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Locomotion of a Slender Floating Hyperredundant Mechanism with Shape Adaptation

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Abstract - We consider the continuous model of a mobile slender mechanism. Rigid body degrees of freedom and deformability of the system are coupled through a Lagrangian weak form that includes control inputs to achieve forward locomotion and shape adaptation. The forward locomotion and the shape adaptation are associated to the coupling with a substrate that models a generic environment in which the robot could be deployed. The assumption of small deformations around rigid body placements allows to adopt the floating reference kinematic description. The weak form is naturally associated to an approximate solution technique based on Galerkin projection on the linear mode shapes of the Timoshenko beam model, that is adopted to describe the body of the robot. Simulation results illustrate the tracking problem when the mechanism is deployed on a smooth surface.

Keywords: Distributed parameters, shape adaptation, dynamics, control.

1. Introduction

Over the last few decades, researchers have studied the bio-mechanics of living organisms with hyper redundant morphologies (Kang et al., 2013; Chirikjian and Burdick, 1995; Atakan et al., 2005). Multi segment and flexible slender robots are generally inspired by biological characteristics in living organisms, and find application in several fields such as health (Menciassi and Dario, 2003; Dario et al., 2003; González-Mora et al., 1999), industrial smart health monitoring (Huston et al., 2004; Esser and Huston, 2005), energy and energy harvesting (Sugawara-Narutaki, 2013; Stevens and Mecklenburg, 2012), to name a few. A Cosserat solid approach has been adopted in (Boyer et al., 2012) to model the dynamics of several kinematically locomoted bio-inspired slender systems. Shape adaptation and path tracking with multi-link manipulators is presented in (Bopearatchy and Hatanwala, 1990; Nanayakkara et al., 2000; Moallem et al., 1997; Xu et al., 2001), where high accuracy path tracking is achieved with high speed systems.

Multibody mobile robots with large number of degrees of freedom can be modeled as one dimensional continua with local Euclideian structure (beam models), due to the slenderness of the system. The vibrations of a flexible manipulator based on the linear Euler-Bernoulli beam model are discussed in (Ower and Van de Vegte, 1987). In this paper we present a model for the forward locomotion and shape adaptation with a slender hyper-redundant mechanism. The mechanism is modeled as a Timoshenko beam in plane motion with natural (force) boundary conditions, which allows a rigid body motion to the system, kinematically described by three degrees of freedom. We assume that the characteristic length of the robot is small as compared to the radius of curvature of the substrate, therefore adopting small deformations kinematics around rigid body placements. This leads to the use of the floating reference frame description (Shabana, 2010, Chapter 5). By reproducing the scenario of a slender robot deployed in a generic environment, the shape-tracking problem is posed in terms of coupling with a substrate. This coupling is realized through a distributed system of spring elements that, in terms of feedback, are represented by a distributed force. The forward locomotion is expressed in terms of the rigid body degrees of freedom tracking a moving point on the substrate. The forward locomotion can therefore be described as a path following problem by employing a Frenet frame intrinsic description of the substrate as a parametrized curve in the two-dimensional environment (Altafini, 2002). The forward locomotion and shape adaptation problems are coupled by posing the problem in a distributed control framework with minimization of a suitable action functional based on the Lagrangian function of the system. A numerical solution based on the Galerkin projection on the linear mode shapes is obtained.

2. Kinematics

We consider the planar motion of a slender robot, with a flexible body modeled as a beam. The material body in the reference configuration has the form of a prism $\mathscr{P}_0$ of $\mathscr{E}$, where $\mathscr{E}$ is the Euclideian three-dimensional space, with associated space of translations $\mathscr{U}$. The reference configuration $\mathscr{P}_0$ is referred to the material coordinates $\mathbf{X} = \{X_1, X_2, X_3\}$ along the orthonormal Cartesian basis $\{\mathbf{E}_1, \mathbf{E}_2, \mathbf{E}_3\}$. The cross section of the beam-like body in the reference configuration is the rigid surface spanned by $\mathbf{E}_2$ and $\mathbf{E}_3$. For an undeformed length ℓ, the coordinate $X_1 \in [0, \ell]$ is the locus of the centroids of the cross sections, and $\mathbf{E}_1$ spans the tangent space to the axis (support) of the beam described by such coordinate.

We want to describe the motion of the robot as composed by a rigid body placement and by a small deformation about the rigid body placement. Therefore we adopt the concept of floating frame that is extensively described in (Shabana, 2010, Chapter 5), and used in (Zehetner and Irschik, 2005) to formulate the problem of vibrations of beams caused by a prescribed rigid motion. The rigid body placement is described by the change of coordinates $\mathbf{x}(\mathbf{X}, t) = \mathbf{d}(t) + \mathbf{R}(\theta(t))\,(\mathbf{X} - \delta\ell\mathbf{E}_1)$

$$\mathbf{x}(\mathbf{X}, t) = \mathbf{d}(t) + \mathbf{R}(\theta(t))\,(\mathbf{X} - \delta\ell\mathbf{E}_1) \tag{1}$$

that maps $\mathbf{X} \in \mathscr{P}_0$ to $\mathbf{x} \in \mathscr{P}_\mathrm{R}$, where $\mathscr{P}_\mathrm{R}$ is the region corresponding to the rigid body placement. The rigid change of coordinates is as usual composed of a rigid body displacement $\mathbf{d}$ that represents the time-varying position of a point in $\mathscr{P}_\mathrm{R}$ with respect to the origin of the fixed reference frame, and by the action of the rotation tensor $\mathbf{R}(\theta)$ defined by (see for example (Chadwick, 1998)) $\mathbf{R}(\theta) = \mathbf{E}_3 \otimes \mathbf{E}_3 + \cos\theta\,(\mathbf{E}_1 \otimes \mathbf{E}_1 + \mathbf{E}_2 \otimes \mathbf{E}_2) - \sin\theta\,(\mathbf{E}_1 \otimes \mathbf{E}_2 - \mathbf{E}_2 \otimes \mathbf{E}_1)$ where $\otimes$ is the tensor product defined by the projection $(\mathbf{u} \otimes \mathbf{v})\mathbf{w} = (\mathbf{v} \cdot \mathbf{w})\mathbf{u}$ for $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ in $\mathscr{U}$, with "$\cdot$" indicating the associated inner product. Moreover, $\delta \in [0, 1]$ defines the point on the axis that is left unaltered by the action of $\mathbf{R}$, so that the rigid body motion is composed of a translation $\mathbf{d}$ and of a rotation around an axis passing through the point with position $\delta\ell\mathbf{E}_1$ with respect to the left boundary of the undeformed body.

The small deformation about the rigid body placement $\mathscr{P}_\mathrm{R}$ is described by a map $\chi : \mathscr{P}_\mathrm{R} \to \mathscr{P}$ that takes points $\mathbf{x}$ and maps them to the point $\chi(\mathbf{x}, t)$ in the current configuration $\mathscr{P}$ as $\chi(\mathbf{x}, t) = \mathbf{x} + \mathbf{U}(\mathbf{x}, t)$, where $\mathbf{U}$ is a small deformation, that consistently with the the linearized planar Timoshenko beam theory (Timoshenko, 1974) is given by $\mathbf{U}(\mathbf{x}, t) = (u(x_1, t) - X_2\psi(x_1, t))\mathbf{e}_1 + w(x_1, t)\mathbf{e}_2$, where $\mathbf{e}_i = \mathbf{R}\mathbf{E}_i$ are rotated orthonormal basis vectors (floating reference frame (Shabana, 2010)) that are used to describe the rigid body placement $\mathscr{P}_\mathrm{R}$, and $x_i = \mathbf{x} \cdot \mathbf{e}_i$. From (1) we have

$$\begin{aligned} x_1 = \mathbf{x} \cdot \mathbf{e}_1 &= \mathbf{d} \cdot \mathbf{e}_1 + (X_1 - \delta\ell)\mathbf{e}_1 \cdot \mathbf{R}\mathbf{E}_1 \\ &= \mathbf{d} \cdot \mathbf{e}_1 + (X_1 - \delta\ell)\mathbf{E}_1 \cdot \mathbf{R}^\mathsf{T}\mathbf{R}\mathbf{E}_1 = \mathbf{d} \cdot \mathbf{e}_1 + (X_1 - \delta\ell) \end{aligned} \tag{2}$$

$$x_2 = \mathbf{x} \cdot \mathbf{e}_2 = \mathbf{d} \cdot \mathbf{e}_2 + X_2\mathbf{e}_2 \cdot \mathbf{R}\mathbf{E}_2 = \mathbf{d} \cdot \mathbf{e}_2 + X_2 \tag{3}$$

where we have used the property $\mathbf{R}^{-1} = \mathbf{R}^\mathsf{T}$ (orthogonality of $\mathbf{R}$) and the definition of transpose $\mathbf{v}_2 \cdot \mathbf{R}\mathbf{v}_1 = \mathbf{v}_1 \cdot \mathbf{R}^\mathsf{T}\mathbf{v}_2$ for any two vectors $\mathbf{v}_1$ and $\mathbf{v}_2$.

In the floating reference frame, the material time derivative is performed by keeping $\mathbf{x}$ constant (Zehetner and Irschik, 2005), and therefore $\mathscr{P}_{\mathrm{R}}$ is treated as the reference configuration with respect to the deformation. This approximation holds due to the hypothesis of small deformations around the rigid body placement. The velocity of a point χ in $\mathscr{P}$ is obtained as its material time derivative, that is therefore given by

$$\dot{\chi} = \dot{\mathbf{d}} + \dot{\theta}\mathbf{W}(\mathbf{x}-\mathbf{d}) + \dot{\mathbf{U}} = \dot{\mathbf{d}} + \dot{\theta}\mathbf{W}((X_1-\delta\ell)\mathbf{e}_1 + X_2\mathbf{e}_2) \tag{4}$$

where $\mathbf{W}$ is the skew-symmetric tensor $\mathbf{W} = \mathbf{e}_2 \otimes \mathbf{e}_1 - \mathbf{e}_1 \otimes \mathbf{e}_2$ which allows to describe the time derivative of unit basis vectors rotating with angular velocity $\dot{\theta}$ as $\dot{\mathbf{e}}_i = \dot{\theta}\mathbf{W}\mathbf{e}_i$; this relation has bees used to derive the third therm on the right hand side of (4). The action of the skew symmetric tensor $\mathbf{W}$ on the floating basis vectors allows to write $\dot{\mathbf{x}}$ as $\dot{\mathbf{x}} = \dot{\mathbf{d}} + \dot{\theta}((X_1-\delta\ell)\mathbf{e}_2 - X_2\mathbf{e}_1)$.

In the floating reference frame the material derivative of $\mathbf{U}$ is given by $\dot{\mathbf{U}} = (\dot{u} - X_2\dot{\psi})\mathbf{e}_1 + \dot{w}\mathbf{e}_2 + \dot{\theta}\mathbf{W}\mathbf{U} = (\dot{u} - X_2\dot{\psi} - \dot{\theta}w)\mathbf{e}_1 + (\dot{w} + \dot{\theta}(u - X_2\psi))\mathbf{e}_2$. The time derivative if the map χ can therefore explicitly be written as

$$\dot{\chi} = \dot{\mathbf{d}} + \left(\dot{u} - X_2\dot{\psi} - \dot{\theta}(w + X_2)\right)\mathbf{e}_1 + \left(\dot{w} + \dot{\theta}(u - X_2\psi + X_1 - \delta\ell)\right)\mathbf{e}_2 \tag{5}$$

Within the hypothesis of small deformations we consider the linearization $(\mathbf{I} + \nabla_{\mathbf{x}}\mathbf{U})^{\mathsf{T}}(\mathbf{I} + \nabla_{\mathbf{x}}\mathbf{U}) \simeq \mathbf{I} + \nabla_{\mathbf{x}}\mathbf{U} + \nabla_{\mathbf{x}}^{\mathsf{T}}\mathbf{U}$, so that the Green-Saint-Venant strain ε reduces to the symmetric part of $\nabla_{\mathbf{x}}\mathbf{U}$, and the gradient is represented in operator form as $\nabla_{\mathbf{x}} = \frac{\partial}{\partial x_j} \otimes \mathbf{e}_j$. Therefore we obtain $\varepsilon = \mathrm{sym}\nabla_{\mathbf{x}}\mathbf{U} = (u' - X_2\psi')\mathbf{e}_1 \otimes \mathbf{e}_1 + \frac{1}{2}(w' - \psi)(\mathbf{e}_2 \otimes \mathbf{e}_1 + \mathbf{e}_1 \otimes \mathbf{e}_2)$, where $(\cdot)'$ means differentiation with respect to x_1.

3. Closed Loop System

In this section we derive the weak form of the governing equations of a slender robot with kinematics described in the previous Section. The weak form allows to state the well posedness of a distributed parameters control problem with inputs suitably included in the variational form.

The kinetic energy of the system is then given by $\mathscr{K} = \frac{1}{2}\int_{\mathscr{P}} \rho\dot{\chi}\cdot\dot{\chi}\mathrm{d}\mathscr{P}$. Given the geometry of $\mathscr{P}$, we consider the Cartesian product structure $\mathscr{P} = \mathscr{A} \times [0,\ell]$, where ℓ is the undeformed length of the beam and $\mathscr{A}$ is the two-dimensional Euclideian point space defining a rigid cross section. The axis of the beam is therefore spanned by the coordinate $x_1 \in [d_1 - \delta\ell, d_1 + \ell(1-\delta)]$, whereas x_2 and x_3 span the cross section. Therefore the integral in $\mathscr{P}$ is accordingly decomposed as $\int_{\mathscr{P}} = \int_{d_1-\delta\ell}^{d_1+\ell(1-\delta)}\int_{\mathscr{A}}$. We introduce $d_i = \mathbf{d}\cdot\mathbf{e}_i$, that are the components of $\mathbf{d}$ in the body reference frame. We now operate the change of coordinate $x_1(X_1) = d_1 + X_1 - \delta\ell$ (see (2)) (with unit Jacobian, as expected being a rigid change of coordinate), and define the material descriptions of different scalar fields involved in the integration $u^\star(X_1,t) := u(x_1(X_1),t) = u(d_1 + X_1 - \delta\ell)$ ($w^\star$ and $\psi^s tar$ are introduced similarly through w and ψ). Since the origins of coordinates $\{X_2, X_3\}$ are the centroids of the cross section (whose locus describes the axis of the beam) we have $\int_{\mathscr{A}} \rho X_2\mathrm{d}\mathscr{A} = 0$, and $\int_{\mathscr{A}} \rho(X_2)^2\mathrm{d}\mathscr{A} = I$, where I is the moment of inertia about X_3 (normal to the plane of motion). We introduce the nondimensional kinematic variables X_1/ℓ, $u^\star/\ell$, and $w^\star/\ell$ with respect to the length ℓ, and the nondimensional forces $b_N\ell/kAG$, $b_Q\ell/kAG$, b_M/kAG, and f_i/kAG. Since no confusion arises, we indicate henceforth the nondimensional fields with the same symbols previously used for the the corresponding dimensional ones. Let $\mathbf{z} = (d_1, d_2, \theta, u^\star, w^\star, \psi^\star)$ be the state vector. With the introduction of the nondimensional parameters

$$\alpha_1 = \frac{Y}{kG}, \quad \alpha_2 = \frac{I}{A\ell^2}, \quad \bar{t}^2 = \frac{\rho\ell^2}{kG} \tag{6}$$

where $\bar{t}$ is a characteristic time, we rewrite the kinetic energy in nondimensional form as $\mathscr{K} = \frac{1}{2}\int_0^1 \dot{\mathbf{z}}^{\mathsf{T}}\mathbf{M}(\mathbf{z})\dot{\mathbf{z}}\mathrm{d}X_1 = \frac{1}{2}\int_0^1 M_{ij}(\mathbf{z})\dot{z}_i\dot{z}_j\mathrm{d}X_1$, where we have adopted the convention of summing repeated indexes in their respective ranges. The 6×6 matrix $\mathbf{M}$ is given by

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & -q_2 & 1 & 0 & 0 \\ 0 & 1 & q_1 & 0 & 1 & 0 \\ -q_2 & q_1 & \alpha_2(1+\psi^\star)^2+q_1^2+q_2^2 & -q_2 & q_1 & \alpha_2 \\ 1 & 0 & -q_2 & 1 & 0 & 0 \\ 0 & 1 & q_1 & 0 & 1 & 0 \\ 0 & 0 & \alpha_2 & 0 & 0 & \alpha_2 \end{pmatrix} \tag{7a}$$

with $q_1 = d_1 + u^\star + X_1 - \delta$ and $q_2 = d_2 + w^\star$. We define the collection of strain components $\bar{\varepsilon} = (u^{\star\prime}, w^{\star\prime} - \psi^\star, \psi^{\star\prime})$, the 3×3 matrix $\mathbf{K} = \mathrm{diag}(\alpha_1, 1, \alpha_1\alpha_2)$, and the collection of forces and torques $\boldsymbol{\tau} = (f_1, f_2, f_3, b_1, b_2, b_3)$, where f_i are duals of rigid body degrees of freedom d_1, d_2, θ, and b_is are duals of deformation fields $u^\star$, $w^\star$, $\psi^\star$, so that the nondimensional potential energy of the system is rewritten as $\mathscr{V} = \int_0^1 \left(\frac{1}{2}\bar{\varepsilon}^\top \mathbf{K}\bar{\varepsilon} - \boldsymbol{\tau}^\top \mathbf{z}\right) \mathrm{d}X_1 = \int_0^1 \left(\frac{1}{2}K_{ij}\bar{\varepsilon}_i\bar{\varepsilon}_j - \tau_i z_i\right) \mathrm{d}X_1$, where the work term $\tau_i z_i$ is transported under the integral by dividing by the nondimensional length of the domain, that in this case is 1.

In order to obtain the weak form of the evolution equations we introduce the Lagrangian function $\mathscr{L}(\mathbf{z}, \dot{\mathbf{z}}, \bar{\varepsilon}, \mathbf{b}, \boldsymbol{\tau}) = \mathscr{K}(\mathbf{z}, \dot{\mathbf{z}}) - \mathscr{V}(\mathbf{z}, \bar{\varepsilon}, \boldsymbol{\tau})$ where, with abuse of notation, we have indicated with $\dot{\{\cdot\}}$ the material time derivative applied componentwise to the collection of states. Here the external forces are interpreted as control inputs to drive the corresponding dual kinematic quantities to desired values. The (strong) governing evolution equations are the cofactors of the variations $\tilde{\mathbf{z}}$, $\tilde{\boldsymbol{\tau}}$, that describe the evolution of the minimizers of the action functional $\int_{t_1}^{t^2} \mathscr{L}\mathrm{d}t$ between two fixed points t_1 and t_2. Minimization of the action functional corresponds to the stationarity of its gradient (Gâteaux derivative) along the variations of its arguments. Here we consider the weak form, that is built by considering the cofactors of all arguments of the Lagrangian function; the weak form is suitable for numerical solution. After time integration by parts the stationarity of the gradient of the action functional gives $\int_{t_1}^{t_2}\int_0^1 \left(-\tilde{z}_k M_{kj}\ddot{z}_j - \tilde{z}_k \frac{\partial M_{kj}}{\partial z_i}\dot{z}_i\dot{z}_j + \tilde{z}_k \frac{1}{2}\frac{\partial M_{ij}}{\partial z_k}\dot{z}_i\dot{z}_j - K_{ij}\tilde{\bar{\varepsilon}}_i\bar{\varepsilon}_j + \tilde{z}_i\tau_i\right) \mathrm{d}X_1\mathrm{d}t = 0$, where consistently with the Hamilton-Kirchhoff variational principle we have assumed that all fields are assigned at times t_1 and t_2, which implies that the boundary terms arising from the integration by parts in time are zero (since the corresponding variations of the fields are zero whenever the fields are assigned). By introducing

$$\mathbf{C}(\mathbf{z}, \dot{\mathbf{z}}) = \begin{pmatrix} 0 & 0 & \dot{q}_2 + \dot{\theta}q_1 & 0 & 0 & 0 \\ 0 & 0 & -\dot{q}_1 + \dot{\theta}q_2 & 0 & 0 & 0 \\ \dot{q}_2 + \dot{\theta}q_1 & -\dot{q}_1 + \dot{\theta}q_2 & -\dot{q}_1 q_1 - \dot{q}_2 q_2 - \alpha_2\dot{\psi}^\star\psi^\star & \dot{q}_2 + \dot{\theta}q_1 & -\dot{q}_1 + \dot{\theta}q_2 & \alpha_2\dot{\theta}\psi^\star \\ 0 & 0 & \dot{q}_2 + \dot{\theta}q_1 & 0 & 0 & 0 \\ 0 & 0 & -\dot{q}_1 + \dot{\theta}q_2 & 0 & 0 & 0 \\ 0 & 0 & \alpha_2\dot{\theta}\psi^\star & 0 & 0 & 0 \end{pmatrix} \tag{8a}$$

By exploiting the arbitrariness of t_1 and t_2 the weak form of the problem is rewritten as

$$\begin{aligned} 0 &= \int_0^1 \left(\tilde{z}_i M_{ij}(\mathbf{z})\ddot{z}_j + \tilde{z}_i C_{ij}(\mathbf{z}, \dot{\mathbf{z}})\dot{z}_j + K_{ij}\tilde{\bar{\varepsilon}}_i\bar{\varepsilon}_j - \tau_i\tilde{z}_i\right) \mathrm{d}X_1 \\ &= \int_0^1 \left(\tilde{\mathbf{z}}^\top \mathbf{M}(\mathbf{z})\ddot{\mathbf{z}} + \tilde{\mathbf{z}}^\top \mathbf{C}(\mathbf{z}, \dot{\mathbf{z}})\dot{\mathbf{z}} + \tilde{\bar{\varepsilon}}^\top \mathbf{K}\bar{\varepsilon} - \tilde{\mathbf{z}}^\top \boldsymbol{\tau}\right) \mathrm{d}X_1 \end{aligned} \tag{9}$$

We consider the distributed coupling with the substrate to be given by normal actions with respect to the axis of the beam, therefore dual of the transverse displacement w. This implies that $b_1 = b_3 = 0$ (no axial distributed forces and no distributed couples). Therefore the external forces vector is redefined as $\boldsymbol{\tau} =$

$(f_1, f_2, f_3, 0, b_2, 0)$, with f_is applied at $x_1 (X_1 = 1)$. The distributed parameters control problem is stated as follows: find a feedback $\boldsymbol{\tau}(\mathbf{z}, \mathbf{z}^{\mathrm{d}}) \in L^2(0,1)$ that stabilizes $\mathbf{z} \in \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times S^1(0,1)$ around the desired trajectory $\mathbf{z}^{\mathrm{d}}$ according to (9) for all $\tilde{\mathbf{z}}$, where S^1 is the usual Sobolev space of functions with first derivative square integrable in $(0,1)$.

4. Galerkin Projection

A finite dimensional projection of the system (9) is obtained by a separation of variables with respect to space and time. Since the rigid body displacement degrees of freedom are functions of time only the separation of variables for $\mathbf{d}$ and θ is trivial. The deformation fields are instead decomposed as $u^\star(X_1, t) = \bar{\mathbf{u}}^\mathsf{T}(X_1)\mathbf{a}(t)$, $w^\star(X_1, t) = \bar{\mathbf{w}}^\mathsf{T}(X_1)\mathbf{b}(t)$, and $\psi^\star(X_1, t) = \bar{\boldsymbol{\psi}}^\mathsf{T}(X_1)\mathbf{c}(t)$, where $\bar{\mathbf{u}} = (\bar{u}_1, \ldots, \bar{u}_n)$, $\bar{\mathbf{w}} = (\bar{w}_1, \ldots, \bar{w}_n)$, and $\bar{\boldsymbol{\psi}} = (\bar{\psi}_1, \ldots \bar{\psi}_n)$ are $n-$dimensional sets of spatial basis functions, and $\mathbf{a} = (a_1, \ldots, a_n)$, $\mathbf{b} = (b_1, \ldots, b_n)$, and $\mathbf{c} = (c_1, \ldots, c_n)$ are time dependent vectors of amplitudes. We introduce the $6 \times n+3$ matrix $\bar{\mathbf{z}}$ (collection of basis functions) and the $n+3$ dimensional collection of amplitudes $\boldsymbol{\zeta}$

$$\bar{\mathbf{z}} = \begin{pmatrix} I_{3\times 3} & 0_{3\times n} \\ & \bar{\mathbf{u}}^\mathsf{T}(X_1) \\ 0_{3\times 3} & \bar{\mathbf{w}}^\mathsf{T}(X_1) \\ & \bar{\boldsymbol{\psi}}^\mathsf{T}(X_1) \end{pmatrix}, \quad \boldsymbol{\zeta} = (d_1, d_2, \theta, \mathbf{a}, \mathbf{b}, \mathbf{c})$$

so that $\mathbf{z} = \bar{\mathbf{z}}\boldsymbol{\zeta}$ and $\tilde{\mathbf{z}} = \bar{\mathbf{z}}\tilde{\boldsymbol{\zeta}}$. Therefore (9) must hold for all $\tilde{\boldsymbol{\zeta}}$, which implies the reduced order system evolution $\boldsymbol{\mu}_{\mathbf{M}}(\boldsymbol{\zeta})\ddot{\boldsymbol{\zeta}} + \boldsymbol{\mu}_{\mathbf{C}}(\boldsymbol{\zeta}, \dot{\boldsymbol{\zeta}})\dot{\boldsymbol{\zeta}} + \boldsymbol{\mu}_{\mathbf{K}}\boldsymbol{\zeta} = \mathbf{F}\left(\boldsymbol{\zeta}, \dot{\boldsymbol{\zeta}}, \mathbf{z}^{\mathrm{d}}, \dot{\mathbf{z}}^{\mathrm{d}}\right)$ of the $3n+3$ coefficients $\boldsymbol{\zeta}$. The $3n+3 \times 3n+3$ operators $\boldsymbol{\mu}_{\mathbf{M}}$, $\boldsymbol{\mu}_{\mathbf{C}}$, and $\boldsymbol{\mu}_{\mathbf{K}}$ and the $3n+3$ load vector $\mathbf{F}$ are given by

$$\boldsymbol{\mu}_{\mathbf{M}}(\boldsymbol{\zeta}) = \int_0^1 \bar{\mathbf{z}}^\mathsf{T}\mathbf{M}(\bar{\mathbf{z}}\boldsymbol{\zeta})\bar{\mathbf{z}}\mathrm{d}X_1, \quad \boldsymbol{\mu}_{\mathbf{C}}(\boldsymbol{\zeta}, \dot{\boldsymbol{\zeta}}) = \int_0^1 \bar{\mathbf{z}}^\mathsf{T}\mathbf{C}(\bar{\mathbf{z}}\boldsymbol{\zeta}, \bar{\mathbf{z}}\dot{\boldsymbol{\zeta}})\bar{\mathbf{z}}\mathrm{d}X_1, \quad \boldsymbol{\mu}_{\mathbf{K}} = \int_0^1 \begin{pmatrix} 0_{3\times 3+3n} \\ 0_{3+3n\times 3} & \bar{\mathbf{K}} \end{pmatrix} \mathrm{d}x_1$$

$$\bar{\mathbf{K}} = \begin{pmatrix} \alpha_1 \bar{\mathbf{u}}'^\mathsf{T}\bar{\mathbf{u}}' & 0_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & \bar{\mathbf{w}}'^\mathsf{T}\bar{\mathbf{w}}' & -\bar{\mathbf{w}}'^\mathsf{T}\bar{\boldsymbol{\psi}} \\ 0_{n\times n} & -\bar{\boldsymbol{\psi}}^\mathsf{T}\bar{\mathbf{w}}' & \alpha_1\alpha_2\bar{\boldsymbol{\psi}}'^\mathsf{T}\bar{\boldsymbol{\psi}}' + \bar{\boldsymbol{\psi}}^\mathsf{T}\bar{\boldsymbol{\psi}} \end{pmatrix}, \quad \mathbf{F}\left(\boldsymbol{\zeta}, \dot{\boldsymbol{\zeta}}, \mathbf{z}^{\mathrm{d}}, \dot{\mathbf{z}}^{\mathrm{d}}\right) = \int_0^1 \tilde{\mathbf{z}}^\mathsf{T}\varphi\left(\bar{\mathbf{z}}\boldsymbol{\zeta}, \bar{\mathbf{z}}\dot{\boldsymbol{\zeta}}, \mathbf{z}^{\mathrm{d}}, \dot{\mathbf{z}}^{\mathrm{d}}\right)\mathrm{d}x_1$$

Basis functions $\bar{\mathbf{u}}$, $\bar{\mathbf{w}}$, and $\bar{\boldsymbol{\psi}}$ are obtained by solving the eigenvalues problem associated with a planar Timoshenko beam. Details about analytic expressions can be found in (Fattahi and Spinello, 2013).

Let $\boldsymbol{\eta}(s) = \eta_1(s)\mathbf{E}_1 + \eta_2(s)\mathbf{E}_2$ be the position in the global frame of a point on the substrate in which the mechanism is deployed, parametrized by the arclength s. Moreover, let $\mathbf{p}(\mathbf{d}, X_1) = \mathbf{x} - X_2\mathbf{e}_2 = \mathbf{d} + (X_1 - 1)\mathbf{e}_1$ be the point on the undeformed axes and $\mathbf{g}$ be a vector with constant components in the body reference frame, so that $\dot{g} = \mathbf{W}\mathbf{g}$. The desired state $\mathbf{z}^{\mathrm{d}}$ is defined by the list $((\boldsymbol{\eta}(s) - \mathbf{g}) \cdot \mathbf{e}_1, (\boldsymbol{\eta}(s) - \mathbf{g}) \cdot \mathbf{e}_2, \theta_{\boldsymbol{\eta}}(s), 0, (\boldsymbol{\eta}(\bar{s}(x_1)) - \mathbf{g}) \cdot \mathbf{e}_2, 0)$, where $\tan\theta_{\boldsymbol{\eta}} = \eta_2'/\eta_1'$ so that $\theta_{\boldsymbol{\eta}}$ is the global orientation of the tangent vector $\boldsymbol{\eta}'$, and $\bar{s}(x_1)$ is the arclength that defines a point on the curve corresponding to the solution of the minimization problem $\bar{s}(X_1) = \arg\min_s \|\mathbf{p}(X_1) - \boldsymbol{\eta}(s)\|$, which is solved by the roots s of the scalar equation $(\mathbf{p}(x_1) - \boldsymbol{\eta}(s)) \cdot \boldsymbol{\eta}'(s) = 0$. Therefore the nonzero components of $\boldsymbol{\tau}$ are determined by the feedback laws corresponding to a PD controller

$$f_i = \left(\kappa_{d_i}\left(\boldsymbol{\eta}(s) - \mathbf{p}(1) - \mathbf{g}\right) + \bar{\kappa}_{d_i}\left(\dot{s}\boldsymbol{\eta}'(s) - \dot{\mathbf{p}}(1) - \dot{\mathbf{g}}\right)\right) \cdot \mathbf{e}_i, \quad i = 1,2 \tag{10a}$$

$$f_3 = \kappa_\theta\left(\theta_{\boldsymbol{\eta}}(s) - \theta\right) + \bar{\kappa}_\theta\left(\theta_{\boldsymbol{\eta}}'(s)\dot{s} - \dot{\theta}\right) \tag{10b}$$

$$b_2 = \kappa_{b_2}\left(\eta_1(x_1)R_{1i} + \eta_2(x_1)R_{2i} - w^\star - \mathbf{g} \cdot \mathbf{e}_2\right) \tag{10c}$$

where κ_{d_i}, κ_θ, and κ_{b_2} are positive proportional gains, and the homonyms quantities with bar are derivative gains; R_{ji} are the components of the two dimensional rotation matrix; $\dot{s}$ can be interpreted as the forward speed of the system, that will be assigned as a driving parameter; and $\theta_{\boldsymbol{\eta}}' = \|\boldsymbol{\eta}''\|$ is the curvature of the substrate.

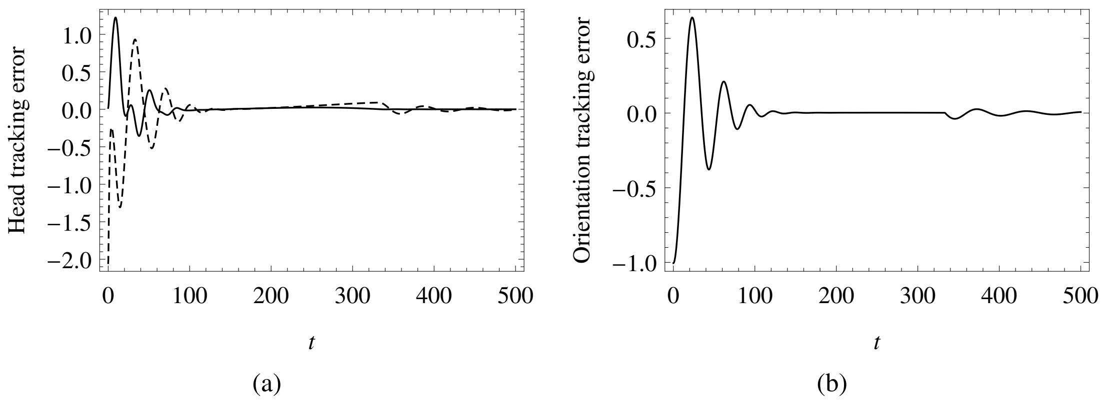

Fig. 1. Time histories of (a) the tracking errors $(\boldsymbol{\eta}(s_t) - \mathbf{p}(1,t)) \cdot \mathbf{e}_i$ (solid line for $i = 1$ and dashed line for $i = 2$); (b) the tracking error $\theta_{\boldsymbol{\eta}}(s_t) - \theta(t)$.

5. Simulation Results

Simulation results are obtained on a simplified system in which the time scale of body deformations is much faster than the time scale of path following rigid body motion, which allows to discard the inertia terms associated with u, w, and ψ; the system is then simulated by solving a static deformation problem around rigid placements at every time step. Deformation fields are approximated with one spatial basis function ($n = 1$). The gap $\mathbf{g}$ is set to zero, and control gains are set to $\kappa_{d_i} = 2$, $\bar{\kappa}_{d_i} = 3$, $\kappa_\theta = 3$, $\bar{\kappa}_\theta = 4$, $\kappa_{b_2} = 10$. The path $\boldsymbol{\eta}$ is parametrized as $\boldsymbol{\eta} = s_t\mathbf{E}_1 + 0.1(s_t - 10)^2(\mathfrak{u}(s_t) - \mathfrak{u}(s_t - 10))\mathbf{E}_2$, where $\mathfrak{u}$ is the unit step function (evaluates to 1 whenever its argument is greater than 0), and $s_t = 0.3t$. Therefore the substrate is an arch of parabola followed by a straight line, see Fig. 2(a).

Time histories of the tracking errors $(\boldsymbol{\eta}(s_t) - \mathbf{p}(1,t)) \cdot \mathbf{e}_i$ and $\theta_{\boldsymbol{\eta}}(s_t) - \theta(t)$ are shown in Figure 1. At time $t = 333$ there is the transition from parabolic to rectilinear path with consequent change of curvature that results into the perturbation of the orientation error; this is explained by the fact that the curvature of the path acts as a disturbance (Altafini, 2002).

Fig. 2(a) shows four snapshots of the system: the initial condition ($t = 0$, at the top); an intermediate state on the parabolic portion of the path ($t = 200$); a state across the point $s_t = 10$ where the change of curvature occurs ($t = 320$), to the final state where the head overlaps to the last defined point of the path ($t = 500$). Fig. 2(b) depicts a zoom of the configuration at time $t = 310$, where it is shown the deformed shape that adapts to the nonzero curvature of the path.

6. Conclusion

We have presented the weak form of a distributed parameters control system that models the path tracking and shape adaptation of a slender mechanism coupled with a smooth surface. The floating reference frame is adopted for the kinematics of the system, based on the hypothesis of small deformations around finite rigid body placements. The path tracking (forward locomotion) and shape adaptation are illustrated through simulation with a parabolic-rectilinear path.

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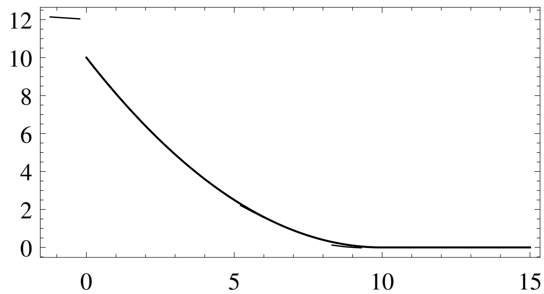
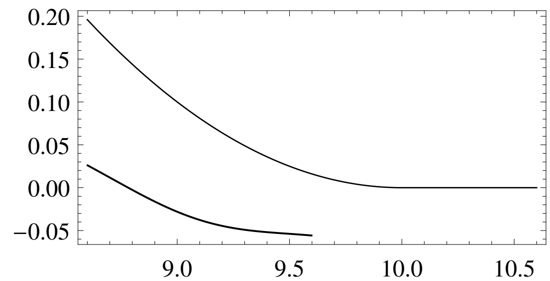


(a)



(b)

Fig. 2. (a) Four snapshots of the system; (b) zoom of the snapshot at $t = 310$ that shows the deformed shape adapted to the nonzero curvature of the path.

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