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Mean and Variance of a Multi-Rate M/G/1 Queuing Model Over Time Correlated Nakagami-*m* Fading Channel

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Abstract - The advent of multimedia applications over wireless networks has necessitated the need for high data rates and a reliable communication over dynamic wireless links. Adaptive Modulation and coding (AMC), Automatic Repeat re-Quest (ARQ) and Forward Error Correction (FEC) have been widely used to overcome the time varying nature of these links and consequently enhancing their performance. When AMC is used, variable number of packets can be transmitted per Coherence Time Interval (CTI) depending on the Transmission Mode (TM). The number of packets transmitted per CTI is a function of the received Signal-to-Noise Ratio (SNR). For time correlated fading, this entails that the service process is non-Markovian. In this paper we derive the first and Second Moments of the service time distribution for an M/G/1 queuing model that employs AMC for TDMA systems over time correlated Nakagami-*m* Fading channel. We consider service times for geometrically distributed message lengths with fixed packet lengths whose arrivals are Markovian. However, individual packet service times are correlated, due to the correlated fading process and affects overall message service time which subsequently determines the delay experienced by a message in a queue awaiting transmission/service.

Keywords: AMC, Mean and Variance of a random Sum of a Random Number, M/G/1 Queuing Model.

1. Introduction

Designing reliable wireless communication networks can be a daunting task. This is because wireless links are characterized by a randomly time varying impulse response, a phenomena called fading. Moreover, the highly stochastic nature of the packet arrival processes to the way they are transmitted/serviced at the transmitter/node and different processes and protocols that they go through at different layers of the protocol stack makes it more challenging in providing efficient and less complex integrated designs that guarantee Quality-of-Service (QoS) over wireless networks.

The rapid growth and usage of multimedia applications with diverse QoS constraints, such as delay, Bit error rate (BER), packet dropping, transmission rate, just to mention a few, over wireless links has led to a enormous interest in providing efficient and reliable wireless communication networks. Adaptive Modulation and Coding (AMC), Link-Layer Forward Error Correction (LLFEC) and Automatic Repeat re-Quest have been used widely to compensate the time-varying nature of the wireless channel (Chockalingam and

Zorzi, ; Xin Wang and Giannakis, 2007; Goldsmith and Chua, 1997). In Adaptive Modulation and Coding, the modulation parameters such as modulation level, coding rate of the FEC for coded modulations, symbol rate and power are allowed to change with the change in the expected SNR value of the transmitted signal (Masashi Naijoh and Kamio, 1997). AMC and ARQ have shown to be powerful link adaptation techniques for improving throughput performance while maintaining a sufficiently small error rate (Qingwen Liu and Giannakis, 2004; Jalil Seifali Harsini and Zorzi, 2011).

As a result of fading and the stochastic characteristic of traffic arrivals and service in wireless links, most analyses made have assumed that these processes are Independent and Identically Distributed (IID) random variables to simplify their work. However, this assumption is not always correct and does not really give accurate insight of the performance evaluation of these communication networks. This is because most of these processes are correlated in nature and this greatly affects the overall performance of a network. For instance, the wireless channel quality at any point in time largely relies on the previous channel state because of a significant amount of correlation in the fading process (A. Chockalingam and Venkataram, 1998). When AMC is used, variable number of packets can be transmitted per CTI depending on the SNR value. As such, for a random length message with fixed packet lengths transmission under AMC over a time correlated fading entails that individual packet service times vary from one CTI to the other. Consequently, the overall message service time is random and an important measure used in designing links for multimedia applications.

In (Yang and Sasankan, 2006), authors have considered a cross-layer design using AMC/ARQ protocols over wireless TDMA systems and have shown that a combined AMC/ARQ system can offer considerable performance improvement over traditional systems even when finite length queuing is considered at the transmitter, however the effects of correlation is not considered. Whereas in (Qingwen Liu and Giannakis, 2004), authors considered queuing effects on AMC over wireless links. Their results optimized packet error rate in the AMC at the physical layer by minimizing packet loss rate and maximizing average throughput when combined with finite length queuing at the link-layer. However, their M/G/1 queue analysis is insufficient in detailing the service process in terms of the mean and variance (Kleinrock, 1975) for time correlated fading process, which is what we are largely concerned with in this paper.

In this paper, we derive the mean and variance of random number of correlated random variables that emanates from analysing the service time distribution of an M/G/1 queuing model with AMC for TDMA systems over time correlated fading Nakagami-m channel. We have obtained the probability generating function of the service time distribution in terms of the number of slots. These results are important in analysing the delay statistics of the M/G/1 model with multi-rate transmission, as a QoS performance measure at the link-layer.

2. The Organization of the Paper

The rest of this paper is structured as follows; in section 3 we introduce the system model and assumptions whilst in 4, we describe the queuing model and present analytical methods of deriving the mean and variance of service time of an M/G/1 Queuing Model in terms of the number of slots under AMC for TDMA systems over time correlated fading. We conclude in section 5.

3. System Model & Assumptions

We consider a point-to point wireless link between a transmitter and a receiver. This model comprises an AMC module at the physical layer. As shown in Fig. 1, an infinite-length queue is implemented at the link-layer end of the transmitter which operates with FIFO service discipline. The AMC selector at the receiver determines proper AMC mode based on the estimated channel quality and feeds the mode back to the transmitter where the AMC controller dictates the number of slots that a packet needs to be transmitted within a frame duration of T_f seconds which is biased by the TM selection and the availability of packets in the queue. For a detailed study of this model, we refer the reader to similar models in (Qingwen Liu and Giannakis, 2004; Yang and Sasankan, 2006; Xin Wang and Giannakis, 2007).

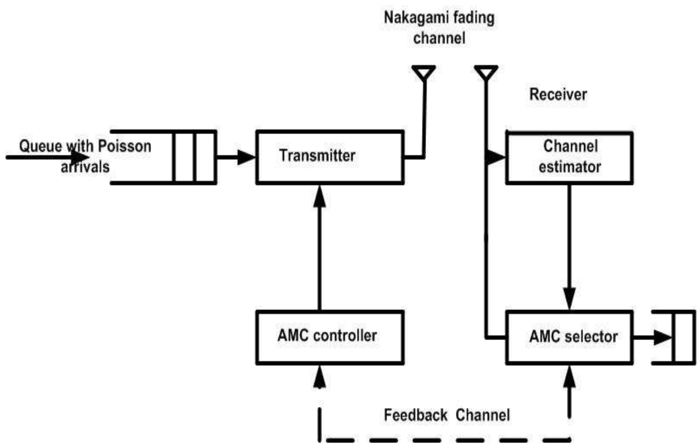

Fig. 1. Queuing and AMC Cross-layer Transmission Model

Our system adheres to the following assumptions;

A1) To model the wireless channel between the transmitter and the receiver, we use Nakagami-m fading channel, whose fading process is assumed to be stationary and ergodic within a *Coherence Time Interval **(CTI)*** of duration T_f sec and allowed to vary from one CTI to another (Qingwen Liu and Giannakis, 2004; Qingwen Liu and Giannakis, 2005; Xin Wang and Giannakis, 2007; Long B. Le and Alfa, 2006; Hossain and Alfa, 2006; Long B. Le and Zorzi, 2007) according to a time correlated random process. As such, the AMC varies from one frame to the other.

A2) We assume that time is slotted, where only one frame can fit within a CTI (Qingwen Liu and Giannakis, 2004; Qingwen Liu and Giannakis, 2005; Long B. Le and Alfa, 2006; Xin Wang and Giannakis, 2007).

A3) We assume that a perfect/reliable Channel State Information is readily available at the transmitter whose feedback channel is error free and without latency. This is achived by employing instantaneous feedback links with powerful error control for feedback information.

To satisfy assumption A1, channel quality can be captured by the received signal-to-noise ratio (SNR) γ. Since, the fading process is assumed to be stationary and ergodic within a *Coherence Time Interval **(CTI)*** of duration T_f sec and allowed to vary from one CTI to another, we adopt the general Nakagami-m model to describe γ statistically. As such, the received SNR γ per frame is a random variable whose Gamma probability density function (pdf) is given as;

$$p_\gamma(\gamma) = \frac{m^m \gamma^{m-1}}{\bar{\gamma}^m \Gamma(m)} \exp\left(-\frac{m\gamma}{\bar{\gamma}}\right) \tag{1}$$

Note that; $\bar{\gamma} := E\{\gamma\}$ is the average received SNR, $\Gamma(m) := \int_0^\infty t^{m-1} \exp(-t)dt$ is the Gamma function and m the Nakagami fading parameter ($m \geq 1/2$). This model is largely adopted in many works, because it can be used for various fading channels by an appropriate selection of the fading parameter m, for instance it includes Rayleigh channel for $m = 1$. Furthermore, a one-to one mapping between the Ricean factor and the Nakagami fading parameter m allows also Ricean channels to be well approximated by Nakagami-m channels (Qingwen Liu and Giannakis, 2005) and references therein.

3.1. Amc

The AMC in this channel model is used to adapt a perfect TM corresponding to a particular γ thereby enhancing data rate whilst maintaining a prescribed PER P_0. Assuming constant power transmission, we denote all available TMs by N and then partition the entire γ into N+1 nonoverlapping consecutive intervals with boundary points at $\{\gamma_n\}_{n=0}^{N+1}$. Fig. 2 shows the partitioning of γ, where R_n is the bit-rate corresponding to a TM n which is selected only when the following equation is satisfied;

$$\gamma \in [\gamma_n, \gamma_{n+1}) \tag{2}$$

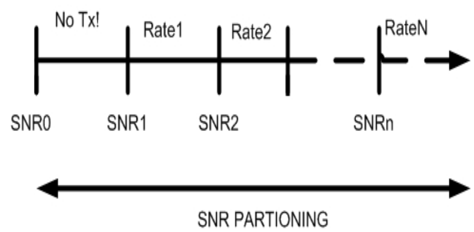

Fig. 2. SNR Partitioning

A detailed analysis of the AMC used here is given by authors in (Qingwen Liu and Giannakis, 2004)

4. Arrival Process

On the transmitter side, we assume that packets arrivals from upper layers follow a Poisson distribution at a rate of $\lambda T_f/s$ and are considered to be independent and identically distributed from slot to slot i.e (independent of both queue and channel states). We let A_t denote the number of packets that arrive per slot, whose probability mass function (p.m.f), mean, variance and probability generating function (pgf) are given as;

$$P(A_t = a) = \begin{cases} \dfrac{(\lambda T_f)^a e^{-\lambda T_f}}{a!}, & \text{if } a \geq 0 \quad (3a) \\ 0, & \text{otherwise} \quad (3b) \end{cases}$$

$$\mu_A = E(A_t) = \lambda T_f \tag{4}$$

$$\sigma_A^2 = E(A_t^2) - [E(A_t)]^2 = \lambda T_f \tag{5}$$

$$\begin{aligned} G_A(z) &= E[z^{A_t}] \\ &= \sum_{a=0}^{\infty} Pr[A_t = a] z^a \\ &= e^{\lambda(z-1)} \end{aligned} \tag{6}$$

Where z is a complex number, such that $|z| \leq 1$.

4.1. Service Process

The use of AMC protocol in our queuing analysis brings about batch services/transmission per CTI. As a result, a geometrically distributed message length with fixed packet lengths entails that individual packets service times are random and correlated, due to the variable TMs and correlated fading process which affects overall message service time. The number of packets served per CTI is a function of SNR. The frame duration varies from one channel state to the other, and when the channel is in state n, it can generally be written as;

$$\frac{R_N l}{R_1 R_n R_s} \tag{7}$$

Where R_n is the rate in bits/symbol and R_s is the channel symbol rate per second (Assumed to be constant). Using the multiple transmission modes as in HIPERLAN/2, IEEE 802.11a and 3GPP standards, we reconstruct transmission modes for uncoded modulation as shown in Table. 1.

Table 1. Transmission Modes with Uncoded M_n-QAM Modulation.

	Mode 1	**Mode 2**	**Mode 3**	**Mode 4**
Modulation	BPSK	**QPSK**	**8-QAM**	**64-QAM**
Rate(bits/sym)	1	2	3	6
a_n	67.7328	73.8279	58.7332	42.5594
g_n	0.9819	0.4945	0.1641	0.0235
γ_{pn}(dB)	6.3281	9.3945	13.9470	22.0340

It is evident that the number of bits carried/symbol R_n for $n = 1,2,3,...,N$ are integer multiples of R_1, Transmission Mode One (TM1) and that the duration of the frame in TM1 is $R_n/R1$ greater than its duration in TM ≥ 2. As such the packet duration for a single active user in a TDMA frame can be rewritten in terms of number of slots as;

$$N_S(n) = \begin{cases} 1, & \text{for } n = N \quad \text{(8a)} \\ \dfrac{R_N}{R_n}, & \text{for } n = 1,2,3,....,N-1 \quad \text{(8b)} \end{cases}$$

Where, $N_S(n)$ equals the number of slots that are needed to transmit a packet for a single user scenario over a TDMA frame and N is the highest TM available. Clearly, only one slot is needed to transmit a packet TM(N) but as many as many as R_N/R_n in channel states $n > N$. Thus the time it takes to transmit the *ith* packet with mode n can be written as;

$$T_i(n) = T_N N_S(n) \tag{9}$$

$T_N = \frac{lbits}{R_N * R_s}$ is the packet duration under transmission mode N.

Table. 2 shows the packet service times, and number of slots needed by an active TDMA single user scenario to transmit a packet using uncoded modulations for TMs with the assumption that each packet has 1080 bits, and the the symbol rate is $1.08M$ (Xin Wang and Giannakis, 2007).

Table 2. Packet Service Time & Slots for Uncoded M_n-QAM Modulation

	Mode 1	**Mode 2**	**Mode 3**	**Mode 4**
Modulation	BPSK	**QPSK**	**8-QAM**	**64-QAM**
Rate(bits/sym)	1	2	3	6
$N_S(n)$	6	3	2	1
Service Time/Slot ($T_i(n)$ (ms))	1	0.50	0.333	0.167

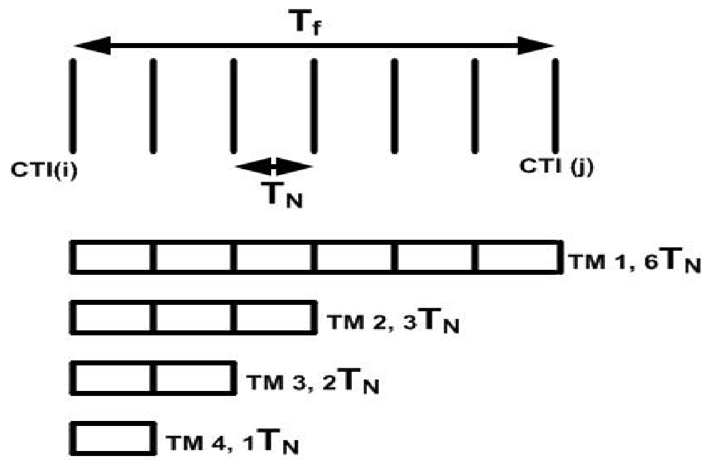

Fig. 3. TDMA Frame Configuration

4.2. Service Time Distribution

Fig. 3 depicts the number of slots needed to transmit a single packet for different TMs. In order to multiplex different active users using the proposed TDMA frame structure, over a single coherence time interval from channel state CTI(i) to CTI(j) whose total frame duration is $T_f = 1ms$, we fix the length of user slots to T_N, which denotes the time it takes to transmit a single packet using the highest rate available, in this case N. As such, the number of active users that can be multiplexed is random variable and is a function of the SNR.

From Fig.3, it is clear that a packet delay in Mode 1 requires $6T_N$ slots, Mode 2 needs $3T_N$ slots and Mode 3 requires $2T_N$ slots whilst only a single slot is used for Mode 4.

Assuming a realistic Poisson arrivals where the number of packet arrivals at the transmitter over a CTI are random which make up messages of M packets whose length is geometrically distributed.

It is evident that the message service time is simply the sum of individual packet service times, as such we have a random sum of correlated random variables of a random message length. The packet service times are correlated due to the effect of time correlated fading channel as per A1.

Therefore we can write the time it takes to transmit a message of M packets using AMC for the proposed TDMA frame structure in terms of the number of slots, which is given as;

$$T_m = \sum_{i=1}^{M} \sum_{N=1}^{N_s(n)} T_N \tag{10}$$

For simplicity we rewrite Eq. (10) as;

$$T_m = \sum_{i=1}^{M} \chi_i \tag{11}$$

where, $\chi_i = \sum_{N=1}^{N_s(n)} T_N$

The following are the assumptions taken in deriving the service time distribution;

1. The message service time. T_m is a random variable.
2. M is a geometrically distributed random variable.
3. χ_i's are non-IID.
4. T_m and M dependent.

The service time distribution probability generating function is given as

$$\begin{aligned} G_{T_m}(z) &= E(z^{T_m}|\chi_0 = 1) \\ &= E[E(z^{\chi_0+\sum_{i=1}^{M}\chi_i}|\chi_1)|\chi_0 = 1] \\ &= zG_\chi(G_{T_{m-1}}(z)) \end{aligned} \tag{12}$$

In order to analyze delay statistics of the M/G/1 queue model, we obtain the mean and the variance of T_m using Adam's and Eve's law.

Since T_m is finite random variable and dependent on M, we use law of total expectation whose mean is given as;

$$\begin{aligned} E(T_m) &= E(E(T_m|M)) \\ &= E(\sum_M E(T_m|M)) \\ &= \mu E(M) \\ &= \mu\mu_M \end{aligned} \tag{13}$$

Let $\mu = E(\chi_i), \sigma^2 = Var(\chi_i)$ and $\rho_{ij}\sigma^2 = Cov(\chi_i, \chi_j)$. We now derive the variance of the service time as follows,

$$Var(T_m) = E(Var(T_m|M)) + Var(E(T_m|M)) \tag{14}$$

but;

$$\begin{aligned} Var(T_m|M) &= Var\left(\sum \chi_i\right) \\ &= MVar(\chi_i) + M(M-1)\rho_{ij}\sigma^2 \end{aligned} \tag{15}$$

$$
\begin{aligned}
Var(T_m) &= E\Big\{MVar(\chi_i) + M(M-1)\rho_{ij}\sigma^2\Big\} + \mu^2 Var(M) \\
&= E(M)\sigma^2 + E(M^2 - M)\sum_{i=1}^{M}\sum_{j=2}^{M}\rho_{ij}\sigma^2 + \mu^2 Var(M) \\
&= E(M)\sigma^2 + \Big\{E(M^2) - E(M)\Big\}\sum_{i=1}^{M}\sum_{j=2}^{M}\rho_{ij}\sigma^2 + \mu^2 Var(M) \qquad (16)
\end{aligned}
$$

Since

$$
Var(M) = E(M^2) - [E(M)]^2 \qquad (17)
$$

We obtain;

$$
\begin{aligned}
Var(T_m) &= \sigma^2 E(M) + \sigma^2\Big\{Var(M) + [E(M)]^2 - E(M)\Big\}\sum_{i=1}^{M}\sum_{j=2}^{M}\rho_{ij} + \mu^2 Var(M) \\
&= \sigma^2\mu_M + \sigma^2\Big\{\sigma_M^2 + \mu_M^2 - \mu_M\Big\}\sum_{i=1}^{M}\sum_{j=2}^{M}\rho_{ij} + \mu^2\sigma_M^2 \qquad (18)
\end{aligned}
$$

Where $\mu_M = E(M)$ and $\sigma_M^2 = Var(M)$ are first and second moments of the message length distribution respectively.

5. Conclusion

In this paper, we have derived analytical expressions for the mean and variance of a random sum of correlated random variables in terms of the number of slots that a message needs to be transmitted for a singles active AMC-TDMA user. Lastly, we obtained the pgf of the arbitrary service time distribution of an M/G/1 with AMC. These expressions are important in analysing the effect of a correlated fading channel on the delay statistics of an M/G/1 Queuing Model that employs Adaptive Modulation and Coding for multimedia applications.

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