Mass Transfer*, 111, Aug., pp. 65–82.

[4] Tam, H., Tam, L., Tam, S., Chio, C., and Ghajar, A. J., 2012. “New optimization method, the algorithms of changes, for heat exchanger design”, *Chinese Journal of Mechanical Engineering, 25(1)*, Jan., pp. 55–62.

[5] H. Pangborn, A. Alleyne, and N. Wu, 2015. “A comparison between finite volume and switched moving boundary approaches for dynamic vapor compression system modeling”. *International Journal of Refrigeration*, 53, May, pp. 101–114.

[6] A. Nash and N. Jain, N. “Dynamic Design Optimization for Thermal Management: A Case Study on Shell and Tube Heat Exchangers.” *Dynamic Systems and Control Conference*, Vol. 59155, American Society of Mechanical Engineers, 2019.

[7] A. Bejan, *Entropy Generation Minimization*, Boca Raton, FL: CRC Press, 1996.

[8] A. Pizzolato, et al. “Local Entropy Generation Analysis of Transient Processes: An Innovative Approach for the Design Improvement of a Thermal Energy Storage with Integrated Steam Generator.” *In Constructal Law & Second Law Conference*, Parma, Italy, 2015.

[9] A. Nash and N. Jain, “Second law modeling and robust control for thermal-fluid systems,” *Dynamic Systems and Control Conference*, Vol. 51906, American Society of Mechanical Engineers, 2018.

[10] Y. Cengel and A. Ghajar, *Heat and mass transfer*, *6th edition*, New York: McGraw-Hill Education, 2019.

[11] S. Sarangi and K. Chowdhury, “On the generation of entropy in a counterflow heat exchanger,” *Cryogenics (Guildf.)*, vol. 22, no. 2, pp. 63–65, 1982.

[12] F. W. Dittus and L. M. K. Boelter, “Heat transfer in automobile radiators of the tubular type” *Int. Commun. Heat Mass Transf.*, vol. 12, p. 3-22, 1985.