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# Distributed Exponential Formation Control of Multiple Wheeled Mobile Robots

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***Abstract*** **-** This paper considers the distributed formation control of multiple nonholonomic wheeled mobile robots with a leader. Distributed tracking control laws are proposed with the aid of results of cascade systems such that the centroid of the states of a group of mobile robot exponentially tracks the leader. Simulation results show the effectiveness of the proposed algorithms.

***Keywords*:** Wheeled mobile robots, cooperative control, formation control, distributed control.

## 1. Introduction

There are lots of applications of wheeled mobile robots in practice and control strategies have been developed for decades. Wheeled mobile robots with nonslip constraints are typically nonhonomic systems. Stabilization of a single nonholonomic system is challenging due to the fact that no smooth pure state feedback control law exists for stabilization. Moreover, no dynamic continuous time-invariant feedback controller is available to render the closed loop system locally asymptotically stable (Pomet 1992). With the efforts of researchers, several approaches have been proposed for stabilizing nonholonomic systems, which can be classified into the following three aspects: discontinuous time-invariant feedback, the time-varying feedback, and the hybrid feedback. For details, see Kolmanovsky and McClamroch (1995) and the references therein.

It is challenging to design tracking controllers for wheeled mobile robots because of their nonlinear features. Samson and Ait-Abderrahim proposed the first tracking controller for a mobile robot in Samson and Ait-Abderrahim (1991). Then a tracking controller was designed through linear approximation for nonholonomic systems in Walsh et al (1994). In D'Andrea-Novel et al (1995), the tracking task of wheeled mobile robots was fulfilled by linearizing both static and dynamic feedback. Fliess et al solved the tracking problem utilizing results of “differentially flat” nonlinear systems Fliess et al (1995). With the aid of backstepping techniques, semi-global tracking controllers were proposed for a chained-form system in Jiang and Nijmeijer (1999). In Jiang (2000), global state and output tracking controllers were proposed for chained-form systems with the aid of Lyapunov techniques. Based on the results of cascade systems, linear tracking controllers were proposed for chained-form systems in Lefeber et al (2000) and Tian and Cao (2007).

Due to the practical requirement of specific tasks, the consensus problem without a leader has been extensively studied in the past decades. In Jadbabaie et al (2003), matrix theory was applied to propose local control laws for fist-order linear discrete-time systems such that the states of multiple systems converge to a constant value. In Ni and Cheng (2010), Laplacian matrix of a communication graph was exploited to propose local control laws for the consensus problem of multiple first-order linear continuous-time systems. In Ren and Beard (2005), consensus algorithms were proposed with relaxed assumption on the communication graphs in Ni and Cheng (2010). Consensus problem with a leader has also been studied systematically and several control laws have been proposed. In Cao and Ren (2010), consensus problems of first-order and second-order linear systems were considered. Distributed controllers were proposed such that the state of each system converge to a desired trajectory within finite time under the condition that the desired trajectory is available to a portion of the group of systems. In Ni and Cheng (2010), leader-following consensus of high-order linear systems was considered over a switching communication topology. Distributed controllers were proposed with the aid of Riccati-inequality-based approach. In Liu and Jia (2010), consensus control of multi-agent systems was considered. Output feedback controllers were proposed with the aid of $H_\infty$ theory. In Scardovi and Sepulchre (2009), consensus of high-order linear systems was considered for time-varying and directed communication topologies. Distributed controllers were proposed with the aid of the observer design approach. In Li et al (2011) and Li et al (2010), consensus of multiple linear systems was considered in an unified viewpoint and a notation of concensus region was introduced. In Meng et al (2012), the leader-following consensus problem for a group of agents with identical linear systems subject to control input saturation was considered. Linear feedback laws were proposed for fixed and switching communication topology. In Cao and Ren (2012a,2012b), consensus of first-order and second-order nonlinear systems was considered. Finite-time control laws were proposed with the aid of a comparison lemma. In Dong (2012), cooperative control of multiple mobile robots was considered. Distributed control laws were proposed with the aid of a consensus approach.

In this paper, we study distributed formation control of multiple nonholonomic wheeled mobile robots with a leader whose state is not available to each system such that the group of robots converges to a desired geometric pattern whose centroid follows the leader. New distributed control laws are proposed based on the results of cascade systems and the properties of persistently exciting signals. Compared to the results in literature, in this paper cooperative tracking control is solved for multiple nonlinear systems. Compared to the results in Dong (2012), a new approach is proposed for cooperative tracking control problem of multiple wheeled mobile robots and the proposed control laws can make the tracking errors uniformly exponentially converge to zero, which is much more applicable in practice.

The remaining parts of this paper are organized as follows. In Section 2, the considered problem is formulated and some preliminary results are presented. In Section 3, distributed tracking controllers are proposed. In Section 4, controllers are proposed for switching communication graphs. In Section 5, simulation results are presented. The last section concludes this paper.

## 2. Problem Statement

It is considered a group of $m$ wheeled mobile robots which move on a horizontal plane. The motion of robot $j$ is described by

$$\dot{x}_j = v_j \cos\theta_j, \quad \dot{y}_j = v_j \sin\theta_j, \quad \dot{\theta}_j = \omega_j \tag{1}$$

where $(x_j, y_j)$ is the position of robot $j$ in a coordinate system, $\theta_j$ is the orientation of robot $j$, $v_j$ is the speed of robot $j$, and $\omega_j$ is the angular speed of robot $j$. The control inputs are $v_j$ and $\omega_j$.

For $m$ systems, each system knows its own state and its neighbors' states by communication and/or sensors. For simplicity, it is assumed that the communications between the systems are bidirectional. If we consider each system as a node, the communication between the systems can be described by an (undirected) graph $G = \{V, E\}$, where $V = \{1, 2, \ldots, m\}$ is a node set, and $E$ is an edge set with unordered pair $(i, j)$ which describes the communication between node $i$ and node $j$. If the state of node $i$ is available to node $j$, node $i$ is called a neighbor of node $j$. The set of all neighbors of node $j$ is denoted by $N_j$. A graph is called connected if for any two different nodes there exists a set of edges which connect the two nodes.

A formation of $m$ robots is defined by a geometric pattern $P$. The pattern $P$ can be described by orthogonal coordinates $(p_{jx}, p_{jy})$ $(1 \le j \le m)$. Without loss of generality, we assume that $\sum_{j=1}^{m} p_{jx} = 0$ and $\sum_{j=1}^{m} p_{jy} = 0$, i.e., the center of the geometric pattern $P$ is at the origin of a local orthogonal coordinate system. It is given a reference trajectory $q_0(t) = (x_0(t), y_0(t), \theta_0(t))$ which satisfies

$$\dot{x}_0 = v_0 \cos\theta_0, \quad \dot{y}_0 = v_0 \sin\theta_0, \quad \dot{\theta}_0 = \omega_0 \tag{2}$$

where $v_0$ and $\omega_0$ are known time-varying functions. The state $q_0$ is assumed to be available to a portion of the $m$ wheeled mobile robots.

Let $q_j = [x_j, y_j, \theta_j]^{\mathrm{T}}$, the control problem considered in this article is defined as follows.

*Control Problem:* Design control laws $v_j$ and $\omega_j$ for system $j$ using its own state $q_j$, its neighbor's state $q_l$, the relative position with its neighbor $(p_{lx}, p_{ly})$ for $l \in N_j$, and the desired trajectory $q_0$ if it is available to the system such that

$$\lim_{t\to\infty} \begin{bmatrix} x_i - x_j \\ y_i - y_j \end{bmatrix} = \begin{bmatrix} p_{ix} - p_{jx} \\ p_{iy} - p_{jy} \end{bmatrix} \tag{3}$$

$$\lim_{t\to\infty} (\theta_i - \theta_0) = 0 \tag{4}$$

$$\lim_{t\to\infty} \left[ \sum_{l=1}^{m} \frac{x_l}{m} - x_0 \right] = 0, \lim_{t\to\infty} \left[ \sum_{l=1}^{m} \frac{y_l}{m} - y_0 \right] = 0 \tag{5}$$

for $1 \le i \ne j \le m$.

In order to solve the defined problem, the following assumption is made on the desired trajectory.

**Assumption 1** *The* $\frac{d^i\theta_0}{dt^i}$ *(*$0 \le i \le 2$*) are bounded and* $\int_t^{t+T} \dot{\theta}_0^2(\tau)d\tau > \mu$ *for some* $\mu > 0$ *and* $T > 0$.

## 3. Cooperative Controller Design

Define the variables

$$z_{1j} = \theta_j, z_{2j} = (x_j - p_{jx})\cos\theta_j + (y_j - p_{jy})\sin\theta_j, z_{3j} = (x_j - p_{jx})\sin\theta_j - (y_j - p_{jy})\cos\theta_j \tag{6}$$

for $0 \le j \le m$, where $k_3 > 0$ and $p_{0x} = p_{0y} = 0$. The transformed state space model is

$$\dot{z}_{1j} = \omega_j \tag{7}$$

$$\dot{z}_{2j} = v_j - \omega_j z_{3j} \tag{8}$$

$$\dot{z}_{3j} = \omega_j z_{2j}. \tag{9}$$

We have the following results.

**Lemma 1** *If* $\lim_{t\to\infty}(z_{1j} - z_{10}) = 0$, $\lim_{t\to\infty}(z_{2j} - z_{20}) = 0$, *and* $\lim_{t\to\infty}(z_{3j} - z_{30}) = 0$ *for* $1 \le j \le m$, *then (3)-(5) hold.*

System (7)-(9) can be considered as a cascade system of (7) and (8)-(9). We first design control law $\omega_j$ for system (7) such that $\lim_{t\to\infty}(z_{1j} - z_{10}) = 0$. For a group of $m$ robots ($1 \le j \le m$), the communication between the robots is described by a graph $G = \{A, E\}$. Given an $m \times m$ constant matrix

$A=[a_{ji}]$ with $a_{ji}=a_{ij}>0$, the Laplacian matrix $L=[L_{ji}]$ of the graph $G$ with weight matrix $A$ is defined by

$$L_{ji}=\begin{cases} -a_{ji}, & \text{if } i\in N_j \text{ and } i\neq j \\ 0, & \text{if } i\notin N_j \text{ and } i\neq j \\ \sum_{l\neq j, l\in N_j} a_{jl}, & \text{if } j=i. \end{cases}$$

For the Laplacian matrix, the following result is useful in this paper.

**Lemma 2 (Dong (2012))** *If the communication graph $G$ is connected, then $(L+\mathrm{d}iag(\mu))$ is a positive definite symmetric matrix, where constant vector $\mu=[\mu_1,\mu_2,\ldots,\mu_m]^{\mathrm{T}}$, $\mu_i\geq 0$ ($1\leq i\leq m$) and at least one of the elements of $\mu$ is nonzero.*

With the aid of Lemma 2, we have the following lemma.

**Lemma 3** *For the $(m+1)$ systems in eqn. (7) ($0\leq j\leq m$), if the communication graph $G$ is connected and the system $0$ is available to at least one of the $m$ systems, the control laws*

$$\omega_j=\xi_{1j}-\sum_{i\in N_j}a_{ji}(z_{1j}-z_{1i})-b_j\mu_j(z_{1j}-z_{10}) \tag{10}$$

$$\dot{\xi}_{1j}=-\sum_{i\in N_j}a_{ji}(\xi_{1j}-\xi_{1i})-b_j\mu_j(\xi_{1j}-\xi_{10})-\rho_{1j}\,\mathrm{sign}\left[\sum_{i\in N_j}a_{ji}(\xi_{1j}-\xi_{1i})-b_j\mu_j(\xi_{1j}-\xi_{10})\right] \tag{11}$$

for $1\leq j\leq m$ guarantee that $\lim_{t\to\infty}(z_{1j}-z_{10})\overset{exp}{=}0$ and $\lim_{t\to\infty}(\omega_j-\omega_0)\overset{exp}{=}0$, where $\xi_{10}=\dot{\theta}_0$, $b_j>0$, $\rho_{1j}$ is sufficiently large, the parameter $\mu_j=1$ if system $0$ is available to system $j$ and $\mu_j=0$ if system $0$ is not available to system $j$.

The proof of Lemma 3 is the same as the proof of Lemma 4 in Dong (2012) and is omitted here for space limitation.

With the aid of Lemma 3 and the results of cascade system, we have the following results.

**Theorem 1** *For the $(m+1)$ systems in eqn. (8) ($0\leq j\leq m$), under Assumption 1, if the communication graph $G$ is connected, then the distributed control laws (10)-(11) and*

$$v_j=-k_1z_{2j}-k_2\omega_jz_{3j}+\omega_jz_{3j}+\xi_{2j} \tag{12}$$

$$\dot{\xi}_{2j}=-\sum_{i\in N_j}a_{ji}(\xi_{2j}-\xi_{2i})-b_j\mu_j(\xi_{2j}-\xi_{20})-\beta_j\,\mathrm{sign}\left[\sum_{i\in N_j}a_{ji}(\xi_{2j}-\xi_{2i})+b_j\mu_j(\xi_{2j}-\xi_{20})\right] \tag{13}$$

for $1\leq j\leq m$ ensure that $(z_{1j},z_{2j},z_{3j})$ uniformly exponentially converges to $(z_{10},z_{20},z_{30})$ and $(\xi_{1j},\xi_{2j})$ exponentially converges to $(\xi_{10},\xi_{20})$, where $\beta_j$ is a sufficiently large positive constant, $k_1>0$, $k_2>0$, and $\xi_{20}=v_0-\omega_0z_{30}+k_1z_{20}+k_2\omega_0z_{30}$.

In Dong (2012), distributed control laws for multiple wheeled mobile robots were proposed such that the state of each system asymptotically converges to a desired state. In this paper, new distributed tracking control laws are proposed with the aid of the results of cascade systems. Moreover, the proposed control laws ensure that the state of each system globally uniformly exponentially converges to a desired state.

## 4. Simulations

To show the effectiveness of the proposed results, simulation has been done for three robots. The desired geometric pattern $P$ is shown in Figure 1. The pattern $P$ can be described by orthogonal coordinates $(p_{1x}, p_{1y}) = (-1, 1.7)$, $(p_{2x}, p_{2y}) = (-1, -1.7)$, and $(p_{3x}, p_{3y}) = (2, 0)$. Assume the reference trajectory is $(x_0, y_0, \theta_0) =$ $(10\sin(0.5t)$, $-10\cos(0.5t)$, $0.5t)$, by (2) $v_0 = 5$ and $\omega_0 = 0.5$. So, Assumption 1 is satisfied.

Assume the communication graph is shown in Figure 2. The cooperative controllers can be obtained by Theorem 1. We chose the control parameters $a_{ji} = 2$, $k_3 = 2$, $b_1 = 2$, $\rho_1 = 2$, and $\rho_2 = 2$. Figure 3 shows the centroid of $x_i$ $(1 \le i \le 3)$ (i.e., $\sum_{j=1}^{3} x_j/3$) and $x_0$. Figure 4 shows the centroid of $y_i$ ($1 \le i \le 3$) (i.e., $\sum_{j=1}^{3} y_j/3$) and $y_0$. Figure 5 shows $(\theta_i - \theta_0)$ $(1 \le i \le 3)$. Figure 6 shows the path of the centroid of the three robots and its desired path. From the simulation (4)-(5) are satisfied. Eqn. (3) is also verified and the response of them is omitted here.

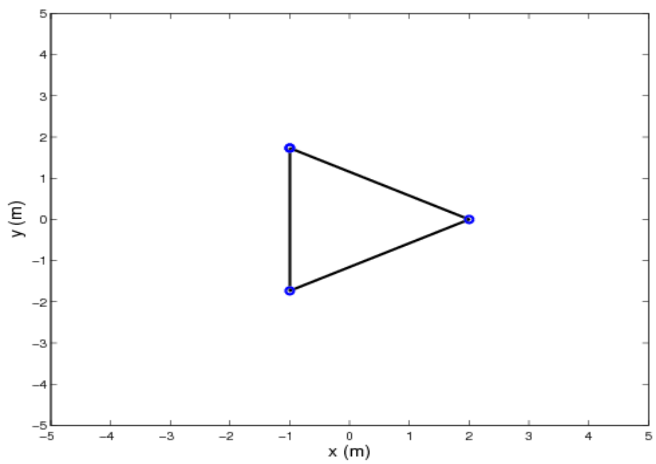


Fig. 1. Desired formation.

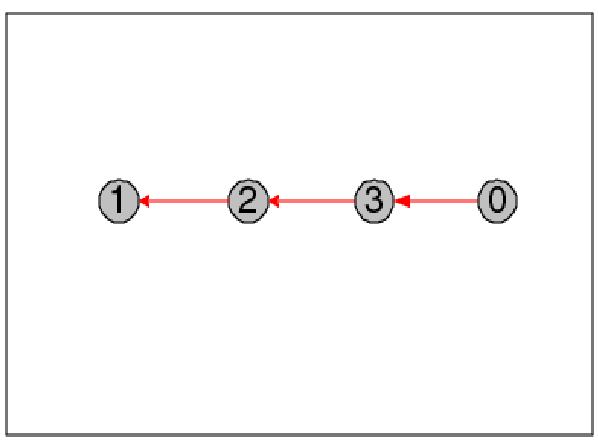


Fig. 2. Information exchange graph *G*.

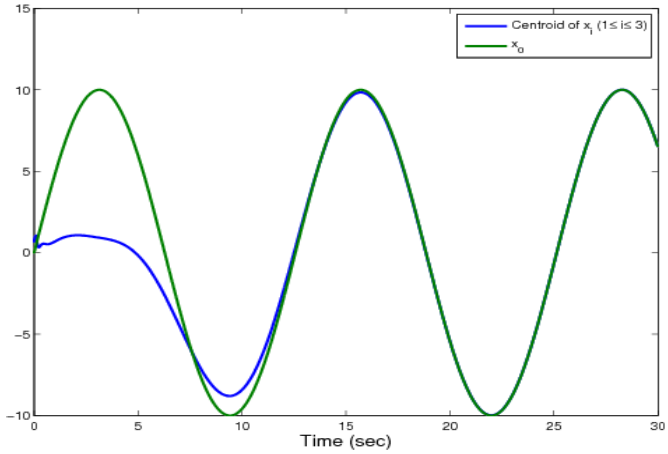


Fig. 3. Response of the centroid of $x_i$ (solid) for $1 \le i \le 3$ and $x_0$ (dashed).

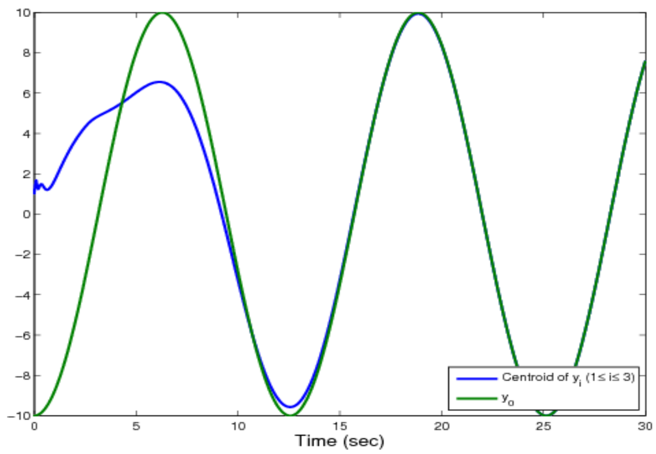


Fig. 4. Response of the centroid of $y_i$ (solid) for $1 \le i \le 3$ and $y_0$ (dashed).

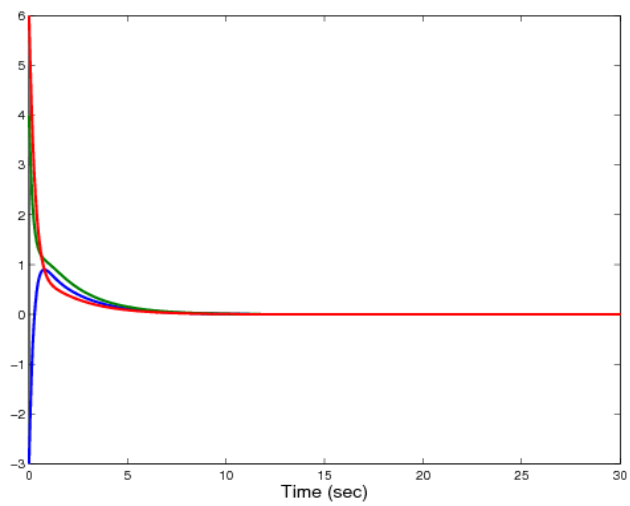


Fig. 6. Response of $(\theta_i - \theta_0)$ for $1 \le i \le 3$.

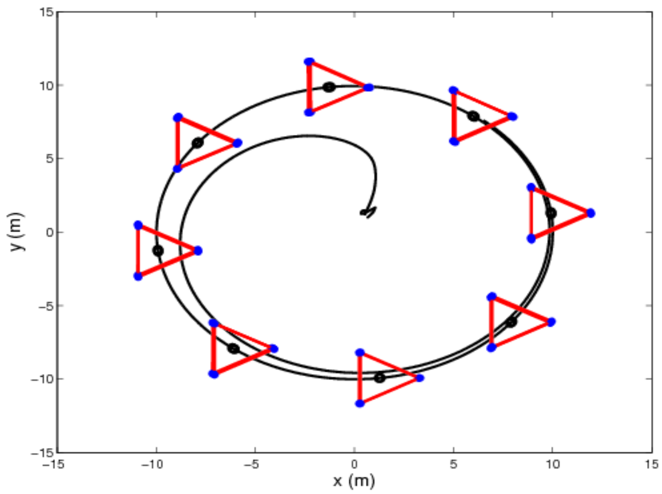

Fig. 6. The path of the centroid of the three robots (dashed line), the desired path (solid line) of the centroid of robots, and formation of the three robots at several moments (red triangles).

## 4. Conclusion

This paper has discussed the formation control of multiple wheeled mobile robots under the condition that a desired trajectory is available to only a portion of the systems. Distributed control laws were proposed with the aid of Lyapunov techniques and results from graph theory. Simulation results show the effectiveness of the proposed control laws. In this paper, the information exchange graph is assumed to be bidirectional. The future work is to extend our results to more general information exchange graphs.


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