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Dynamic Analysis of a Rimless Wheel on Randomly Generated Rough Terrain

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Abstract - Some modern walking robots are capable of walking over very rough terrain. However, these robots often require significantly more energy to do so than a human. Passive-dynamic walking robots are often quite energy-efficient, but are usually only able to walk over smooth surfaces. In this work, we examine the dynamics of a 2D rimless wheel, an often studied simple model for walking. This paper analyzes the motion of a rimless wheel on slanted ground with a roughness level that is significantly smaller than the dimensions of the rimless wheel itself. The roughness of the ground is randomly generated but bounded in magnitude. The minimum angle of inclination required for a rimless wheel to walk down both smooth and rough ramps is determined. The rimless wheel is capable of walking down a rough surface but requires higher angles of inclination than when it is walking on a smooth surface. For the rimless wheel we examined with 5 legs, the minimum slope required for a rough surface is 12.4% higher than that required for a smooth surface, and for 10 legs, the minimum slope for a rough surface is 40.83% higher than the smooth surface.

Keywords**:** Rimless wheel, smooth surface, rough surface, slope, passive dynamic, walking.

1. Introduction

All walking devices will, by necessity, have plastic collisions between their feet and the ground, since foot bounce is not a characteristic of walking gaits. For generic plastic collisions, energy is not conserved, and there is an abrupt change in the system velocities. The simplest “walking” robot that has ground collisions is a rimless wheel (RW), whose dynamics have often been studied Coleman and Ruina (1997). McGeer (1990), the pioneer of passive dynamic walking, also examined the dynamics of this simple 2D rimless wheel walking down an incline or ramp and determined how the steady-state speed of the wheel changes as a function of parameters and ramp angle. As shown by Asano (2012), the motions of the rimless wheel down a ramp are periodic and asymptotically stable. The dynamics of several variants of the standard 2D rimless wheel have been studied. For example, Jian (2010)) examined a rimless wheel that had been modified to have asymmetric flat feet. Essentially this modification to the wheel introduces a doubling of the number of legs with a 2-cycle periodic radial spacing. Again though, like most dynamic studies of the rimless wheel the system is analysed for motion down smooth, non-bumpy, ramps.

One of the often stated advantages of legged locomotion is the ability to traverse rough terrain since, unlike wheels, only intermittent ground contact is needed. In this paper, we explore the dynamics of a 2D rimless wheel traversing a rough surface. However, a rimless wheel on a rough surface does not have a strictly periodic motion since the angular position of the leg at the start of the step will be different at the end of step due to the roughness of the ground but it can have long periods of continuous walking (Byl and Tedrake 2009). In this paper, we use a randomly generated rough terrain which varies about a mean slope angle. Some work has been done on the stability of the rimless wheel on a rough terrain by Byl and Tedrake (2008). They studied the dynamics of the simple rimless wheel on slanted ground whose slope changes at each new impact and is determined by randomly using a Gaussian distribution. They have pointed out that systems with long periods of continuous walking are metastable systems as they neither fall in the completely stable category nor in the unstable category. In our analysis the roughness of the ground also changes randomly at each new impact but we use a different description of ramp roughness (which leads to fundamentally different long term system behaviour) and a uniform probability distribution for the roughness.

One of the reasons for the interest in studying a walking systems response to foot impacts is because, for these ideal models, foot impacts are the only method for energy loss in the system. In addition, it has been shown by Chatterjee *et al.* (2002) and Gomes and Ruina (2011) that even for a system subject to plastic collisions, it is possible to carefully design a device which can avoid this mechanism of energy loss (at least for smooth terrain). Namely, matching the velocity of the contact point at the time of impact. Improved understanding of the energy losses due to collisions can improve our understanding of how to design these walking systems to be more efficient over rough terrain.

In this paper we study the slope requirements of a rimless wheel on a smooth surface as well as a rough surface. We also analyse the average number of steps a rimless wheel can complete for a certain a slope of the slanted ground and for a given number of legs.

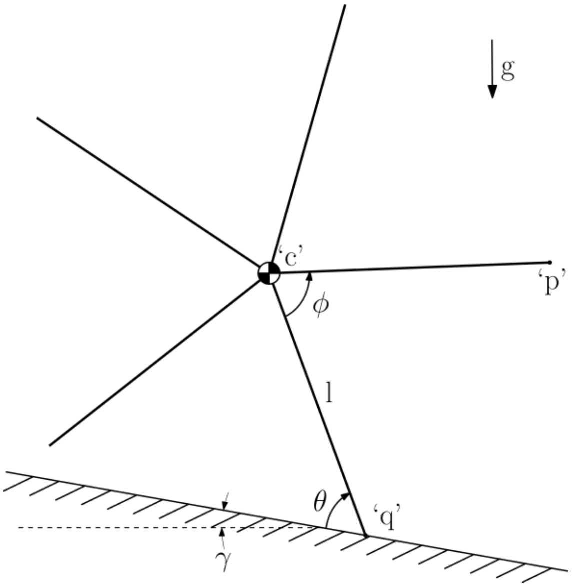

Fig. 1. Rimless wheel on a slanted ground with a smooth surface.

2. System Model

A rimless wheel can be thought of as a wagon wheel without the outer rim. The spokes are evenly spaced starting from the central hub and have equal lengths. The "feet" of the rimless wheel are at the end of the spokes. Fig. 1 shows a schematic of the rimless wheel on a smooth surface. The centre of mass is located at the central hub. The motion of the simple rimless wheel is in two dimensions. Gravity is the only source of energy used by the rimless wheel to walk down a slanted ground. Perfectly plastic collisions are assumed to occur when the foot impacts the ground. The foot, which is in contact with the ground, is called the stance foot and the foot, which is about fall on the ground, is called the swing foot or, after contact, the new stance foot. The transition from the old stance foot to new stance foot is assumed to be instantaneous. The post collision angular velocity is calculated using the fact that angular momentum is conserved at the point where the swing foot contacts the ground. The model also assumes that the stance foot does not slip and only lifts off the ground when the swing foot strikes the ground.

The motion of the rimless wheel is described in two phases. The two phases are:

a) The swing phase: Here the wheel acts as a rigid-body pendulum. From the free-body diagram in Fig. 2 the equation for this phase is given in Eqn. 1.

$$\ddot{\theta} = \frac{-mgl\cos(\theta+\gamma)}{I_o} \tag{1}$$

The initial conditions are angular position of the stance leg θ and angular velocity $\dot{\theta}$. θ is fixed, since we start the system with both feet on the ground, and depends on the number of legs. θ is calculated as follows:

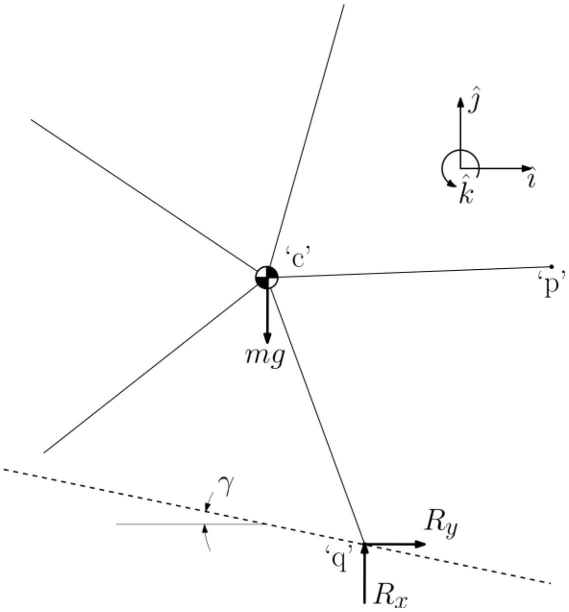

Fig. 2. Free body diagram of the rimless wheel on a slanted ground with a smooth surface.

$$\Phi = \frac{360}{number\ of\ legs} \tag{2}$$

$$\theta = \frac{180 - \Phi}{2} \tag{3}$$

b) The strike phase: Here the swing foot comes in contact with the ground and it becomes the new stance foot. Angular momentum of the system is conserved about the impact point. The velocity after the collision is different than the velocity just before the collision and can be determined by Eqn. 4.

$$\dot{\theta}_{after} = \dot{\theta}_{before}\,\frac{I_{cm}}{ml^2 + I_{cm}} \tag{4}$$

Numerical solutions to these equations were determined using a 4th order Runge–Kutta integration method (ODE45 in MATLAB).

Table 1 lists the system parameters used in the simulations, which corresponds to a physical prototype for a related system.

Table 1. System parameters and their values.

Parameters	Symbol	Value	Units
Mass of the rimless wheel hub	m	1.778	(kg)
Gravity	g	9.81	(m/s^2)
Leg length	l	0.425	(m)
Initial angular velocity of the rimless wheel	$\dot{\theta}$	10	(rad/s)
Moment of inertia about the centre of mass of the rimless wheel	I_{cm}	0.0959	(kg m/s^2)
Number of legs	-	5,6,7,8,9,10	-

The roughness of the terrain is generated using a pseudorandom number with a uniform probability distribution. The roughness, shown in Fig. 3&4, (δ_n) is bounded between +/- 1/20 of the leg length.

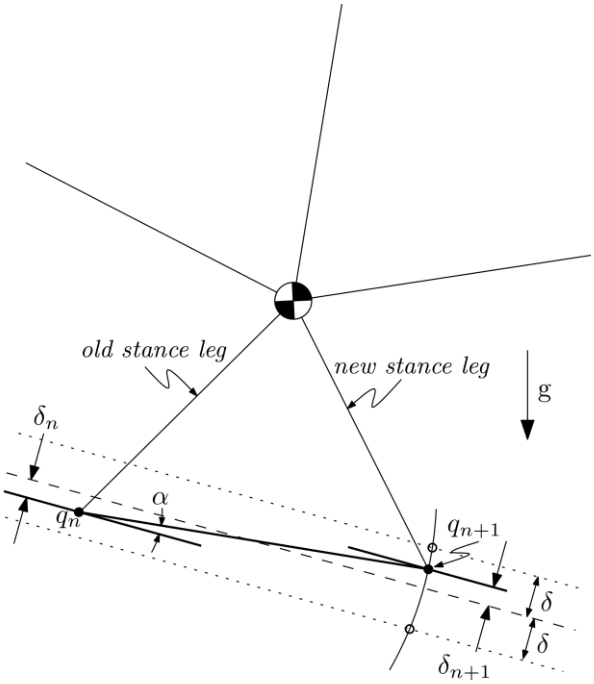

Fig. 3. A Rimless wheel on slanted ground with a rough surface. The maximum roughness, above or below the median slope of the ramp (indicated by the dashed line) is δ.

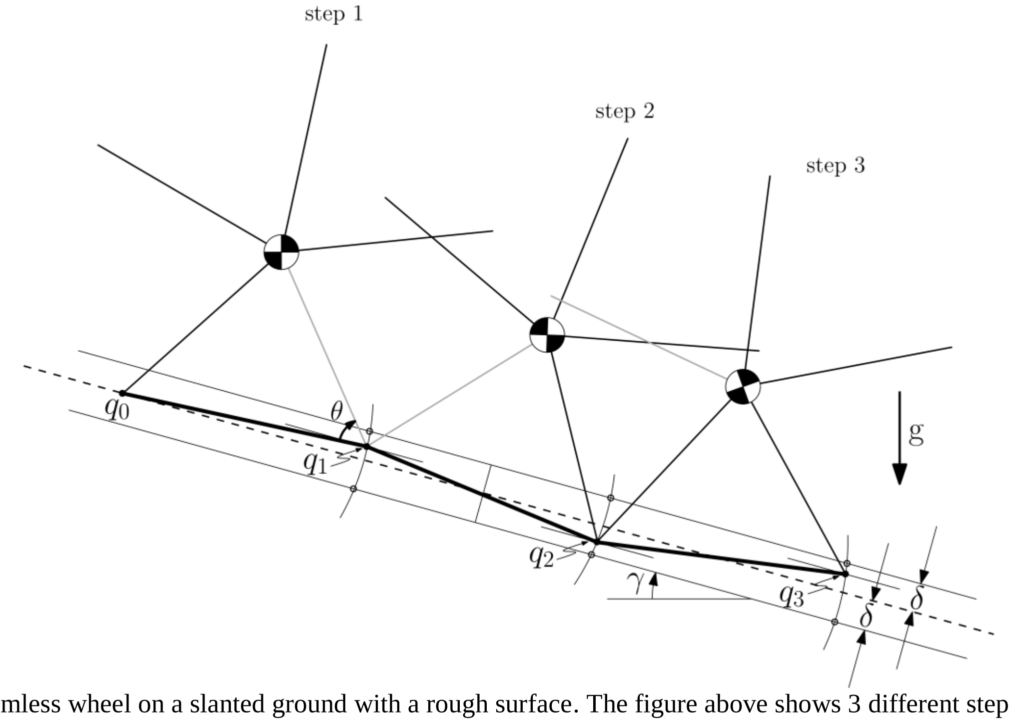

Fig. 4. Rimless wheel on a slanted ground with a rough surface. The figure above shows 3 different steps taken by the rimless wheel. For each step the roughness of the slanted ground is given by q_1, q_2, q_3. The maximum roughness above or below the slanted ground is denoted by δ which for our simulation is $1/20$ of the leg length.

Note that this definition of roughness is fundamentally different from the random-slope-at-each-step definition used by Byl and Tedrake (2008). One way in which it differs is that it strictly limits the "worst-case" scenarios for several steps. For example, assume the random distribution results in several successive steps with the largest deviation the model allows. For the model of roughness we use, the first step will be shallow (low ramp angle), but the succeeding steps will all have the mean slope (it will just be displaced, perpendicular to the slope, by δ_{max}. However, if we use Byl and Tedrake's model several successive steps with maximum deviation in ramp angle will result in several steps with a constantly lower ramp angle than the mean.

3. Results

3.1. Minimum Angle of Inclination Required by the Rimless Wheel on a Slanted Ground with Smooth and Rough Surfaces

A computer simulation was created to determine the minimum angle of inclination (slope) required by the rimless wheel on a smooth surface for a given number of legs. The minimum slope was measured through simulated trials. Note that for a smooth surface, there is a defined minimum slope below which the rimless wheel cannot sustain motion. This minimum slope depends on the number of legs of the device but also its mass and inertia properties. The static slope curve shown in Fig.5, is the slope on which the rimless wheel can walk down without any initial angular velocity. This happens when the line of centre of gravity lies vertically above on the stance foot. In our simulations, all the geometric and mass parameters were kept constant except for the number of legs.

The rimless wheel was made to walk on a randomly generated rough terrain on the slanted ground. Fifty trials were performed for each different slope. The range of slopes for a given number of legs were chosen such that its least slope was the minimum slope required by the rimless to walk down the smooth surface and the maximum of the slope range is the one on which the rimless wheel can take at least 50 steps in all the 50 trials. The roughness of the terrain was maintained between -l/20 meters to +l/20 meters. As seen in Fig.5, the rimless wheel on the rough terrain requires larger slope to complete fifty steps as compared to the rimless wheel on a smooth surface. The line of static slope as expected is above all the other lines. Interestingly, the line of the minimum slope for the rough surface crosses the line of the static slope. Also, the difference between the minimum required slope for different number of legs, for both a smooth surface as well as a rough surface decreases as the number of legs increases.

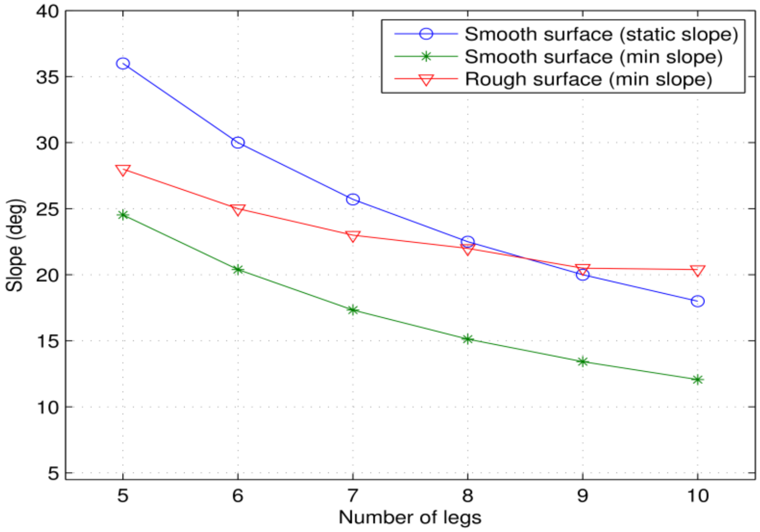

Fig. 5. Minimum slope required for a given a number of legs to complete 50 steps on a smooth surface as well as on a rough surface. The initial angular velocity of the rimless wheel was considered to be 10rad/s. Note that the lines connecting data points can be misleading, we are not implying that we are interested in solutions for non-integer numbers of legs, but the interpolation lines improve the clarity of the trend.

3.2. Average Number of Steps on a Rough Surface

The number of steps completed by a rimless wheel on a rough terrain strongly depends on the amount of roughness and the slope of the slanted ground. The roughness at every step is randomly generated and can be different at every step. Hence, the number of steps completed for a certain number of legs and slope for every trial may or may not be different. If the roughness is at its peak, that is, either -l/20 meters or +l/20 meters and the slope is not enough, then the rimless wheel will not be able to complete a step. Fig. 5 shows that as the slope increases, the average number of steps taken by the rimless wheel also increases.

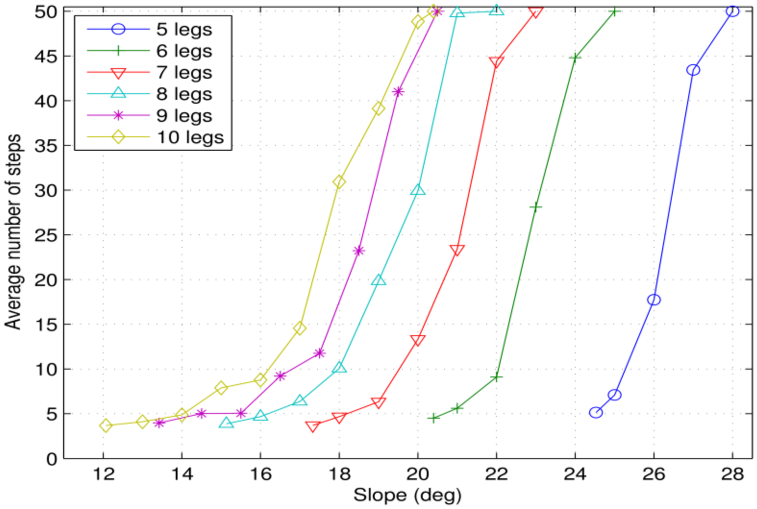

Fig. 6. Relation between the average number of steps completed by a rimless wheel walking a down a rough terrain for a given number of legs and slope. Fifty trials were performed for each different slope and fifty steps for each trial. Even though the relation is not linear, a similar pattern is followed by all the wheels of varying leg number.

4. Conclusion

This paper analyses the motion of the rimless wheel walking down a slanted ground with both, a smooth surface and a rough surface. The roughness of the ground is generated randomly. The equations of motion for the rimless wheel on both the rough surface and smooth surface are the same. The minimum slope required for the rimless wheel to complete 50 steps is more when the surface is rough then when the surface is smooth. The minimum slope required by a five legged rimless wheel on a rough surface is 12.4% higher than the slope required on a smooth surface. The difference in percentage of minimum slope, on a rough surface and on a smooth surface, increases as the number of legs increases. For a ten legged rimless wheel the difference is about 41%. It was found that the number of steps a rimless wheel can take on a rough surface highly depends on the slope and roughness of the slanted ground.

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