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A Nonlinear Dynamic Model for Single-axle Wheelsets with Profiled Wheels and Rails

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Abstract **-** In this paper, the approach and preliminary results of the author's attempt to formulate a nonlinear dynamic simulation model that can be used to effectively and accurately predict the critical speed of railway vehicles are presented. Focusing on the modelling of the single-axle wheelset, a critical, yet challenging component, the paper discusses a number of important issues which include, but not limited to, representations of the wheel and rail geometries, determination of the contact points between the wheels and rails, kinematics of the wheelset and the creepages at the contact points, determination of the creep forces and moments, and the equations of motion. Results are presented that demonstrate the capability of the present model to investigate cases involving real wheel and rail profiles (instead of the idealized conical wheel and knife-edged rail) and not subject to the limitation of small yaw and roll motions. These preliminary results show that the impact of assuming small roll and yaw motions, and in particular, using idealized conical wheel and knife-edged rail is rather significant. This, in turn, demonstrates the need in properly formulating the nonlinear dynamic model of the wheelset. The next phase of the research will be to fine-tune the model and to extend it to a multi-axle truck, and eventually to the entire rail freight car.

Keywords**:** Nonlinear dynamics, rail wheelset, creep forces and moments, critical speed.

1. Introduction

The history of rail vehicles dates back to 18th century Europe. Today, rail transport plays a very important role in the transportation network, moving goods, heavy goods in particular, and people all over the world.

The interests in railway dynamics, for research as well as practical applications, have grown for decades. For example, dynamic stability was examined as early as the 1960's (de Pater, 1960; Wickens, 1965), and research into practical wheel-rail contact seems to begin in the 1970's (Kalker, 1973).

Of particular interest is the phenomenon known as hunting. Hunting refers to the self-excited lateral-yaw oscillation of the wheelset which takes place when the vehicle speed reaches and surpasses a certain speed, known as the critical speed. Once this critical speed is reached, the wheelset sways from side to side, causing it to rotate about the vertical axis. Hunting can cause wear to the wheels and rails. The large lateral force occurring during hunting can lead to derailment.

Since the 1990's hunting has been examined in the light of bifurcation of nonlinear systems (Ahmadian and Yang, 1998; Knudsen et. al. 1994). This paper presents the approach and preliminary results of the author's attempt to formulate a nonlinear dynamic simulation model that can be used to effectively and accurately predict the critical speed of railway vehicles. As such, the focus has been a single-axle wheelset, a critical, yet perhaps the most challenging, component to model.

The modelling of a single wheelset includes a number of important issues. One of such issues is, the determination of the forces and accelerations involved. This, in turn, necessitates, (a) the effective and accurate descriptions of the wheel and rail geometries; (b) the effective and accurate determination of contact points between the wheels and rails; (c) the accurate evaluation of the kinematics of the wheelset, and the creepages at the contact points; and (d) the effective and accurate determination of the creep forces and moments.

Existing methods of finding the contact points are mostly in the category of solution of nonlinear equations. For instance, de Pater (1988) proposed a set of 6 nonlinear equations for two-dimensional contact cases, and a set of 30 nonlinear equations for the three-dimensional cases. The iterative method employed by Yang (1993) was a combination of solving nonlinear equations and path-tracking. It was capable of finding single- as well as double-point contact. Of special interest to the present research is the search scheme adopted by Wang (1984). In this scheme, the wheelset was given a vertical lift and a lateral shift over the tracks. The roll angle was then incremented to find the roll angle such that the minimal distances between wheels and rails on both sides were technically equal. Because of incrementing the roll angle, Wang's search scheme was not computational effective. As will be seen in sub-Section 2.2, the present approach is that of optimization in which the roll angle is the independent variable and the objective is to minimize the difference of the minimal distances. The objective function is formulated in such a way that it can deal with single- as well as double-point contacts.

In terms of creep forces and moments, the Master's thesis by McFarlane (2009) has found the Polach nonlinear theory of creep to be accurate and computationally efficient, after benchmarking 5 theories including Johnson and Vemeulen's nonlinear theory, Kalker's linear theory, Heuristic nonlinear theory, and Polach nonlinear theory. It is worth mentioning that creep forces and moments are one of the most important aspects of wheel-rail interactions. They result from the relative motion between the wheel and rail. The extent of such relative motion is measured by creepage, which is defined as the difference between ideal or pure rolling (where the velocities of the wheel and rail at their contact point are equal), and the deviation of such. In the present work, the Polach nonlinear theory is also employed. It is however noted that it does not include the creep moments.

As to the equations of motion, which is another important issue in the modelling of a single wheelset, the wheelset is regarded as a rigid body. Garg and Dukkipati (1984) demonstrated the steps in reducing the usual 6 equations of motion to 2. They were concerned with the lateral and yaw motions, the so-called lateral dynamics. One key step was the determination of the normal forces at the contact points in terms of the vertical acceleration and the acceleration of the roll motion. Simplified equations of motion for idealised wheel and rail, and for small yaw and roll motions, were also given in Garg and Dukkipati (1984). The present work considers only the lateral dynamics. However, real wheel and rail profiles are used instead of the idealized conical wheel and knife-edged rail. In addition, the wheelset is not limited to having small yaw and roll motions.

The remainder of the paper is organized as follows. Section 2 deals with some modelling aspects. Due to space limitation, only the highlights are presented. Section 3 is concerned with numerical studies and concluding remarks are given in Section 4.

2. Modelling Aspects

In this section, the highlights of a number of important issues involved in the nonlinear dynamics of a single-axle wheelset will be presented. They are given in 5 sub-sections.

2. 1. Real Wheel and Rail Profiles

In the context of this paper, new wheel and rail profiles will be used. They are real in the sense that they have not been idealized to simple geometric shapes. The profiles will change once in service for a length of time, due to wear and tear. Such a case is not considered in this phase of the research. Figure 1 shows the profiles of an AAR 1:20 wheel and a UIC60 rail, where AAR and UIC stand for Association of American Railroads and International Union of Railways, respectively. The profiles are first represented by discrete data. After curve fitting, they are represented Basis splines (B-splines).

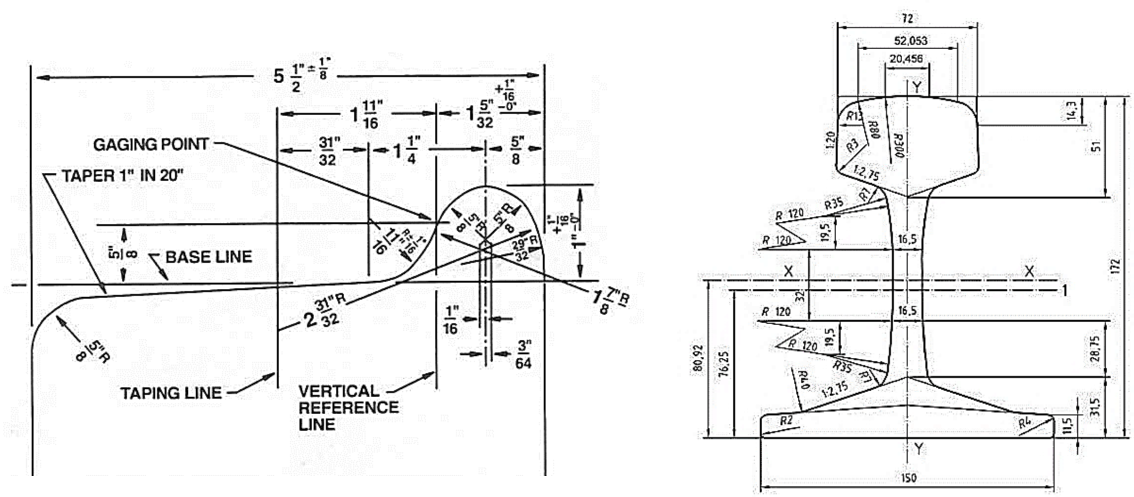
2. 2. Optimization Scheme for Determination of Contact Points

As mentioned in Section 1, Wang's search scheme (Wang, 1984) serves as the basis of the presently employed optimization scheme. As shown in Figure 2, the wheelset is given a vertical lift (the Z_s in Figure 2, which is arbitrary but rather large, say, 100 mm) and a lateral shift (which equals the lateral displacement y) such that the center of gravity (C.G.) of the wheelset is shifted to O^* from O. For any given roll angle φ, the minimal distance between left wheel and left rail profiles (geometrically, this will be shortest vertical bar amongst vertical bars drawn between the profiles), and that between right wheel and right rail profiles, can be easily evaluated. In Wang's search scheme, the roll angle φ was incremented until a roll angle was found that gave two technically equal minimal distances.

In the present research, such an angle is determined through optimization. In this optimization scheme, roll angle is the independent variable. For the case of single-point contact, the objective is to minimize the absolute difference in the two minimal distances. For double-point contact, the objective becomes to minimize the weighted sum of three absolute differences in minimal distances. The unconstrained optimization problem is then solved by the Nelder-Mead method.

The above optimization is repeated for a number of lateral displacement *y*. Resulting plots of roll angle versus *y*, vertical displacement at C.G. versus *y*, rolling radius at contact point(s) versus *y*, and contact angle versus *y*, are given in Figure 3. It is noted that the track gage is set to 1432 mm. No double-point contact is detected but the left wheel is seen to make contact with the left rail in the tread as well as the flange regimes.

Subsequently these plots are represented, after some curve fitting, by piece-wise polynomials of various degrees. These polynomials collectively will be incorporated into the nonlinear dynamics model, see Section 3.



dimensions in inches (1 inch = 25.4 mm)

dimensions in millimeters

Fig. 1. Wheel Profile of AAR 1:20 (left) and Rail Profile of UIC60 (right)

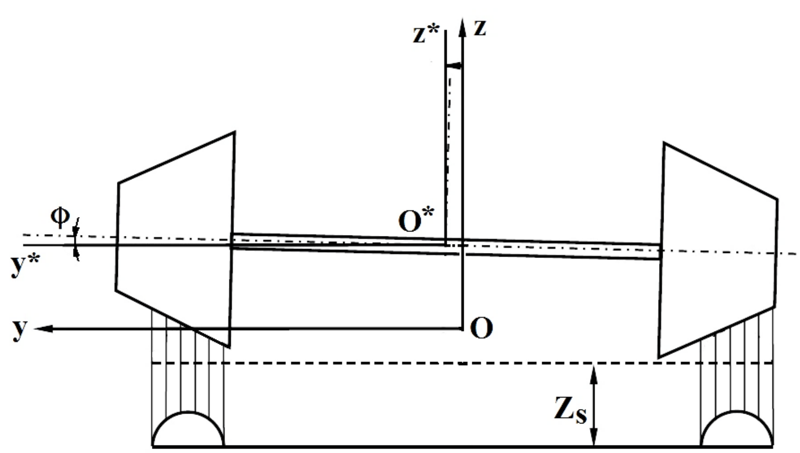
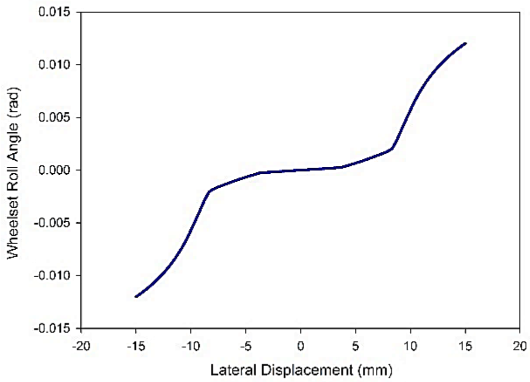
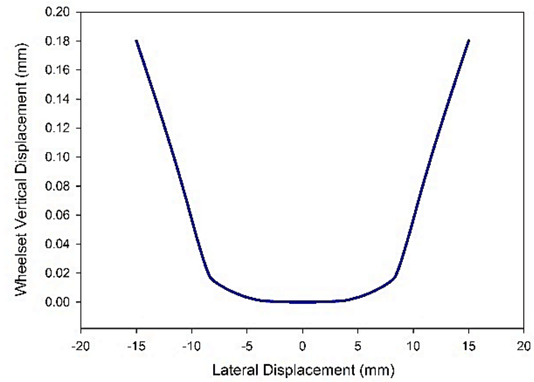


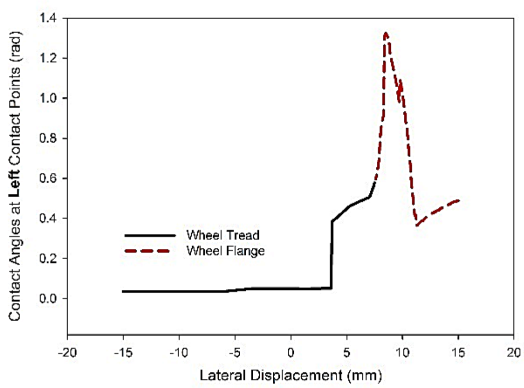
Fig. 2. A Schematic for Wang's Search Scheme (1984)



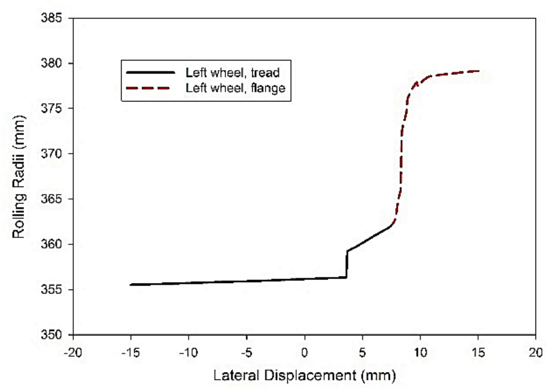
(a) roll angle



(b) vertical displacement



(c) contact angle



(d) rolling radius

Fig. 3. Representative Plots of Geometric Contact Parameters (AAR 1:20 with UIC60, track gage = 1432 mm)

2. 3. Kinematics of the Wheelset

A number of references deal with the kinematics of the wheelset (for example, Shabana, et al., 2008; Garg and Dukkipati, 1984; Petersen and Hoffmann, 2003). Typically, the general case and the case of small yaw and roll motions are presented. In the present work, the formulas given by Petersen and Hoffmann (2003), for the general case where yaw and roll motions may not be small, are implemented for its conciseness. For brevity, the details of Petersen and Hoffmann's formulas will be omitted. It should be mentioned that despite their differences in presentation, the above listed references represent essentially the same kinematics.

2. 4. Creepages, Creep Forces and Moments

Again, a number of references deal with the creepages at a contact point, see for example, Shabana, et al. (2008), Garg and Dukkipati (1984) and Petersen and Hoffmann (2003). When the assumption of small yaw and roll motions is invoked, the creepages can typically be explicitly expressed. Otherwise, creepages will be written in terms of components of some vectors. This, in turn, makes it easy to implement in computer coding. Similar to the kinematics of the wheelset, the formulas given by Petersen and Hoffmann (2003) for the general case where yaw and roll motions may not be small are employed in the present work. It should also be mentioned that the above listed references represent essentially the same creepages.

In terms of creep forces and moments, McFarlane (2009) has found the Polach nonlinear theory of creep to be accurate and computationally efficient. Consequently, the present work employs the Polach nonlinear theory. It should be noted that it does not include creep moments.

2. 5. Equations of Motion

The wheelset, as a rigid body, has 6 degree-of-freedom (DOF), see Figure 4. Based on Garg and Dukkipati (1984), the two equations of motion for the lateral dynamics of the wheelset are, for the case of single-point contact

$$m_w\ddot{y} = F_{Ly} + F_{Ry} + N_{Ly} + N_{Ry} + F_{sy} \tag{1}$$

$$I_{wx}\ddot{\psi} = -I_{wy}\left(\frac{V}{r_0}\right)\dot{\varphi} + R_{Lx}\left(F_{Ly} + N_{Ly}\right) - R_{Ly}F_{Lx} + R_{Rx}\left(F_{Ry} + N_{Ry}\right) - R_{Ry}F_{Rx}$$
$$+M_{Lz} + M_{Rz} + M_{sz} \tag{2}$$

where m_w, I_{wx}, and I_{wy} denote the mass of the wheelset, and the mass moments of inertia of the wheelset about the x, and y axes, respectively. Symbols y, φ and ψ mean the lateral displacement, roll motion and yaw motion, respectively. V is the forward speed of the wheelset, and r_0 is the (nominal) rolling radius. In the meantime, F_{sy} and M_{sz} are the suspension force and moment including spring and damping forces and moments. Further, N means a normal force, F a creep force, and M a creep moment. For such forces and moment, the first subscript indicates its location being at the L(eft) or R(ight) contact points, and the second subscript indicates its direction being along the x, y or z axes. Finally, R is the position vector draw from the C.G. of the wheelset to a contact point. The subscripts of R follow the convention used with N, F and M.

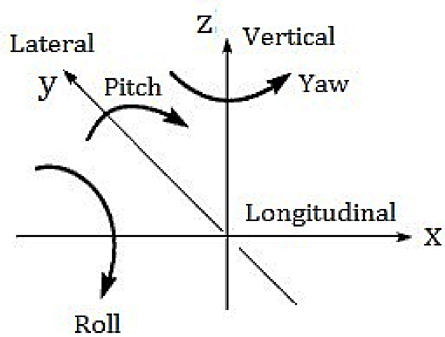


Fig. 4. The Six Degrees-of-Freedom of the Wheelset.

The determination of creep forces and moments requires the knowledge of the normal forces. The steps to evaluate the dynamic normal forces are as follows (McFarlane 2011). Firstly, inertial force F_d and moment M_d are defined

$$F_d = m_w\ddot{z} + W_a - F_{Lz} - F_{Rz} - F_{sz} \tag{3}$$

$$M_d = I_{wx}\ddot{\varphi} - I_{wy}\left(\frac{V}{r_0}\right)\dot{\psi} - R_{Ly}F_{Lz} + R_{Lz}F_{Ly} - R_{Ry}F_{Rz} + R_{Rz}F_{Ry}$$
$$-M_{Lx} - M_{Rx} - M_{sx} \tag{4}$$

Next, normal forces are expressed in terms of their respective magnitudes and unit vectors

$$\mathbf{N_L} = N_L\mathbf{n_l}\,,\ \ \mathbf{N_R} = N_R\mathbf{n_r} \tag{5a}$$

or

$$\begin{Bmatrix} N_{Lx} \\ N_{Ly} \\ N_{Lz} \end{Bmatrix} = N_L \begin{Bmatrix} n_x^l \\ n_y^l \\ n_z^l \end{Bmatrix}, \quad \begin{Bmatrix} N_{Rx} \\ N_{Ry} \\ N_{Rz} \end{Bmatrix} = N_R \begin{Bmatrix} n_x^r \\ n_y^r \\ n_z^r \end{Bmatrix} \tag{5b}$$

Finally, N_L and N_R are solved from

$$\begin{bmatrix} n_z^l & n_z^r \\ R_{Ly} n_z^l - R_{Lz} n_y^l & R_{Ry} n_z^r - R_{Rz} n_y^r \end{bmatrix} \begin{Bmatrix} N_L \\ N_R \end{Bmatrix} = \begin{Bmatrix} F_d \\ M_d \end{Bmatrix} \tag{6}$$

Note that the W_a in Eq. (3) is the pay load on the wheelset. In evaluating F_d and M_d via Eq. (3), the respective velocity and acceleration values at the previous step (which are known) are used instead of the respective unknown values at the current step.

Finally, it should be mentioned that the equations of motion involved in the lateral dynamics of a single-axle wheelset for the case of double-point contact has been presented in McFarlane (2009) as well as Mohan (2003). They are not presented here for brevity.

3. Numerical Results

A schematic of the single-axle wheelset under consideration is shown in Figure 5. Table 1 gives the numerical values used in the computational simulation. Note that there are three sets of values assigned to k_x and k_y. They are known as, from the top set to the bottom set, set 1, 2 and 3, respectively.

Three simulation codes are used.

- The Matlab code developed by Mohan (2003). This code has employed the small roll and yaw motions assumption, Johnson and Vemeulen's nonlinear creep theory. In addition, the first term on the right-hand side of Eqs. (3) and (4), respectively, have been omitted. Finally, idealized wheel and rail are assumed (in a similar manner to sub-Section 2.3.1 of Mohan 2003). Results from this code will be denoted as "Mohan" in later discussions.
- A Matlab code developed by the author that implements the kinematic and creepage formulation by Petersen and Hoffmann (2003), without the small roll and yaw limitation. The equations of motion are per sub-Section 2.5 above. Idealized wheel and rail are assumed. Results from this code will be denoted as "Idealized".
- A Matlab code developed by the author that implements the kinematic and creepage formulation by Petersen and Hoffmann (2003), also without the small roll and yaw limitation. The equations of motion are per sub-Section 2.5 above. Profiled wheel and rail are assumed. Piece-wise polynomial representation of contact geometric parameters (Figure 3) is incorporated. Results from this code will be denoted as "Profiled".

Table 1. Parametric Values Used in Simulation*

Parameter	Value	Parameter	Value	Parameter	Value
m_w	1751 kg	I_{wx}	761 kg.m^2	I_{wy}	130 kg.m^2
r_0	355.6 mm	g_a	716 mm	b	610 mm
k_x	2.85×10^5 N/m	k_y	1.84×10^5 N/m	λ	0.05
	2.85×10^6 N/m		1.84×10^6 N/m		
	9.12×10^6 N/m		5.84×10^6 N/m		

* λ is the conicity.

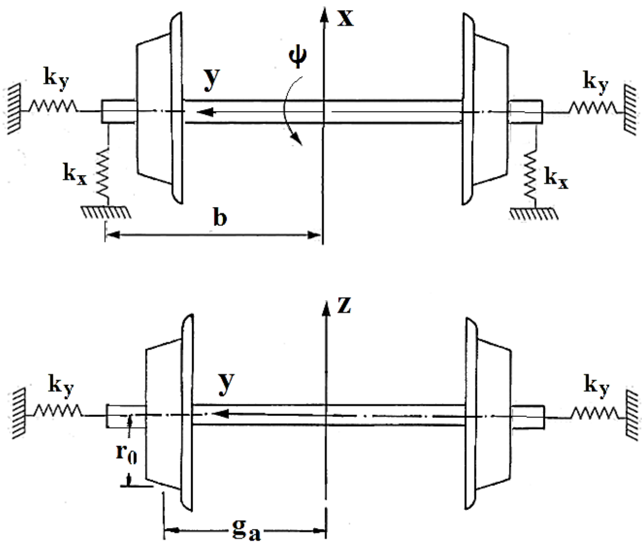


Fig. 5. A Schematic for Single-Axle Wheelset.

The objective at the current time is to determine the critical velocity. To be precise, this critical velocity is only the so-called "linear critical speed". The computed critical speeds are tabulated in Table 2.

Table 2. Computed Critical Speeds.

Code	k_x and k_y		
	Set 1	Set 2	Set 3
Mohan	37.1 m/s	117.2 m/s	221.0 m/s
Idealized	37.0 m/s	116.9 m/s	210.0 m/s
Profiled	38.8 m/s	115.4 m/s	187.8 m/s

Comparing the critical speeds obtained by "Mohan" and by "Idealized", it is seen that the small roll and small yaw assumption has a noticeable impact on the critical speed, yielding a higher value. The impact is more pronounced with stiffer springs. The effect of the terms such as $m_w\ddot{z}$ of Eq. (3) and $I_{wx}\ddot{\phi}$ of Eq. (4) seems negligible, as the same (or technically the same) critical speeds as those listed in second row of Table 2 have been found when $m_w\ddot{z}$ and $I_{wx}\ddot{\phi}$ are omitted in the computations.

Using "idealized" wheel and rail, on the other hand, may have a more significant impact on the critical speed as oppose to using the real wheel and rail. The impact, specifically, seems to depend on the parameters of the wheelset. Although a -4.6% relative error is found with the Set 1 values in k_x and k_y, the relative error becomes greater than 10% when the springs are stiffer, as in Set 3. It is also seen that, the "Idealized" way of evaluating the critical speed typically gives upper bound solution compared with the "Profiled" way, when practical (i.e., higher) values of spring constants are considered.

4. Conclusion

In this paper, a nonlinear dynamic simulation model that can be used to effectively and accurately predict the critical speed of a single-axle railway wheelset is presented. A number of important issues are discussed including, but not limited to, descriptions of the wheel and rail geometries, determination of contact points between the wheels and rails, kinematics of the wheelset and creepages at the contact points, determination of the creep forces and moments, and the equations of motion. Results are presented that demonstrate the capability of the present model to investigate cases involving real wheel and rail profiles (instead of the idealized conical wheel and knife-edged rail) and not subject to the small yaw and roll angles. These preliminary results show rather convincingly why small roll and yaw motions may not be assumed, and in particular, why idealized conical wheel and knife-edged rail on the critical speed should be avoided. This, in turn, calls for the need to properly formulate the nonlinear dynamic model of the wheelset. It is known that the dynamic response of railway vehicle systems can be very sensitive to changes in the system parameters (Shabana et al., 2008). Therefore, the next phase of the research will fine-tune the model. The model will be extended from a single-axle wheelset to a multi-axle truck, and from trucks to the entire rail freight car.

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