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Modelling and State Dependent Riccati Equation Control of an Active Hydro-Pneumatic Suspension System

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Abstract **-** In this study, a controller for the nonlinear model of an active Hydro-Pneumatic (HP) suspension system is developed. The active HP suspension system model is incorporated into the quarter car model. A linear model with state dependent matrices of the nonlinear quarter car model is derived. A State Dependent Riccati Equation control (SDRE) is used to attenuate sprung mass acceleration, suspension deflection, and tire deflection. The performance of the controller is examined in both frequency and time domains. Active HP suspension system is simulated with sinusoidal inputs at discrete amplitudes and frequencies and the approximate frequency response functions are obtained. Moreover, active HP suspension system is simulated with random road inputs and the root mean square values of the responses are calculated to evaluate the performance of the controller. The results show that the active suspension successfully and simultaneously decreases the sprung mass acceleration, suspension deflection, and tire deflection around body bounce frequency and thus improved ride comfort and road holding are obtained.

Keywords**:** Hydro-Pneumatic suspension, active control, State Dependent Riccati Equation.

1. Introduction

Passive suspension systems can satisfy ride comfort and handling performances to a limited extend. To improve the performances of the suspension, various active and semi-active suspensions have been proposed. In this study, a quarter car model with a nonlinear HP suspension is developed. In the basic HP suspension system, a gas volume provides the suspension spring characteristics and the flow of oil through an orifice provides the suspension damping characteristics. By changing the orifice opening area, a semi-active HP suspension system can be developed. Similarly, by pumping oil into the system or by extracting oil from the system, an active HP suspension system can be developed.

In literature, there are numerous studies about the active control of the conventional suspension systems. Models of active systems in these studies are mostly linear and thus linear control methods can be applied to determine the desired active force. In some of the studies, hydraulic actuators which provide active force are included in the model and thus actuator dynamics are also taken into account. Basic functions of the suspensions, suspension control methodologies, and the theoretical and practical limitations of the controllable suspensions were examined in the study of the Karnopp and Heess (1991). Elmadany (1990) proposed an optimal linear active suspension system controlled by a multivariable integral control. Alleyne et al. (1993) examined linear and nonlinear control methods in active suspension systems. The desired active force was obtained using skyhook control methods and the realization of this desired control force from hydraulic actuator was performed using Proportional, Integral, and Derivative (PID) control, feedback linearization control, and sliding control. They showed that active suspension with a nonlinear control method provided better performance than the linear control. Pilbeam and Sharp

(1996) examined different active suspension system configurations with preview control together with power consumptions. Thompson and Chaplin (1996) studied the inner loop force control in active suspensions control.

On the other hand there are only a few studies about the active control of the quarter car model with HP suspension system which appears in the literature. El-Demerdash and Crolla (1996) studied the active and slow-active HP suspension systems with preview control. A linear HP suspension system was obtained and linear optimal control approach was used to obtain active suspension. Gao et al. (2005) linearized a HP suspension model and then obtained a reduced model to be used in the control applications. The objective of the controller was disturbance rejection and body leveling control. Linear quadratic control was used for the disturbance rejection. For leveling control, a PID control which used the suspension deflection as the feedback was designed. Shi et al. (2009) linearized a HP suspension model using feedback linearization method and then applied the sliding mode control to the linearized HP model to get an active HP suspension system.

State Dependent Riccati Equation (SDRE) control is a systematic way of designing a nonlinear controller (Mracek and Cloutier, 1998). It is similar to Linear Quadratic Regulator (LQR) control and is suitable for application to nonlinear control problems. Firstly, the linear model with state dependent matrices (SDM) of the nonlinear system is constructed and then the cost function with SDM is specified. The SDRE is solved online to find the control input minimizing the cost function. Moreover, nonlinear estimation problem can also be handled by the SDRE method in such a way that the state dependent observer gain is found online from the solution of the SDRE. In literature, there are studies about the SDRE control and estimation. Banks et al. (2007) studied the application of SDRE in nonlinear control and estimation problems. An extensive review of the SDRE control was presented in the study of Çimen (2008).

In this study, modeling and control of an active HP suspension is examined. Firstly an active HP suspension system is modeled, and then incorporated into quarter car model. Then, the linear model with SDM of the nonlinear quarter car model is obtained. SDRE control method is applied to the nonlinear quarter car model to attenuate the sprung mass acceleration, suspension deflection, and tire deflection. The performances of the controllers are examined with the obtained frequency response plots of the sprung mass acceleration, suspension deflection, and tire deflection and with the time domain plots. By applying the SDRE control as a nonlinear control method, all properties of the well-known LQR control can be exploited.

2. Active Hydro-Pneumatic Suspension System Model

The general layout of the quarter car model with active HP suspension system is given in figure 1. In this study, the control input to the active suspension is assumed as the flow rate to the second chamber. The hydraulic circuit which produces this control input is not modeled and therefore is not taken into account in the active control. A detailed analysis of the passive HP suspension system is given in the study of Joo (1991).

Equation of motion for the main piston is:

$$P_1 A_p - Mg - F_f = M\ddot{z}_p \tag{1}$$

where P_1 is the pressure in the first chamber, A_p is the piston area, M is the sprung mass, g is the constant of gravitational acceleration, z_p is the sprung mass displacement, and F_f is the Coulomb friction between the piston and cylinder. Equation of motion for the floating piston is:

$$(P_3 - P_2 - P_{Atm})A_p - M_{fp}g - F_{fp} = M_{fp}\ddot{z}_{fp} \tag{2}$$

where P_3 is the absolute gas pressure at the third chamber, P_2 is the oil pressure at the second chamber, P_{Atm} is the atmospheric pressure, M_{fp} is the floating piston mass, z_{fp} is the floating piston displacement, and F_{fp} is the Coulomb friction between the cylinder and the floating piston. Assuming incompressible oil, the continuity equation for first and second chambers is:

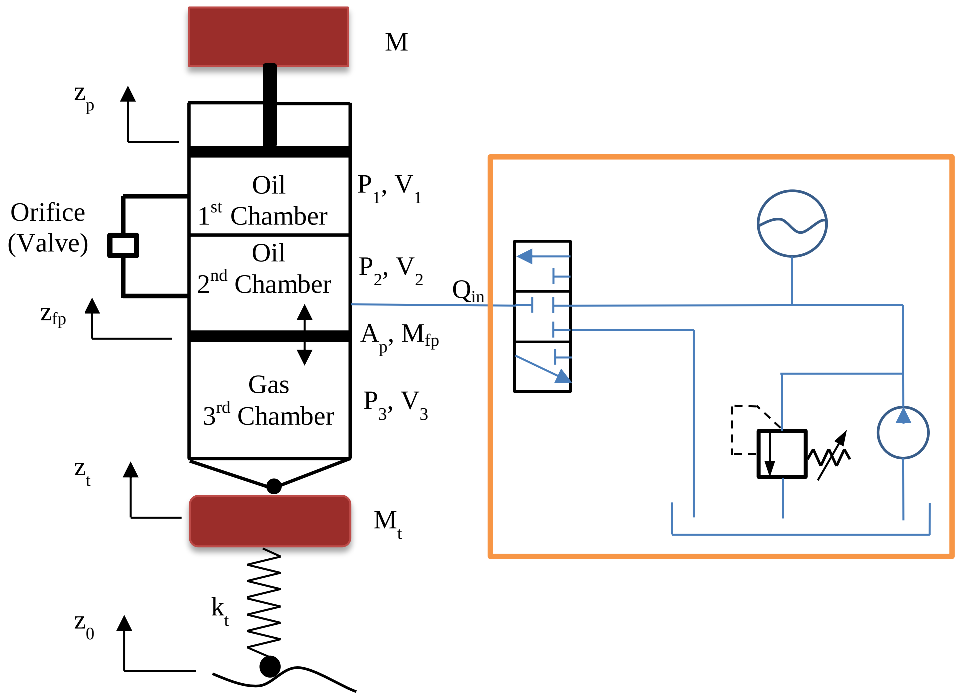

Fig. 1. Active HP suspension.

$$\dot{z}_{fp} = \dot{z}_p - \frac{Q_{in}}{A_p} \tag{3}$$

where Q_{in} is the flow rate which is the control input. The flow rate of oil Q passing through the orifice is:

$$Q = A_p(\dot{z}_p - \dot{z}_t) = A_v C_D \sqrt{\frac{2}{\rho}|P_2 - P_1|}sign(\dot{z}_p - \dot{z}_t) \tag{4}$$

where A_v is the orifice opening area, C_D is the orifice loss factor, z_t is the unsprung mass displacement, and ρ is the oil density. The absolute gas pressure in third chamber can be found using the polytropic gas model as:

$$P_3 = \frac{P_{30}V_{30}^{\gamma}}{[V_{30} + A_p(z_{fp} - z_t)]^{\gamma}} \tag{5}$$

where P_{30} is the absolute gas pressure at static equilibrium, V_{30} is the volume of third chamber at static equilibrium, and γ is the polytropic gas coefficient. Equation of motion for the unsprung mass can be written similarly.

The equations derived so far can be used to simulate the active HP suspension system. The next step is the formation of the state dependent linear model of the active HP suspension system. To simplify the equations, floating piston mass and frictions between the pistons and the cylinder can be neglected. Moreover, a fifth degree polynomial model is fitted to relative gas force as:

$$(P_3 - P_{Atm})A_p - Mg = a_1 z_{fpt} + a_2 z_{fpt}^2 + a_3 z_{fpt}^3 + a_4 z_{fpt}^4 + a_5 z_{fpt}^5 = f_1 z_{fpt} \tag{6}$$

where a's are the polynomial coefficient, z_{fpt} is relative displacement of the floating piston as defined in Eq. (9), and f_1 is the state dependent gas stiffness. Similarly, oil damping force F_{oil} can be written as:

$$F_{oil} = A_p \left[\frac{A_p(\dot{z}_p - \dot{z}_t)}{A_v C_D}\right]^2 \frac{\rho}{2} sign(\dot{z}_p - \dot{z}_t) = f_2(\dot{z}_p - \dot{z}_t) \quad (7)$$

where f_2 is the state dependent oil damping coefficient. The states are defined as:

$$x_1 = \dot{z}_p \quad (8)$$
$$x_2 = z_{fp} - z_t = z_{fpt} \quad (9)$$
$$x_3 = z_t - z_0 \quad (10)$$
$$x_4 = \dot{z}_t \quad (11)$$
$$x_5 = z_p - z_t \quad (12)$$

where z_0 is the road displacement input. After simplifications the state equations take the form:

$$\dot{x}_1 = \frac{1}{M}(-f_2 x_1 + f_1 x_2 + f_2 x_4) \quad (13)$$
$$\dot{x}_2 = x_1 - x_4 - \frac{Q_{in}}{A_p} \quad (14)$$
$$\dot{x}_3 = x_4 - \dot{z}_0 \quad (15)$$
$$\dot{x}_4 = \frac{1}{M_t}(f_2 x_1 - f_1 x_2 - k_t x_3 - f_2 x_4) \quad (16)$$
$$\dot{x}_5 = x_1 - x_4 \quad (17)$$

where k_t is the tire stiffness and M_t is the unsprung mass.

3. Controller Design

In SDRE control, the first step is to get a state space model in which the matrices are functions of states. By this way, a linear structure of the nonlinear model is obtained and the nonlinearities in the model are fully captured by state dependent parameter values. Then the cost function to be minimized is formed using the SDM. Thus Riccati equation is also state dependent and is solved online along the trajectory of states. Therefore, a state dependent control law is obtained. After the state dependent linear model of the HP suspension system is obtained, its control can be achieved by SDRE control. All states in the system are assumed to be estimated. In active suspension control, there are three performance criteria to be satisfied, which are

1) Minimum sprung mass vertical acceleration for ride comfort,
2) Minimum suspension deflection for suspension packaging, and
3) Minimum tire deflection for road holding.

While satisfying these performances, the control input should be bounded. Therefore, the cost function to be minimized is

$$J(x) = \frac{1}{2}\int_0^\infty \left[q_1 \ddot{z}_p^2 + q_2(z_p - z_t)^2 + q_3(z_t - z_0)^2 + RQ_{in}^2\right] dt \quad (18)$$

where q_1, q_2, q_3, and R are the weighting coefficients.

The performance of the controller is examined both in frequency domain and in time domain. In the frequency domain, the model is simulated by sine inputs with different frequencies and amplitudes and the approximate frequency response functions (FRF) are obtained for the corresponding road input. At each frequency, sprung mass acceleration, suspension deflection, and the tire deflection FRFs are calculated as the ratio of the root mean square (rms) of the responses to the rms of the road velocity input.

HP suspension system is a nonlinear system and thus it does not have a unique FRF. However, trials have shown that the active controller can improve the performance variables for different input amplitudes. Here the primary aim of the active controller is the ride comfort and thus the controller is designed accordingly by adjusting the weighting coefficients in the cost function. The system parameters used in the simulations as well as weighting factors and the input are given in table 1. The FRFs obtained from simulations are given in figures 2-4.

Table 1. System, weighting, and input parameters.

System Parameters			**Weighting Factors**	**Input**
M=1500 kg	V_{30}=0.0019 m^3	F_f=40 N	q_1=1	Amplitude 0.1-0.01 m
A_v=2e-4 m^2	γ=1.4	F_{fp}=20 N	q_2=100	
C_D=0.8	M_t=153 kg	M_{fp}=1 kg	q_3=1	Frequency 0.4-20 Hz
A_p=0.007 m^2	k_t=6e5 N/m	ρ=800 kg/m^3	R=1e5	

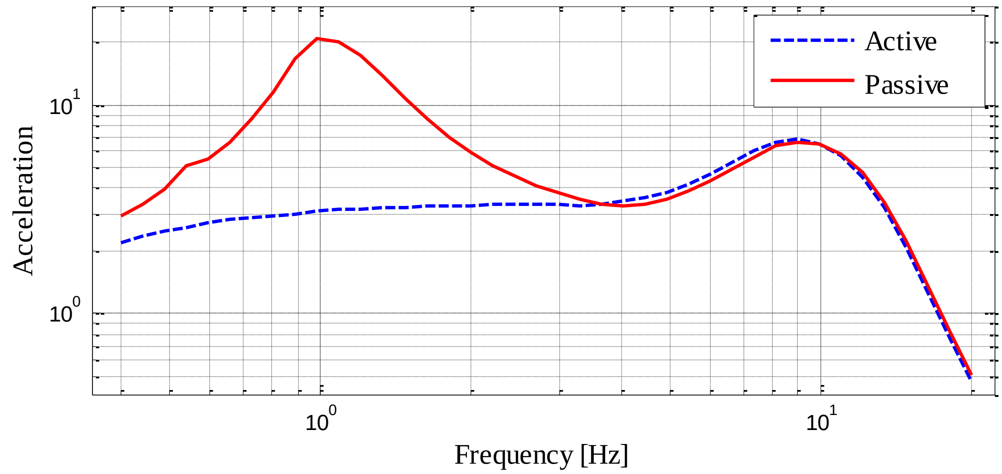

Fig. 2. Sprung mass acceleration FRF for passive and active HP suspension.

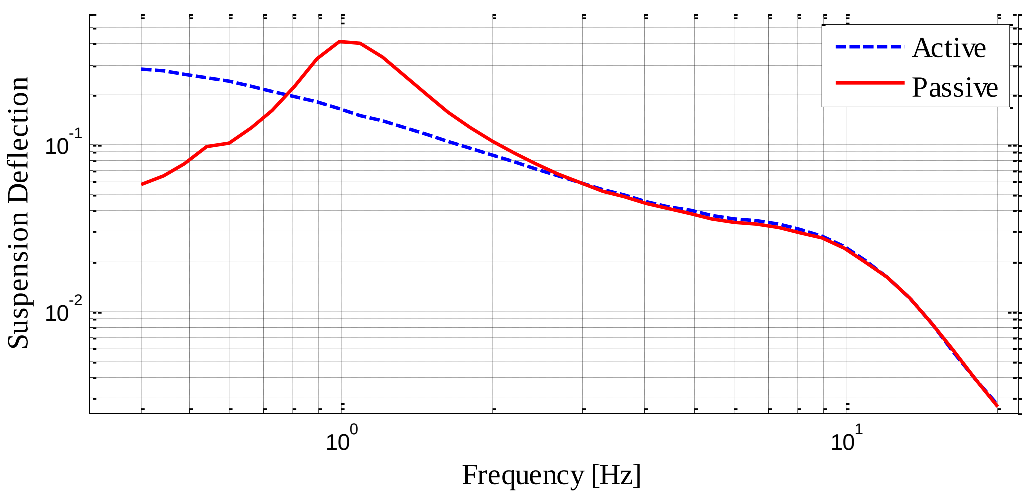

Fig. 3. Suspension deflection FRF for passive and active HP suspension.

Figure 2 shows that active suspension attenuates the sprung mass acceleration around body bounce frequency considerably. Above 4 Hz, active and the passive suspension performances are similar and active suspension cannot improve the sprung mass acceleration.

As figure 3 illustrates, active suspension decreases the suspension deflection around body bounce frequency. At very low frequencies active suspension has inferior results with respect to passive suspension. Above 3 Hz, performances of the active and the passive suspensions are nearly the same.

The result for the tire deflection is similar to result for the sprung mass acceleration. As figure 4 shows, at low frequencies, active suspension reduces the tire deflection considerably. Nearly above 3 Hz, active suspension deteriorates the performance and passive suspension gives better result slightly.

Therefore, active suspension attenuates the sprung mass acceleration, suspension deflection and the tire deflection at low frequencies, mainly around body bounce frequency of the vehicle. However at lower frequencies active suspension increases the suspension deflection and thus it degrades the suspension deflection. Moreover, at wheel hop frequency, active suspension cannot attenuate the all three performance variables.

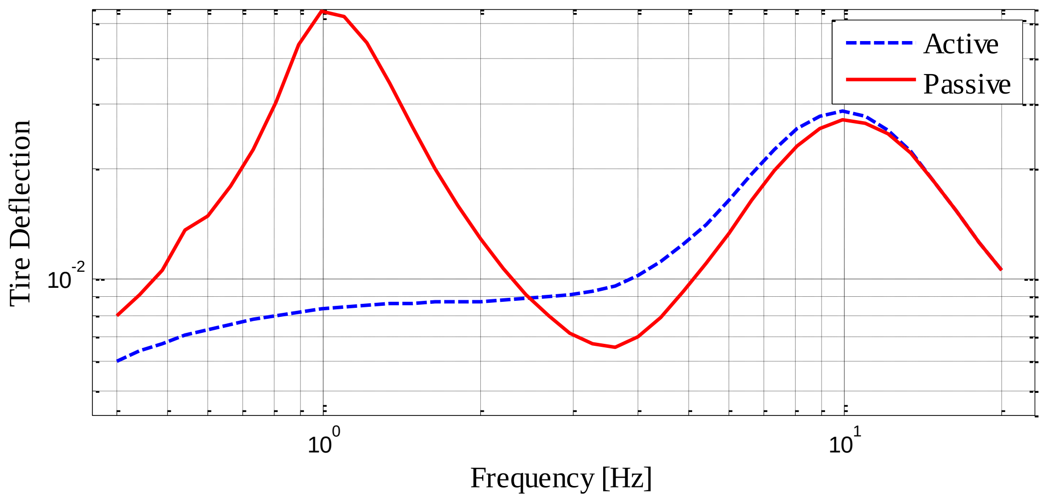

Fig. 4. Tire deflection FRF for passive and active HP suspension.

The performance of the active controller can also be evaluated using random road profile input. rms values for the sprung mass acceleration, suspension deflection, tire deflection, and the flow rate are calculated and given in table 2. The random road profile input used in the simulation is representative of a course road as shown in figure 5. Simulations results are given in figure 6.

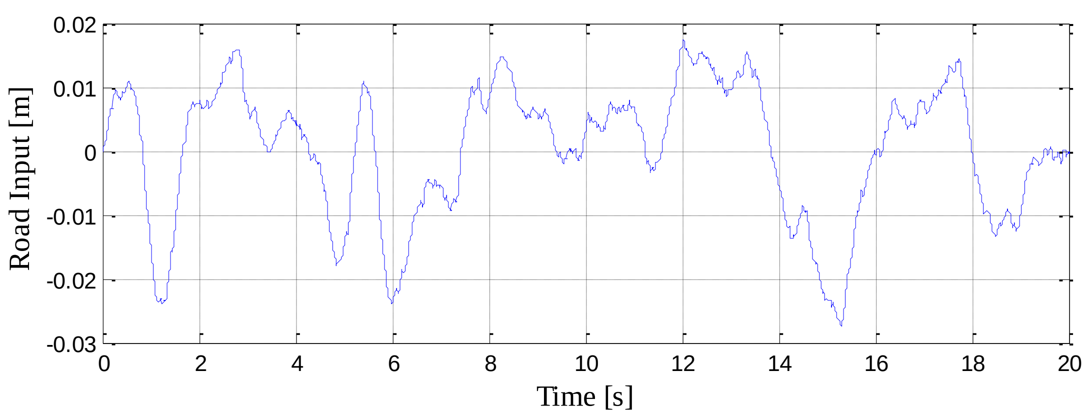

Fig. 5. Random road displacement input.

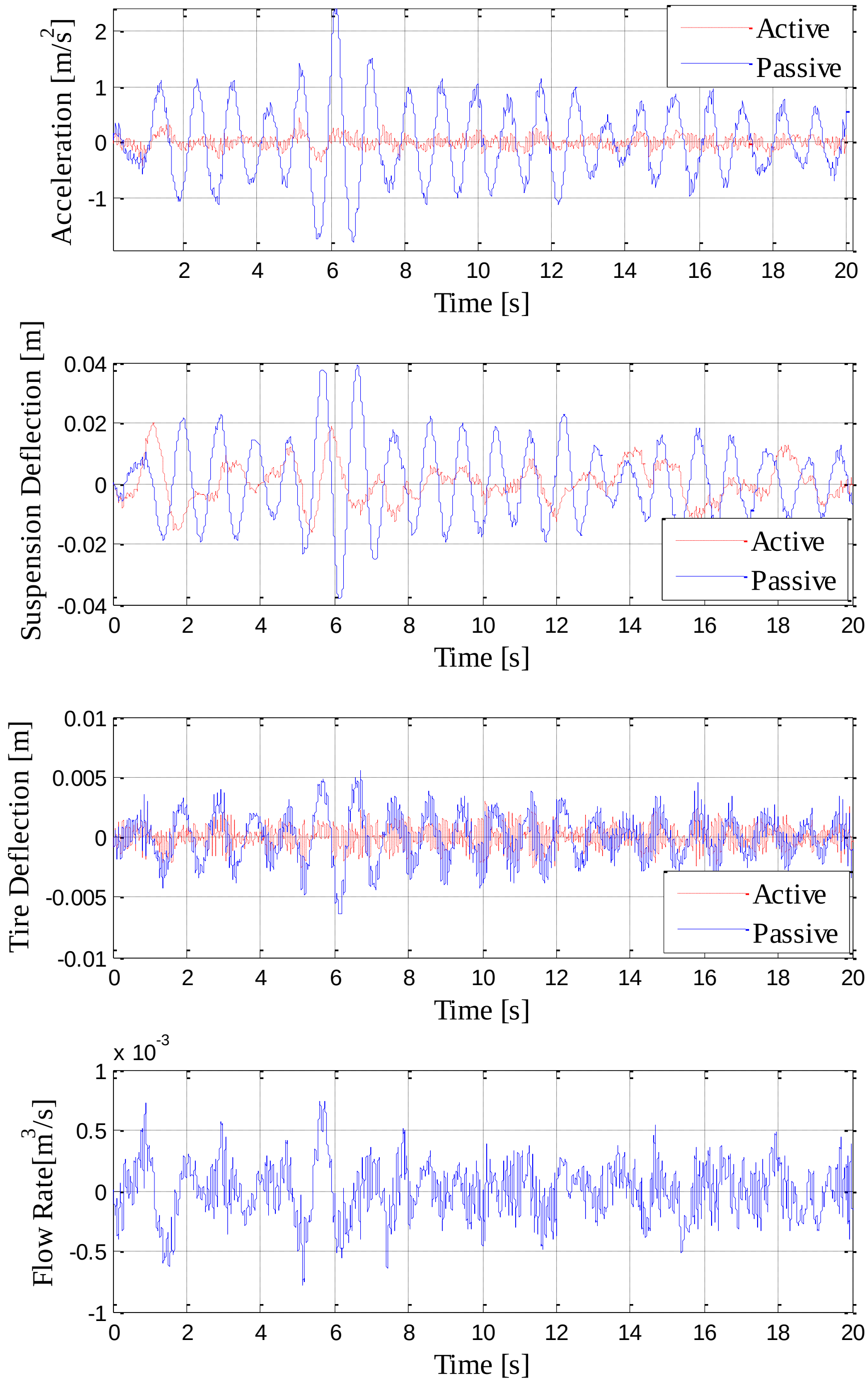

Fig. 6. Simulation results for random road displacement input for active and passive suspension.

Table 2. rms values of responses for active and passive suspensions.

	Active	**Passive**
Vertical Acceleration [m/s^2]	0.1086	0.6840
Suspension Deflection [m]	0.0065	0.0130
Tire Deflection [m]	8.72e-04	0.0019
Flow Rate [m^3/s]	2.24e-04	

Simulation results show that, active suspension control reduces the sprung mass acceleration, suspension deflection, and the tire deflections simultaneously for random road profile input. Particularly, for sprung mass acceleration, active suspension reduces the rms value of the sprung mass acceleration considerably. Therefore, SDRE control can be applied to active HP suspension system to improve the ride performance successfully. The rms value of the flow rate for the given road profile is sufficiently low for a practical implementation.

4. Conclusion

In this study a systematic method of designing active controller for an HP suspension system is developed to simultaneously attenuate sprung mass acceleration, suspension deflection, and tire deflection arising from road disturbances and thus improved ride comfort and road holding are obtained. Firstly, the state dependent linear model of the active HP suspension system is derived. Then using SDRE approach a nonlinear controller is designed for the active suspension. Sprung mass acceleration, suspension deflection, and tire deflection can be substantially attenuated at frequencies around body bounce frequency of the vehicle, yet at lower frequencies, suspension deflection increases. On random roads, the sprung mass acceleration is reduced for active suspension with low control effort. Therefore, the results from this study show that the proposed active suspension control can be implemented to a vehicle with HP suspension successfully.

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