# Neuro-adaptive Formation Maintenance and Control of Nonholonomic Mobile Robots

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**Abstract** - In this paper, the problem of fleet navigation with formation control and maintenance under leaderfollower strategy has been addressed using simultaneous localization and mapping (SLAM)-based navigation unit and artificial potential field approaches. Agents are nonholonomic ground robots. In the proposed approach, the leader or virtual leader is assumed to be the center of the formation and its navigation is guaranteed using SLAMbased controller. On the other hand, potential field control strategy is used to position all followers in a certain formation around their leader. In this work, we represents the case where the dynamic is completely unknown. In such case, an on-line NN-based adaptive model estimator is used to approximate the robot nonlinear dynamic and guarantee the tracking of a desired path. In both cases, the control strategy adds an inner loop to the system's configuration. Simulation results demonstrate the performance of the proposed approach.

Keywords: Leader-follower, fleet formation, localization, potential fields, neural networks.

## 1. Introduction

In recent years, cooperative multi-robot systems attracted the interest of the researchers because of the many advantages they offer compared to a single robot system. For example, multiple-agents robots can estimate their position faster and more accurately due to their ability to exchange information related to their positions, whenever they are in range and the connectivity is guaranteed. Cooperative control has diversified applications in many areas. They can be deployed for transportation, sensing, or military missions. In many situations, the mobile robots have to work cooperatively to accomplish certain tasks or actions. Several design strategies for controlling such a group of mobile robots have been proposed in the literature. The fleet formation problem is addressed in different stages depending on the desired mission. The first stage, agents should reach formation, while the fleet navigates to track a desired trajectory (formation maintenance). Stage 3 aims at ensuring at all times a collision-free mobility of the different agents (collision avoidance).

This paper addresses the problem of a collision-free fleet formation control and maintenance under leader-follower strategy for non-holonomic autonomous robot vehicles using simultaneous localization and mapping (SLAM) for the navigation control unit. The proposed framework is composed of two stages and will be built by integrating robot's group formation based on the artificial potential fields combined with SLAM navigation system. In this work the leader represents a real leader or a virtual leader. The later situation can occur for instance during target tracking of a moving target (see for example Ma and Hovakimyan, 2013). In our approach, the first stage addresses the localization of a mobile agent at each time step when the group's leader or target follow its trajectory based on the current observation and the last available knowledge about the navigation environment. This stage is achieved using simultaneous localization and mapping (SLAM) on the group leader.

Many approaches have been investigated for formation control and maintenance. Artificial potential fields is one of the most used techniques. In the recent decade many control strategies for controlling the formation shape of a fleet of robots were proposed based on the potential fields approach. Song and Kumar (2002) analyzed the force equilibrium to show how potential fields can be used for different

shapes generation and formation control, also they adopted a control framework for controlling a fleet of robots for handling cooperative tasks. Chaimowicz, et al.(2005), addressed an approach for shape formation and control for a group of robots manipulation to arbitrary shapes. Hsieh and Kumar (2006) extend Chaimowicz approach and developed decentralized controllers for group desired shape formation. Sabattini et al. (2011) presented a novel decentralized control approach for mobile robots formation, their strategy is not only to handle robots shape formation, but also to determine the robot's positions. However, our approach will extend Lorenzo Sabattini et al (2011) to make one of the robots lead the others in unknown environment, and at the same time all the agents in the fleet will keep their formation shape based on the potential fields. The whole system will be integrated to satisfy Song and Kumar framework (2006), to handle and manipulate cooperative tasks.



Fig. 1. A nonholonomic car-like mobile robot.

The objective of this work is to design a framework for navigation and control of a fleet of nonholonomic robots required to handle cooperative tasks. The proposed solution will be a combination between two techniques. The first one is navigation system, which will be installed by running the SLAM algorithm on the group's leader. The second is the group formation shape controllers. This controller will be derived based on simple potential fields approach to handle the group interactions.

We assume that the leader, which is running SLAM algorithm, will be used as the field center of the group of 'n' robots, and the followers will localize themselves around the leader and keep their formation based on attractive and repulsive potentials. Our approach extends the design in presented in Lorenzo Sabattini et al (2011) and Song and V. Kumar, (2002). This work contributes to the literature in many fronts:

- 1. The new framework will extend the work in Lorenzo et al (2011), from point mass holonomic agents to nonholonomic vehicles.
- 2. This framework allows one of the robots to lead the others in unknown environment, assuming that the leader configuration is used as the group field center. At the same time all the agents in the fleet will keep their formation shape based on the potential fields.
- 3. The potential field is developed to obtain several formation schemes (star, square, and pentagon) for a group of nonholonomic mobile robots.
- 4. Assuming an unknown system dynamics, a neural network based-control will be used to estimate the system dynamics and tracking the inner loop errors

This paper is organized as follows. Section 2 covers system modelling and its necessary assumptions. section 3 presented the proposed frame work design. Finally, simulation results for illustrating the behavior of proposed design is presented in section 4.

## 2. System Modeling and Assumptions

In this section we present the system's model which can be found in Fierro and Lewis. (1997). In addition, we list the needed assumptions. The nonholonomic mobile robot with 2-dimensional coordination space system and subjected to m constraints can be described in general coordinates (q) and



Fig. 2. The leader sensing range  $F_i$ ,  $F_j$  followers.

its Euler Lagrange form can be expressed as

$$M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + F(\dot{q}) + G(q) + \tau_d = B(q)\tau - A^T(q)\lambda,$$
(1)

Where

$$\begin{split} \mathsf{M}(\mathsf{q}) &\in \mathcal{R}^{n \times n} : \text{Symmetric PD inertia matrix.} \\ \mathsf{V}_{\mathsf{m}}(\mathsf{q}, \dot{\mathsf{q}}) &\in \mathcal{R}^{n \times n} : \text{Centripetal and corioles matrix.} \\ \mathsf{F}(\dot{\mathsf{q}}) &\in \mathcal{R}^{n \times 1} : \text{Surface friction.} \\ \mathsf{G}(\mathsf{q}) &\in \mathcal{R}^{n \times 1} : \text{Gravitational vector.} \\ \tau_{\mathsf{d}} &\in \mathcal{R}^{n \times 1} : \text{Unknown disturbance.} \\ \mathsf{B}(\mathsf{q}) &\in \mathcal{R}^{n \times r} : \text{Input transformation matrix.} \\ \tau &\in \mathcal{R}^{n \times 1} : \text{Input vector.} \\ \mathsf{A}^{\mathsf{T}}(\mathsf{q}) &\in \mathcal{R}^{m \times n} : \text{Is matrix associated with constraints.} \\ \lambda &\in \mathcal{R}^{m \times 1} : \text{Constraint forces vector.} \\ \text{If we consider all kinematic equality constraints are not time-dependent, which follows that} \end{split}$$

$$A(q)\dot{q} = 0 \tag{2}$$

Let S(q) be a full rank matrix of a set of smooth and linearly independent vector fields in null space of A(q), i.e.,

$$S^{\mathrm{T}}(q)A^{\mathrm{T}}(q) = 0 \tag{3}$$

If we consider (2) and (3), it will be possible to find a vector time function  $v(t) \in \mathbb{R}^{n-m}$  such that

$$\dot{\mathbf{q}} = \mathbf{S}(\mathbf{q})\mathbf{v}(\mathbf{t}) \tag{4}$$

The position of the robot shown in figure (1) is in an inertial Cartesian frame {0, X, Y} and it is presented by the vector  $\mathbf{q} = [\mathbf{x}, \mathbf{y}, \theta]^{\mathrm{T}}$ . The kinematic equations of the motion (4) can be presented in terms of linear and angular velocities by

$$S(q) = \begin{bmatrix} \cos(\theta) & -d\sin(\theta) \\ \sin(\theta) & d\cos(\theta) \\ 0 & 1 \end{bmatrix}$$
(5)

$$\mathbf{v} = \begin{bmatrix} \mathbf{v} \\ \boldsymbol{\omega} \end{bmatrix} = \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix} \tag{6}$$

$$\dot{\mathbf{q}} = \begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{y}} \\ \dot{\mathbf{\theta}} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -d\sin(\theta) \\ \sin(\theta) & d\cos(\theta) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix}$$
(7)

The matrices that define the model dynamics based on figure (1) are expressed in (1) where,

$$M(q) = \begin{bmatrix} m & 0 & md \sin(\theta) \\ 0 & m & -md \cos(\theta) \\ md \sin(\theta) & -md \cos(\theta) & I \end{bmatrix}$$
(8)

$$V_{i}(q, \dot{q}) = \begin{vmatrix} -md\dot{\theta}^{2}\cos(\theta) \\ -md\dot{\theta}^{2}\sin(\theta) \\ 0 \end{vmatrix}$$
(9)

$$G(q) = 0 \tag{10}$$

$$B(q) = \frac{1}{r} \begin{bmatrix} \cos(\theta) & \cos(\theta) \\ \sin(\theta) & \sin(\theta) \\ R & -R \end{bmatrix}$$
(11)

$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \tag{12}$$

#### 2.1. Assumptions

1. Boundedness:  $M_i(q)$ , the norm of  $V_m(q, \dot{q})$ , and  $\tau$  are bounded.

2. Skew Symmetric: The matrix  $M_i(q) - 2M_{m_i}(q, \dot{q})$  is skew symmetric such that.

 $x^{T} \left( M_{i}(q) - 2V_{m_{i}}(q, \dot{q}) \right) x = 0.$ 

The whole system consists of two cascaded stages. The first one is the robot system which includes the system dynamics control. The second stage is the potential fields based formation controller. These two stages are combined together in cascaded form. The general structure of the integrated system is presented in figure (3).

#### 2.2. Shape Formation

Each robot has a sensing range equal to  $S_r$ , figure (2), and each robot can detect its neighbors' position within sensing area. All nonholonomic robots in the fleet need to navigate in a desired polygon shape. These robots localize themselves as followers around a moving leader. The distance between each two neighboring followers should be equal to  $L \leq S_r$ . The radius of the circumcircle of the desired polygon is equal to r. The coordinates of the group moving center is  $x_c \in \mathbb{R}^2$  which can be generated by using the SLAM approach on the group leader. As shown by Lorenzo et al (2011) from the basics of the geometry r will be.

$$\gamma = \frac{L}{2\sin(\pi/n)} \tag{13}$$

#### 2.3. Potential Fields

The proposed system is a nonholonomic robot model such as car-like robot. In this section we will give complete attractive and repulsive potential field analysis for such a system. The potential analysis here will include  $(x, y, \theta)$  instead of (x, y) holonomic model analysis.

### 2.3.1. Attractive Potential

In the case of nonholonomic mobile robot, if the interest is  $(x, y, \theta)$ :

Let, 
$$V_{c_i} = \frac{1}{2} K_c (R_{c_i} - r)^2$$
 (14)

$$P_{\text{att}} = -\nabla_{P_i} V_{c_i}(P_i) \tag{15}$$

where,

$$\nabla_{P_i} = \frac{\partial}{\partial P_i} , R_{ci}(t) = \|P_i(t) - P_c\|, \qquad P_i(t) = \begin{bmatrix} x_i(t) \\ y_i(t) \\ \theta_i(t) \end{bmatrix}, \text{ and } P_c = \begin{bmatrix} x_c \\ y_c \\ \theta_c \end{bmatrix},$$

 $\left\|\cdot\right\|$  stands for the Euclidean norm. We have

$$P_{att} = -\nabla_{p_i} V_{ci}(P_i) = \frac{\partial V_{ci}}{\partial P_i} = -\left(\frac{\partial V_{ci}}{\partial R_{ci}}\frac{\partial R_{ci}}{\partial P_i}\right)$$
(16)

Deriving (14) with respect to R<sub>ci</sub>, gives

$$\frac{\partial V_{c_i}}{\partial R_{c_i}} = K_c (R_{c_i} - r)^2$$
(17)

Similarly,

$$\frac{\mathbf{R}_{\mathrm{ci}}(\mathbf{t})}{\partial \mathbf{P}_{\mathrm{i}}} = \frac{1}{\mathbf{R}_{\mathrm{ci}}} [\mathbf{P}_{\mathrm{i}}(\mathbf{t}) - \mathbf{P}_{\mathrm{c}}]^{\mathrm{T}},$$
(18)

Using (17) and (18) in (16) leads to,

$$P_{att} = -\frac{1}{R_{ci}} \left[ K_c (R_{c_i} - r) \right] \left[ (P_i(t) - P_c) \right]^T$$
(19)

## 2.3.2. Repulsive Potential

The repulsive potential between two neighbouring followers can be expressed as:

$$P_{\text{rep}_{ij}} = -\nabla_{P_i} V_{ij}(P_i, P_j)$$
<sup>(20)</sup>

Define  $\overrightarrow{R_{f_{ij}}}(t)$  as the vector relating agent I to agent j with  $\left\|\overrightarrow{R_{f_{ij}}}\right\| = R_{f_i}$ , we have  $R_{f_i} = -R_{f_j}$ .

Let, 
$$R_{f_i}(t) = \|P_i(t) - P_j(t)\|, i = 1, 2, ..., N. j = 1, 2, ..., N.$$
  
 $V_{ij} = \begin{cases} \frac{1}{2} K_a (R_{f_i} - L)^2, & R_{f_i} \le L \\ 0, & Otherwise. \end{cases}$ 
(21)

where L is the minimum distance to separate between two agents.

$$\frac{\partial R_{f_i}}{\partial (P_i, P_j)} = \left[ \frac{1}{R_{f_i}} \left( P_i(t) - P_j(t) \right)^T + \frac{1}{R_{f_j}} \left( P_j(t) - P_i(t) \right)^T \right]$$
(22)

$$\frac{\partial V_{ij}}{\partial R_{f_i}} = K_a (R_{f_i} - L)$$
(23)

using the chain rule we get,

$$\frac{\partial V_{ij}}{\partial (P_i, P_j)} = \frac{\partial V_{ij}}{\partial R_{f_i}} \frac{\partial R_{f_i}}{\partial (P_i, P_j)}$$
(24)

substituting (22),(23) in (24) leads to the repulsive potential to be given by

$$P_{rep} = -K_a (R_{f_i} - L) \frac{1}{R_{f_i}} \left[ (P_i(t) - P_j(t))^T - (P_j(t) - P_i(t))^T \right]$$
(25)

## 3. The Proposed Framework Design

In this section, we will discuss the formation maintenance under the assumption that all system dynamics are unknown. On-line NN weight tuning algorithms guarantee tracking a desired path. The follower will localize itself on its desired trajectory based on potential fields forces. For unknown robot dynamics, the proposed framework is composed of two stages. The first one is based on potential fields for formation control and maintenance as described in the previous section. The second one is the NN based robot controller to guarantee tracking of the desired path. Figure (3) shows the general structure of the system.

The robot has to follow its desired trajectory  $q_d$ . The tracking error is the difference between the desired path trajectory  $q_d$  and the estimated trajectory q. (Lewis, F. et. al. 1998)

$$\mathbf{E}(\mathbf{t}) = \mathbf{q}_{\mathbf{d}}(\mathbf{t}) - \mathbf{q}(\mathbf{t}) \tag{26}$$

where

$$q_d = \begin{bmatrix} x_f \\ y_f \end{bmatrix}$$
, and  $q = \begin{bmatrix} x \\ y \end{bmatrix}$  (27)

The filtered tracking error is given by

$$\mathbf{r}(\mathbf{t}) = \dot{\mathbf{e}} - \mathbf{l}'\mathbf{e} \tag{28}$$



Fig. 3. General structure of the proposed system.

 $\Gamma > 0$ : is a PD design parameter matrix.

By using the rigid body dynamics in general inertial frame as follow:

$$M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + F(\dot{q}) + G(q) + \tau_d = B(q)\tau$$
(29)

and differentiating equation (60). The dynamics of the robot can be written in terms of filtered error as

$$M\dot{r} = -V_m r - \tau + f + \tau_d \tag{30}$$

where f: is the nonlinear robot function which equal

$$f(x) = M(q)(\ddot{q}_d - \vec{\Gamma}\dot{e}) + V_m(q, \dot{q})(\dot{q}_d + \vec{\Gamma}e) + G(q) + F(\dot{q})$$
(31)

To compute f(x) we need to define the vector x as

$$x = \begin{bmatrix} e^T & \dot{e}^T & q_d^T & \dot{q}_d^T & \ddot{q}_d^T \end{bmatrix}^T$$
(32)

where x can be measured.

The appropriate controller can be used for the path following is derived by putting

$$\tau = \hat{f}(x) + K_c r \tag{33}$$

where  $K_c = K_c^T$ : is the gain matrix, and  $K_c r$  is the outer PD tracking loop.  $\hat{f}(x)$ : is the estimate of f(x). Then by using the controller (28), the closed loop error dynamics is given by:

$$M\dot{r} = -(K_c + V_m)r + \tilde{f} + \tau_d \tag{34}$$

The functional estimated error f is:

$$\tilde{f} = f - \hat{f} \tag{35}$$

The controller (33) will minimize the tracking error based on selecting an appropriate gain value for  $K_c$  and estimating  $\hat{f}$ . The error r(t) and the control signals should be bounded. The structure of this controller is given in figure (3). This controller uses PD control in the outer tracking loop and Neural network control in the inner loop to estimate the robot function f(x). The NN controller structure will be selected as

$$\tau = \widehat{W}\sigma(\widehat{V}^T x) + K_c r \tag{36}$$

Tuning weights of NN can be selected by using equation (34) and selecting  $K_c$  which can stabilize the filtered tracking error r(t).

For whole system to be stable the outer formation loop must be slower than the inner robot stabilization loop. The NN estimation error must be bounded and the tracking error also must be bounded then the system will be stable.

#### 4. Simulation Results

In this section we will report the simulation results of the leader motion and followers localization around their leader. The robot system is implemented in MATLAB. The robot parameters used in this study are m = 10kg, r = 0.05 m, R = 0.5 m, I = 5kg-m<sup>2</sup>; d = 0.8m.



Figure.(5).Group formation control response based on potential fields and NN.

The overall simulation results of the leader-follower group formation and whole fleet navigation are shown in the figures below. From figure (4), we can see how the group of three followers can localize themselves and create a star shape around their leader.



Fig. 4. a) Leader at (1.6,1.7), followers start from (1,1). b) Leader at (4.2,2.7), followers start from (1,1).

Figure (5.a) shows that the fleet of three nonholonomic follow their leader based on the potential fields and NN control. Also figures (5.b) shows fleet of five robots. The complete navigations map along the group desired path is shown in figure (6), which is giving us the desired formation.

#### 5. Conclusions and Future Work

In this paper, a framework for controlling and maintaining leader-follower nonholonomic robots multi-agent fleet formation is presented. Euler-Lagrange modeling framework has been used to make the approach applicable for other types of vehicles. The leader-follower group formation, maintenance, and navigation controller is developed based on artificial potential field strategy and SLAM-based navigation. The attractive and repulsive potential fields are used to control nonholonomic robots' positions and hold them on the desired path with reference to their leader. The leader generates its navigation map using the simultaneous localization and mapping (SLAM) approach. Also the designed framework can be used effectively for formation control and maintenance for any n nonholonomic robots. The paper addressed the case of unknown robot dynamics in the case when the on-line NN weight tuning algorithms guarantee bounded tracking error dynamics. The designed framework can be used effectively for formation control for any type of robots (i.e., heterogeneous systems). The designed controllers are verified and supported by numerical Matlab simulations. This paper considers the nonholonomic robots leader-follower formation in 2-D space. This system would be extended in future work to 3-D space. In this case, the control will be more complicated because the size of the degree of freedom will be

increased. Future work can also consider dynamic landmarks. For instance submarines do not have known landmarks and SLAM as it is described here cannot be used for the leader.



The fleets map"unknown dynamics & NN control

Fig. 6. The map of fleet of three agents along its navigation with their leader under.

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