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On the Sliding Mode Control of a Magnetic Levitation System Case: Thomson's Jumping Ring

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Abstract - The trajectory tracking problem in the Thomson's Jumping Ring is addressed and solved using the properties of flat systems and a sliding mode control strategy. It is shown that the non-linear Thomson's Jumping Ring model can be linearized around an equilibrium point obtaining a new linear model that is differentially flat, with incremental flat output. Taking the advantage of flatness properties, the implementation of a sliding mode (SM) based control scheme is feasible. Numericals simulations show the performance of the proposed control scheme.

Keywords: Sliding mode control, Thomson's Jumping Ring, differential flatness, magnetic levitation.

1. Introduction

Magnetic levitation is a method by which an object is suspended in the air with no support other than magnetic fields. The fields are used to reverse or counteract the gravitational pull and any other counter accelerations. Maglev can create frictionless, efficient, far-out-sounding technologies. The principle of magnetic levitation has been known for over 100 years, when American scientists Robert Goddard and Emile Bachelet first conceived of frictionless trains.

One of the first major applications of magnetic levitation was in supporting airplane models into wind tunnels. Researchers had found that mechanical support structures sometimes interfere with airflow enough to produce more drag than the drag force on the model. The solution developed by Gene Covert and his MIT colleagues in the 1950's was magnetic levitation (although they called it a magnetic suspension and balance system).

Even though magnetically levitated trains have been the focus of multiple global concerns, the technology is not limited to train travel (Yaghoubi et al., 2012). Maglev usages from point of view engineering science can be categorized and summarized as follows:

- Transportation engineering (Yaghoubi, 2008), (Yaghoubi, 2011).
- Aerospace engineering (Fiske et al., 2006), (Hull et al., 2002), (McNab, 2003).
- Biomedical engineering (Noh et al., 2008), (Qian and Jing, 2008), (Wu et al., 2011).
- Chemical engineering (Mirica et al., 2010).

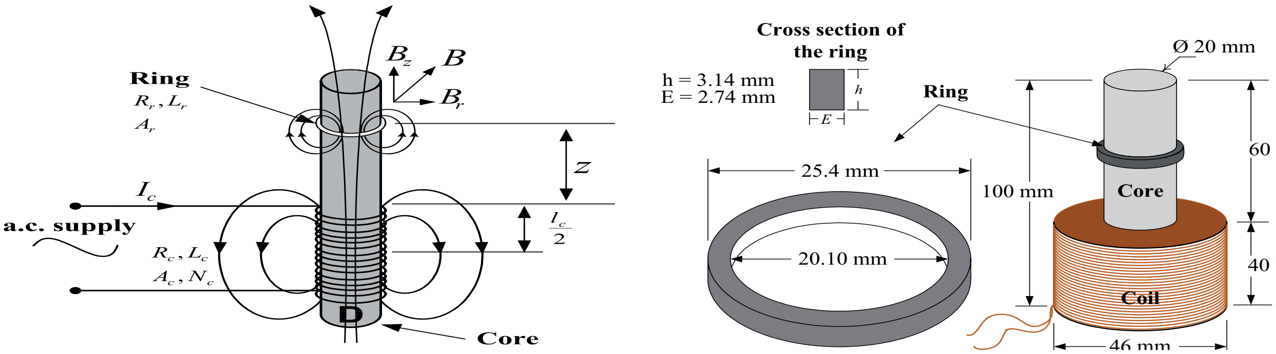
Nowadays Maglev helps to circulate blood in human chests, manufactures integrated circuits with multimillion-dollar photolithography (Kim and Trumper, 1998) systems, measures fine dimensions with subatomic resolution, enhances wind-tunnel and plasma research, melts and mixes reactive high-temperature metals, simulates the sense of touch in haptics systems, cools our laptop computers, enriches uranium and other isotopes in centrifuges, stores energy in spinning flywheels, and floats spinning rotors with low friction in countless rotating machines around the world (Livingston, 2011).

In general, a Maglev system can be classified according to the type of considered forces, as an attractive or repulsive system (Wang et al., 1991). These systems are, usually, highly nonlinear and open loop unstable and therefore they require a specific control scheme in order to get a stable closed-loop operation.

An interesting Maglev systems due to their high non-linearity is the "Thomson's Jumping Ring" (Barry and Casey, 1999), (Tjossem and Cornejo, 2000), created by Elihu Thomson (1853-1937). It consists of an induction coil with a core of a ferromagnetic material and a ring placed in the core (see Figure 1 for a detailed diagram). When an alternating current circulates in the coil, an electric current is induced in the ring. The magnetic field, due to this induced current, opposes the magnetic field induced by the coil producing a total repulsive force making the ring levitate above the core of the coil. (García-Antonio et al., 2011).

In (Moazen et al., 2012) two different PID control methods are described to control of Maglev Guiding System for Linear Elevator. The design of an on-line levitation and propulsion control for a magnetic-levitation transporting system is shown in (Rong-Jong et al., 2010). They incorporate a total sliding-mode control strategy into particle swarm optimization (PSO) to form an on-line control framework with varied inertial weights for preserving the robust control characteristics and reducing the chattering control phenomena of the sliding-mode control. In (García-Antonio et al., 2011) path tracking and synchronization of two Thomson's ring powered by alternating current are achieved by considering a modification of a control strategy presented initially on the field of mobile robotics (Sun et al., 2009), together with a second order sliding mode technique. Finally, in (Ramírez-Neria et al., 2013) the problem of tracking a smooth time-varying reference trajectory for Thomson's ring is addressed from the perspective of Active Disturbance Rejection Control.

The goal of this paper is to design a control law using the properties of flatness and the theory of sliding mode control in order to reach trajectory tracking in the Thomson's Jumping Ring. The paper is organized as follows: Section 2 presents the Thomson's Jumping Ring model and the problem statement. Section 3



(a) (b)

Fig. 1. a) Schematic diagram of the Thomson's jumping ring b) System main dimensions

shows the control strategy based on a slide mode control that solves the trajectory tracking in this kind of systems. The performance of the proposed control strategy using numerical simulations is shown in Section 4. Finally, in Section 5 some conclusions are given.

2. Thomson's Ring Model

2.1. The Non-linear Model

Consider the Thomson's jumping ring system, shown in Figure 1, given by

$$m\frac{d^2}{dt}z = -mg + \frac{K}{Z_c'^2 z}V_c^2 \tag{1}$$

where z is the distance measured from the end of the core to the ring plane, m is the mass of the ring, g is the acceleration of gravity, Z_c' is an impedance magnitude related to the ring and V_c is the amplitude of a sinusoidally modulated input voltage applied to the circuit. The ring is free to move up and down along the core with zero friction between itself and the core. The core is made from a solid piece of ferrous metal, while the ring is made from a non-magnetic electrical conductor (i.e. aluminium).

In the work of (García-Antonio et al., 2011) a simplified model of Thomson's jumping ring is studied. It may noticed the approximated model (1) does not involve periodic functions of time as input control, this allow the design of control strategies for position regulation or trajectory tracking. The parameter V_c^2 acts as the control input to the system denoted, henceforth by u. The strictly positive parameter K exhibits a complex dependence on the magnetic field, the dimensions of the circuit and the position of the ring from the end of the coil. In this work, its value is taken to be a constant, precisely determined at a nominal equilibrium condition. In actual operation, specially under time-varying trajectory tracking conditions, the control input gain of the system exhibits a noticeable variation which cannot be easily measured.

2.2. Flatness and the Linearized Model of Thomson's Jumping Ring System

A constant equilibrium point for the ring's position is obtained from (1) as

$$z = \bar{z}, \quad \dot{z} = \dot{\bar{z}} = 0, \quad \bar{u} = \frac{mZ_c'^2\bar{z}g}{K} \tag{2}$$

The tangent linearization of (1) around the equilibrium point (2) is given by,

$$m\ddot{z}_\delta = \left(\frac{K}{Z_c'^2\bar{z}}\right)u_\delta - \left(\frac{K\bar{u}}{Z_c'^2\bar{z}^2}\right)z_\delta \tag{3}$$

with $z_\delta = z - \bar{z}$, $u_\delta = u - \bar{u}$, being the incremental variables of the linearized model. Clearly $\dot{z}_\delta = \dot{z}$ and $\ddot{z}_\delta = \ddot{z}$. Naturally, higher order terms (h.o.t.) are being neglected. The linear model (3) is differentially flat, with incremental flat output denoted by F_δ and is given in this case by the following expression:

$$F_\delta = z_\delta \tag{4}$$

Indeed, all the system variables in the linear model are expressible as differential functions of the incremental flat output (Sira-Ramírez and Agrawal, 2004)

$$z_\delta = F_\delta \tag{5}$$

$$\dot{z_\delta} = \dot{F}_\delta \tag{6}$$

$$u_\delta = \left(\frac{mZ_c'^2\bar{F}_\delta}{K}\right)\ddot{F}_\delta + \frac{\bar{u}}{\bar{F}_\delta}F_\delta \tag{7}$$

The linearized system is clearly equivalent to the following input-output model

$$\ddot{F}_\delta = \left(\frac{K}{mZ_c'^2\bar{F}_\delta}\right)u_\delta - \left(\frac{K\bar{u}}{mZ_c'^2{\bar{F}_\delta}^2}\right)F_\delta \tag{8}$$

The nominal incremental trajectory for the flat output is given by

$$\ddot{F}_\delta^*(t) = \left(\frac{K}{mZ_c'^2\bar{F}_\delta}\right)u_\delta^*(t) - \left(\frac{K\bar{u}}{mZ_c'^2{\bar{F}_\delta}^2}\right)F_\delta^*(t) \tag{9}$$

2.3. Problem Formulation

We propose an arbitrary smooth position reference trajectory $z^*(t)$ for the ring position z, using the advantage of flatness properties device and a slide mode controller law so that the incremental variable flat output $F_\delta = z - \bar{z}$ governed by (8), tracks the desired incremental position $F_\delta^*(t) = z^*(t) - \bar{z}$ irrespectively of the gravity effects and of the exogenous perturbation inputs possibly affecting the ring position.

3. Slide Mode Control Strategy

From the incremental reference trajectory for the flat output (9) and the input-output system (8) the reference trajectory tracking incremental error $e_{F\delta} = F_\delta - F_\delta^*$ coincides with the tracking error $z - z^*(t)$. In the same way, the second derivative $\ddot{e}_{F\delta} = \ddot{F}_\delta - \ddot{F}_\delta^* = \ddot{z} - \ddot{z}^*$. Also $e_{u\delta} = u_\delta - u_\delta^* = u - \bar{u} - (u^*(t) - \bar{u}) = u - u^*(t)$. The dynamics of tracking error $\ddot{e}_{F\delta}$ satisfies

$$\ddot{e}_{F\delta} = \left(\frac{K}{mZ_c'^2\bar{F}}\right)e_{u\delta} + \left(\frac{K\bar{u}}{mZ_c'^2{\bar{F}_\delta}^2}\right)(F_\delta - F_\delta^*) \tag{10}$$

We adopt the following perturbed simplified model for of the tracking error dynamics

$$\ddot{e}_{F\delta} = \left(\frac{K}{mZ_c'^2\bar{F}}\right)u + \tilde{\psi}(t) \tag{11}$$

where $\tilde{\psi}(t)$ represents state depend expressions, all the higher order derivatives terms (**H.O.T.**) neglected by linearization, the possibly un-modeled dynamics, and external unknown disturbances affecting the system. We lump all this terms, into this time-varying function represented by

$$\tilde{\psi}(t) = -\left(\frac{K}{mZ_c'^2\bar{F}_\delta}\right)u^*(t) - \left(\frac{K\bar{u}}{mZ_c'^2\bar{F}_\delta^2}\right)F_\delta + \textbf{H.O.T} \tag{12}$$

Taking advantage of flatness properties, the implementation of a sliding mode (SM) based control scheme is feasible [see (Sira-Ramírez, 1992)]. We define, for dynamics (9) the following sliding surface:

$$\sigma(t) = (\dot{F}_\delta - \dot{F}_\delta{}^*(t)) + \lambda(F_\delta - F_\delta^*(t)) \tag{13}$$

Where λ is a real constant such that the polynomial $s+\lambda$ is Hurwitz . Taking the time derivative of $\sigma(t)$ yields

$$\dot{\sigma}(t) = \left(\frac{K}{mZ_c'^2\bar{F}_\delta}\right)u - \left(\frac{K\bar{u}}{mZ_c'^2\bar{F}_\delta^2}\right)F_\delta^* + \tilde{\psi}(t) + \lambda(\dot{F}_\delta - \dot{F}_\delta^*) \tag{14}$$

From the dynamic model (1) we can establish that downwards accelerated motions, beyond "1g" are not into performance of the system, due to Thomson's jumping ring is classified as a repulsive force, accelerated motion only can occur through an external perturbation. Negative values of the input V_c produce the same effect as its absolute value. Hence, the assumption, $V_c \geq 0$, therefore $u \geq 0$ is inherent to the problem. Note that free fall corresponds with zero applied input voltage in u. Thus the sliding mode control is specified as follows

$$u = \frac{1}{2}\left(\frac{mZ_c'^2\bar{F}_\delta}{K}\right)W(1 - sign(\sigma(t))) \qquad W > 0 \tag{15}$$

In order to reduce the chattering effect, the following control law is proposed:

$$u = \frac{1}{2}\left(\frac{mZ_c'^2\bar{F}_\delta}{K}\right)W\left(1 - \frac{\sigma(t)}{\|\sigma(t)\| + \varepsilon}\right) \tag{16}$$

we substitute the slide mode control (16) in the time derivative of $\sigma(t)$ equation (14)

$$\dot{\sigma}(t) = \frac{1}{2}W\left(1 - \frac{\sigma(t)}{\|\sigma(t)\| + \varepsilon}\right) + \psi(t) \tag{17}$$

Where $\psi(t) = -\left(\frac{K\bar{u}}{mZ_c'^2\bar{F}_\delta^2}\right)F_\delta^* + \tilde{\psi}(t) + \lambda(\dot{F}_\delta - \dot{F}_\delta^*)$. To guarantee that the tracking error tends to zero as time evolves, the parameter $\varepsilon = 0.0005$ has to be a small positive scalar and W must be selected such that $W > sup_t \|\psi(t)\|$. For the sliding surface $\sigma(t) = 0$, the gain was chosen such that the following desired characteristic polynomial was matched

$$s + \lambda = s + \omega_c^2 \tag{18}$$

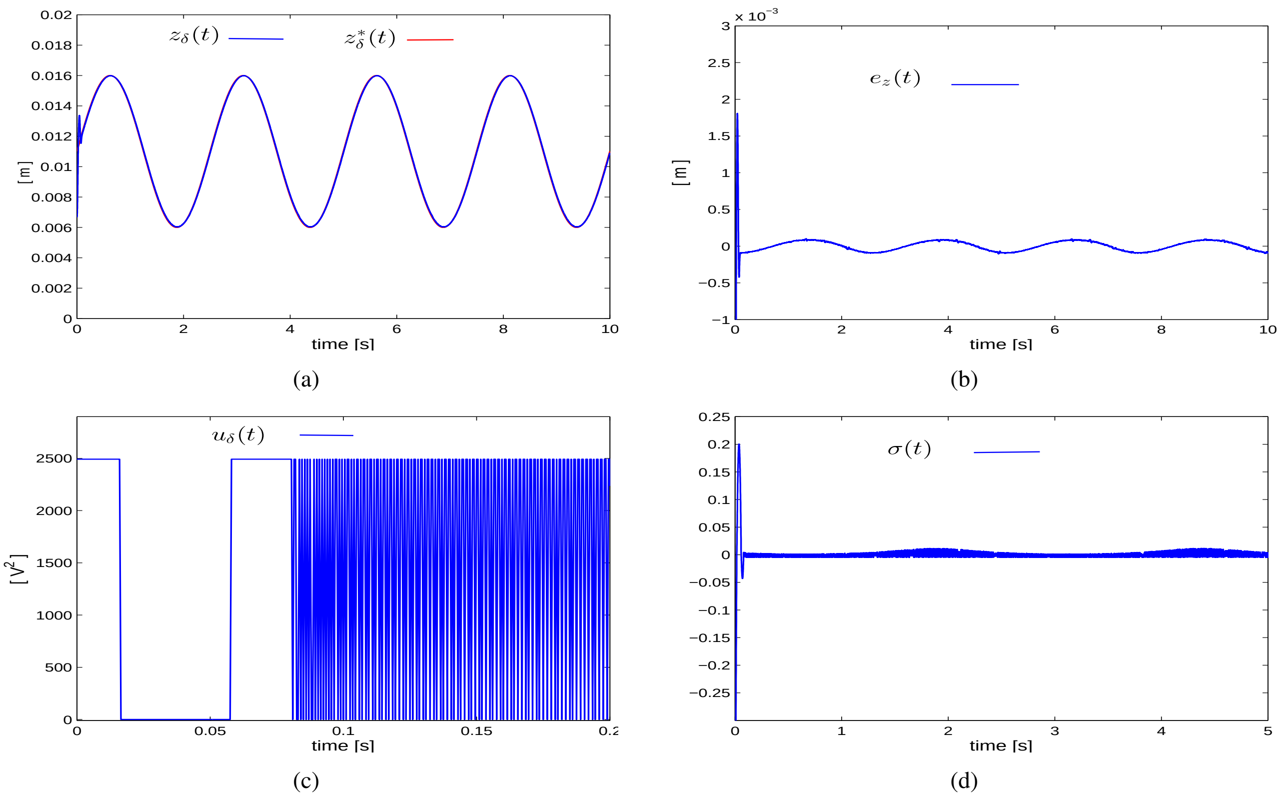
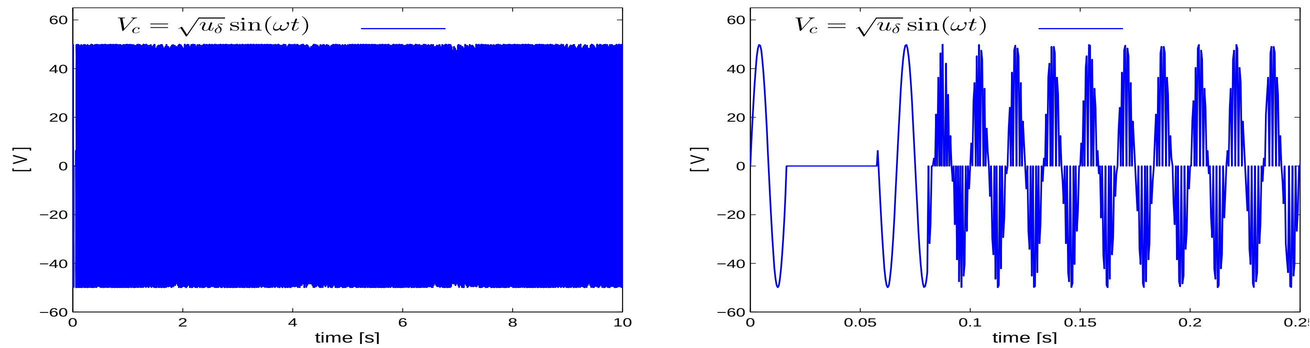


Fig. 2. a) Ring position reference trayectory tracking performance b) Reference trajectory tracking error surface c) Zoom of control input signal d) Sliding surface

4. Numerical Simulation

In this section the proposed control law is simulated. The dimensions of the Thomson's ring considered in the dynamic model are given in Figure 1(b). The parameters associated with the model of the electric circuit were obtain of (García-Antonio, 2011) and (García-Antonio et al., 2011). Terms such as: Friction, air resistance, parasitic effects etc. are not specifically modeled in (1). The main coil has a primary winding of 1200 turns of AWG copper wire of 0.574[mm] of diameter. The wire is wrapped around a stainless steel core. The ring to be levitated is an aluminum ring of mass $m = 1.44x10^{-3}$[kg]. The position-depending, variation of the gain parameter $\frac{K}{Z_c'^2 z}$ and gravitational acceleration force g, are presented in the plant. The value of K and Z_c' were obtained setting the nominal equilibrium height as $\bar{z} = 20$[mm], while using a corresponding constant voltage amplitude of $V_c = 56$[V]

The following position reference trajectory $F^*_\delta(t) = 11 + 5\sin(\omega t)$[mm] with $\omega = \frac{2\pi}{5}$[rad] was implemented in software. The controller coefficient were set as coming from the desired characteristic polynomial (18), $\lambda = 10$ and $W = 10$. The tracking performance is depicted in Figure 2(a) where the desired reference trajectory is superimposed to the actual ring position trajectory. The control scheme proposed is remarkably robust with respect to such unknown variations of the input gain $\frac{K}{Z_c'^2 z}$ it can be consider as a perturbation. The corresponding output trajectory tracking error $e_F = z_\delta - z^*_\delta(t)$ is shown in Figure 2(b). As it can be seen, the trajectory tracking error is uniformly absolutely bounded by a small neighborhood of zero. Figure 2(c) shows a zoom of the resulting control input u, where we can observe that $u(t)$ remains nonnegative for all time, the sliding surface is depicted into Figure 2(d).



(a) (b)

Fig. 3. a) Input voltage in the electric circuit b) Zoom of Input

Figure 3(a) shows the voltage represented by the modulated envelope of a sinusoidal signal with $f = 60$ [Hz], $\omega_f = 2\pi f$[rad], this voltage is applied to the coil. Figure 3(b) depicts a zoom of the input voltage showing the sinusoidal character of the carrier signal.

5. Conclusion

The combination of flatness and slide mode control allows for the efficient solution of a challenging trajectory tracking problem in a Thomson's Jumping Ring, The fact that the tangent linearization of this system, around an arbitrary equilibrium point is flat, allows the use of slide mode control. Some simulation results of the proposed controller are presented in order to validate this control strategy, the controller presented is robust with respect variations of the input gain of the system.

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