Proceedings of the International Conference of Control, Dynamic Systems, and Robotics
Ottawa, Ontario, Canada, May 15-16 2014
Paper No. 104

On Sliding Mode Control of Flexible Cable Based on Differential Flatness

Gilberto Ochoa, Mario Ramírez-Neria, Viridiana Rodriguez-Pompa, Feliciano Martinez-García and Sabrina Vega-Zepeda
Universidad Politécnica del Valle de México, Division of Mechatronics
Av. Mexiquense s/n, esquina Av. Universidad Politécnica, Col. Villa Esmeralda, Estado de México, México
mramirezn@ctrl.cinvestav.mx; gochoa79@gmail.com; viridiana.rp@gmail.com;
feliciano.martinezg@gmail.com; imecsabrive@gmail.com

Abstract - In this paper a control strategy, based in both sliding modes and differential flatness is proposed to regulate the vertical motion of a flexible cable. The dynamics of the system are represented with a simplify model, that consists in three lumped masses linked by springs. Due to the differential flatness property that the system present, a variety of control schemes can be applied, particularly: sliding modes. By using this scheme, the aim of control is to achieve an output trajectory tracking without oscillations, employing only position measurements and approximations of the time derivatives. The system performance is validated by some numerical simulations.

Keywords: Sliding modes control, differential flatness, flexible cable.

1. Introduction

Flexible cables are a kind of special engineering structures. They are used in overhead transmission lines with high voltage, aerial cables for transport, elevator, cranes, as actuators in some robotics joints, planar robots , large span structures in bridges and other civil engineering structures (Ahmadi-Kashani, 1989), (Sahay, 1989). When we move the cable on its vertical axis (as in the case of elevators or cranes) (K. Jun-Koo, 2000), undesirable vibrational movement can be appeared, due to the elasticity property that the cable present. This condition affect in a great manner when a task of tracking an specified trajectory has to be accomplished. In this case those oscillations need to be avoided.

The concept of differential flatness was firstly addressed by Fliess and colleges (Fliess, 1995). A system is called differentially flat if we can find a set of output variables, called flat outputs, such that the states and the variable inputs can be determined from these variables (without integration). This property allows us to find a differential parametrization of the inputs, states and outputs in terms of the flat outputs and its time derivatives. An interesting property that the linear time-invariant systems posses is that the concept of controllability and the differential flatness coincides. Another advantage about the differential flatness property is that due to the structure of the differential parametrization the not modeled dynamics, as well, as the nonlinear effects can be considered as disturbances.

The flexible cable can be modeled as a set of springs and masses. This simplified dynamic model is differentially flat and since this property is analogue to the controllability, a great variety of control approaches can be applied in order to achieve a tracking trajectory. Among the great diversity of control schemes that can be applied to the system, the sliding modes technique stands out because of its simplicity of implementation and easy understanding. Another desire characteristic of this approach is that is robust to a wide range of perturbations, parameter variations and model uncertainties. Sliding mode control was developed in the former Soviet Union. Names like Emelyanov, Utkin, Itkis are associated with earlier developments of the theory (Emelyanov, 1967), (Itkis, 1976), (Utkin, 1992). The basic idea behind the sliding mode control is to applied a discontinuous feedback control law or a switching strategy to the input of the system in order to drive the trajectory system to a desire surface, called sliding surface, in the state space. The historical areas of applications of sliding mode control are: Aerospace systems (L. Xiangdong, 2011), Robotics (L.M. Capisani, 2010), Automotive system (Hyunsup and Hyeongcheol, 2011) and Power Electronics (A. Franco-Gonzáles, 2007) .

In this contribution the flatness of flexible cable and slide mode control strategy for tracking trajectory tasks is considered. The paper is organized as follows: in Section 2 the dynamic model and differential flatness are described. The design of the sliding mode controller for the system is developed in Section 3. In Section 4 the corresponding numerical simulations are presented. Concluding remarks ends the contribution.

2. Modeling and Control of a Flexible Cable

2.1. Flatness of a Flexible Cable

In this section a model of a flexible cable is considered. The model can be represented by an equivalent system that consist of 3 lumped masses linked by springs, see Figure 1. The first mass is directly actuated by the input force, while the remaining are under-actuated. The mathematical model is represented by the following dynamics equations

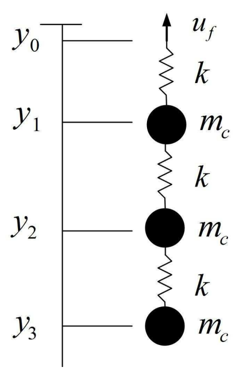


Fig. 1. Cable flexible equivalent system

$$m_c\ddot{y}_1 = m_c g - k(y_1 - y_0) + k(y_2 - y_1) \tag{1}$$
$$m_c\ddot{y}_2 = m_c g - k(y_2 - y_1) + k(y_3 - y_2) \tag{2}$$
$$m_c\ddot{y}_3 = m_c g - k(y_3 - y_2) \tag{3}$$

where m_c is the mass of a section of the cable, k denote the stiffness coefficient of the springs, and $y_i, i = 0,1,2,3$ is the vertical displacement of a section of the cable.

It is considered that the control input is the force applied to variable y_0, that is $u_f = ky_0$. The controlled output is the vertical motion of the third mass y_3. System (1)-(3), can now be expressed as

$$\ddot{y}_1 = g - \frac{k}{m_c} y_1 + \frac{1}{m_c} u_f + \frac{k}{m_c}(y_2 - y_1) \tag{4}$$

$$\ddot{y}_2 = g - \frac{k}{m_c}(y_2 - y_1) + \frac{k}{m_c}(y_3 - y_2) \tag{5}$$

$$\ddot{y}_3 = g - \frac{k}{m_c}(y_3 - y_2) \tag{6}$$

Following (Fliess, 1995), it can directly shown that, for system (4)-(6), the flat output is given by the displacement of the third mass, that is $y_3 = F$, and a differential parametrization can be obtained

$$y_3 = F \tag{7}$$

$$y_2 = \frac{m_c}{k}\ddot{F} - \frac{m_c}{k} g + F \tag{8}$$

$$y_1 = \frac{m_c^2}{k^2} F^{(4)} + 3\frac{m_c}{k}\ddot{F} + F - 3\frac{m_c}{k} g \tag{9}$$

$$u_f = \frac{m_c^3}{k^2} F^{(6)} + 5\frac{m_c^2}{k} F^{(4)} + 6 m_c \ddot{F} + kF - 6 m_c g \tag{10}$$

that yields to the equilibrium relations

$$\bar{y}_3 = \overline{F} \tag{11}$$

$$\bar{y}_2 = \overline{F} - \tfrac{m_c}{k} g \tag{12}$$

$$\bar{y}_1 = \overline{F} - 3\tfrac{m_c}{k} g \tag{13}$$

$$\bar{u} = k\overline{F} - 6 m_c g \tag{14}$$

From (4)-(6), we can also arrive to the following set of relations for the higher order derivatives of the flat output, note that they appears in terms of the state variables

$$F = y_3 \tag{15}$$

$$\dot{F} = \dot{y}_3 \tag{16}$$

$$\ddot{F} = \frac{k}{m_c} y_2 + g - \frac{k}{m_c} y_3 \tag{17}$$

$$F^{(3)} = \frac{k}{m_c}\dot{y}_2 - \frac{k}{m_c}\dot{y}_3 \tag{18}$$

$$F^{(4)} = \frac{k^2}{m_c^2} y_1 - 3\frac{k^2}{m_c^2} y_2 + 2\frac{k^2}{m_c^2} y_3 \tag{19}$$

$$F^{(5)} = \frac{k^2}{m_c^2}\dot{y}_1 - 3\frac{k^2}{m_c^2}\dot{y}_2 + 2\frac{k^2}{m_c^2}\dot{y}_3 \tag{20}$$

These relations are useful to write the feedback controller. From (10), the system is equivalent to the input-output model of the form

$$F^{(6)} = \frac{k^2}{m_c^3} u_f - 5\frac{k}{m_c} F^{(4)} - 6\frac{k^2}{m_c^2}\ddot{F} - \frac{k^3}{m_c^3} F + 6\frac{k^2}{m_c^2} g \tag{21}$$

2.2. Problem Formulation

Now, we propose a rest-to-rest maneuver for the flat output nominal trajectory $F^*(t)$, starting at an initial position $F^*(t_{initial}) = \bar{F}$ and resting at a final one $F^*(T) = F_{final}$. It is desirable that the position of the last mass arrives to its final destination without oscillations.

3. A Slide Mode Control

Equation (21) can be formulated as a simplified perturbed model for the underlying flexible cable system, that is

$$F^{(6)} = \frac{k^2}{m_c^3}u + \delta(t) \tag{22}$$

Here $\delta(t)$ represent a function that depends on the states and the higher order derivatives of the flat output. The not modeled dynamics, and the unknown external perturbations (that may affect the system performance) are denoted by $\eta(t)$. The uncertain terms are lumped into this time-varying function represented by

$$\delta(t) = -5\frac{k}{m_c}F^{(4)} - 6\frac{k^2}{m_c^2}\dot{F} - \frac{k^3}{m_c^3}F + 6\frac{k^2}{m_c^2}g + \eta(t) \tag{23}$$

Due to the flatness property of the system, a control scheme based on sliding modes (SM) is feasible, see (Sira-Ramirez, 1992),(A. J. Koshkouei, 2005). To this end we first define, for dynamics (22), the following sliding surface

$$\begin{aligned}\sigma(t) =&(F^{(5)} - F^{(5)*}) + \kappa_1(F^{(4)} - F^{(4)*}) + \kappa_2(F^{(3)} - F^{(3)*}) + \kappa_3(\ddot{F} - \ddot{F}^*) + \kappa_4(\dot{F} - \dot{F}^*)\\ &+ \kappa_5(F - F^*)\end{aligned} \tag{24}$$

where $\kappa_1, \kappa_2, \kappa_3, \kappa_4, \kappa_5$ are a set of real constants, that are selected in such a way, that the polynomial

$$p(s) = s^5 + \kappa_1 s^4 + \kappa_2 s^3 + \kappa_3 s^2 + \kappa_4 s + \kappa_5 \tag{25}$$

is Hurwitz. Now, we compute the time derivative of $\sigma(t)$, that lead us to the expression

$$\begin{aligned}\dot{\sigma}(t) =&\frac{k^2}{m_c^3}u_f + \delta(t) - F^{(6)*} + \kappa_1(F^{(5)} - F^{(5)*}) + \kappa_2(F^{(4)} - F^{(4)*}) + \kappa_3(F^{(3)} - F^{(3)*}) + \kappa_4(\ddot{F} - \ddot{F}^*)\\ &+ \kappa_5(\dot{F} - \dot{F}^*)\end{aligned} \tag{26}$$

Then, the sliding mode control is specified as follows

$$u_f = -\frac{m_c^3}{k^2}W sign(\sigma(t)), \qquad W > 0 \tag{27}$$

and, in order to reduce the chattering effect in (26), the control law (27), is proposed as

$$u_f = -\frac{m^3}{k^2}W\frac{\sigma(t)}{\|\sigma(t)\| + \mu} \tag{28}$$

By substituting (28) in (26), we arrive to

$$\dot{\sigma}(t) = -W\frac{\sigma(t)}{\|\sigma(t)\| + \mu} + \psi(t) \tag{29}$$

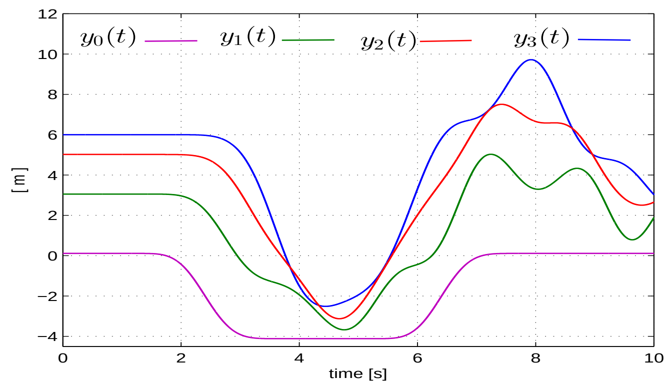


Fig. 2. Flexible cable performance without control

where $\psi(t)$ is defined as

$$\psi(t) = \delta(t) - F^{(6)*} + \kappa_1(F^{(5)} - F^{(5)*}) + \kappa_2(F^{(4)} - F^{(4)*}) + \kappa_3(F^{(3)} - F^{(3)*}) + \kappa_4(\ddot{F} - \ddot{F}^*) + \kappa_5(\dot{F} - \dot{F}^*)$$

Here μ is a small positive scalar parameter with a value selected as $\mu = 0.0005$, and W must be chosen such that

$$W > \sup_t \|\psi(t)\| \tag{30}$$

this condition must holds, in order to ensure that the tracking error tends to zero as time evolves. For the sliding surface $\sigma(t) = 0$, the set of gains κ_i were chosen such that the polynomial (25) match with the desire polynomial

$$q(s) = (s+p)(s^2 + \zeta\omega s + \omega^2)^2 \tag{31}$$

4. Numerical Simulation

In this section numerical simulations of the system are presented. The flexible cable parameters used for the simulation are: the mass $m = 0.1[kg]$, the stiffness coefficient $k = 1[N/m]$ and $g = 9.81[\text{m}/s^2]$. We establish a rest-to-rest reference F^*, for the flat output $F = y_3$, it was set as a smooth polynomial interpolation between initial and final desired values for the last mass. The initial conditions (11)-(14), at $t = 0[s]$, are the following: $\bar{y}_3 = \bar{F} = 6[m]$, $\bar{y}_2 = 5.0190[m]$, $\bar{y}_1 = 3.0570[m]$, $\bar{u}_f = 0.1140[N]$. The reference trajectory at $t = 0[s]$ was set to be the position $F^* = 6[m]$ then, in $t = 1[s]$, the system was moved to the position $F^* = 2[m]$ accomplish the reference in 3 seconds. At $t = 4[s]$ the system remains in this reference position for a period of 1 second, then in $t = 5[s]$ the system was moved to the initial position $F^* = 6[m]$ in 3 seconds.

Figure 2 shows the performance of flexible cable without control, where the position y_0 follow a reference trajectory specified by $y_0^*(t) = F^*(t) - 5.8860[\text{m}]$. When the task is completed, we can observe undesirable oscillations in positions y_1, y_2 and y_3 .

Now the control law is simulated, in order to show its effectiveness. The design coefficients for the sliding surface κ_i were set as coming from the desired characteristic polynomial $q(s)$ (31), selected by means of the parameters: $\omega = 5$, $p = 5$, $\zeta = 1$. The controller gain was elected as $W = 5000$. The initial conditions for the system, at $t = 0[s]$, are the following: $y_3(0) = F(0) = 6.05[m]$, $y_2(0) = 4.95[m]$, $y_1(0) = 2.95[m]$ and $u_f = 0[N]$

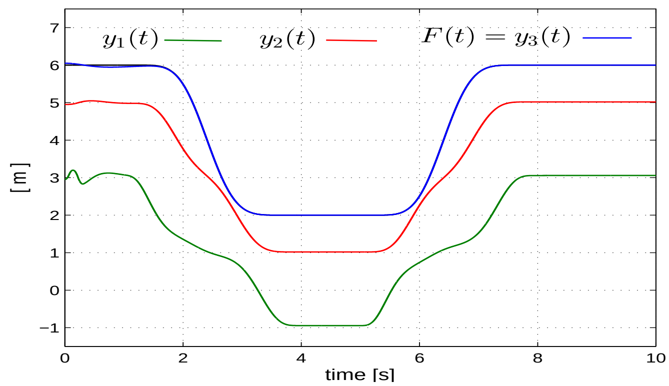
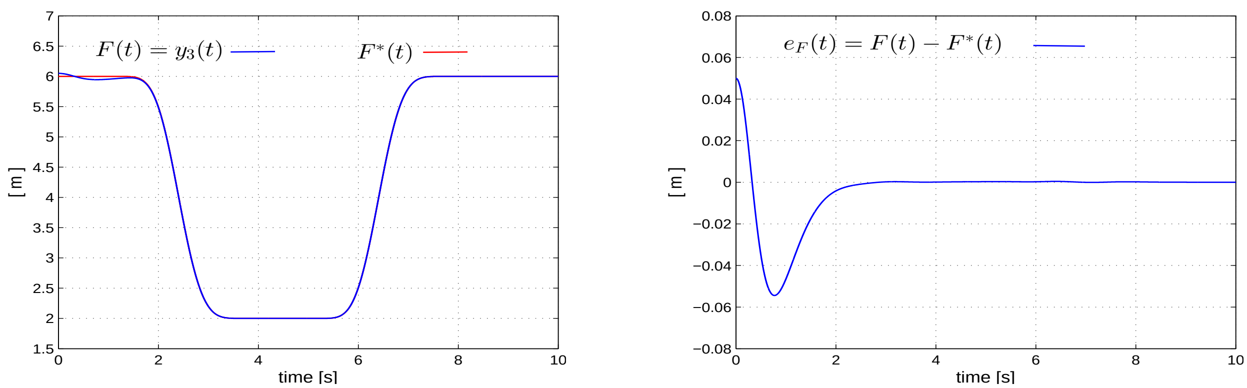


Fig. 3. Tracking performance of flexible cable with slide mode controller based in differential flatness



(a) (b)

Fig. 4. a) Tracking performance of the slide mode controller based in differential flatness b) Tracking error signal

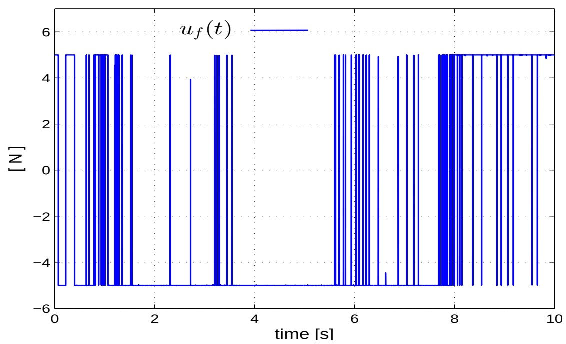
The time evolution of the desired position $F^*(t)$, as well as, the controlled trajectory of the flat output F are shown in Figure 4(a). It becomes evident the suitable tracking quality of the scheme, where, the position of the last and under-actuated mass, carried out the trajectory tracking task without oscillations.

In Figure 4(b), the tracking error $e_F = F^* - F$ is depicted. Here we can observe that the position error is restricted to a small vicinity of the origin and, in the steady state, its uniformly bounded.

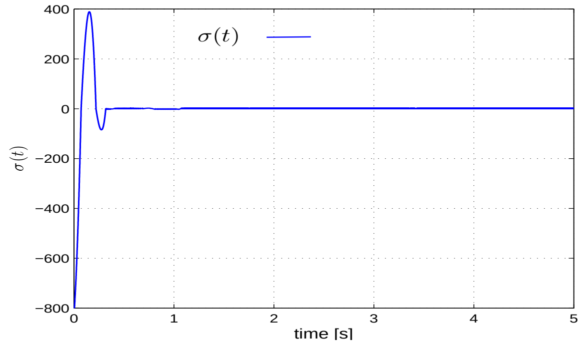
The control input force applied to the system is depicted in Figure 5(a) and the sliding surface in Figure 5(b)

5. Conclusion

In this contribution the control of the position of a flexible cable was addressed. A simplified model was proposed that is based in three lumped masses linked with springs, where the second and third of the masses are under-actuated. Based on the flatness property that the system present, a sliding mode control is proposed in order to achieve the trajectory tracking minimizing the oscillations that the flexible cable could present, principally in the last under-actuated mass.



(a)



(b)

Fig. 5. a) Control input b) Sliding surface

References

A. Franco-Gonzáles, R. Marquez, H. S. R. (2007). On the generalized-proportional-integral sliding mode control of the ”boost-boost” converter. *4th International Conference on Electrical and Electronics Engineering*, pages 209–212.

A. J. Koshkouei, K. J. Burnham, A. S. I. Z. (2005). Diferenctial flatness of sliding mode nonlinear system. pages 670–674.

Ahmadi-Kashani, K. (1989). Vibration of hanging cables. *Computers & Structures*, 21(5):699–715.

Emelyanov, S. (1967). *Variable Structure Control Systems*. Nauka, Moscow.

Fliess, M., L. J. M. P. R. P. (1995). Flatness and defect of non-linear systems: introductory theory and applications. *International Journal of Control*, 61:1327–1361.

Hyunsup, K. and Hyeongcheol, L. (2011). Height and leveling control of automotive air suspension system using sliding mode approach. *IEEE Transaction on Vehicular Technology*, 60(5).

Itkis, U. (1976). *Control Systems of Variable Structure*. Wiley, New York.

K. Jun-Koo, S. S.-K. (2000). Vertical vibration control of elevator using estimated car acceleration feedback compensation. *IEEE Transactions on Industrial Electronics*, 47(1):91–99.

L. Xiangdong, C. Zhen, R. X. (2011). Time-varying sliding mode control for spacecraft attitude maneuver with angular velocity constraint. *Chinese Control and Decision Conference*, pages 670–674.

L.M. Capisani, A. Ferrera, A. G. (2010). Robust interaction control via first and second order sliding modes of planar robotic manipulators. *49th IEEE nternational Conference on Decision and Control*, pages 209–212.

Sahay, C. (1989). Vibration of overhead transmission lines. *Shock Vibration Digest*, 21(5):8–13.

Sira-Ramirez, H. (1992). On the sliding mode control of nonlinear systems. *System and Control Letters*, 19:303–312.

Utkin, V. I. (1992). *Sliding modes in control and optimization*. Edition Springer Verlag.