Proceedings of the International Conference of Control, Dynamic Systems, and Robotics
Ottawa, Ontario, Canada, May 15-16 2014
Paper No. 73

Comparison and Verification of Several Stiffness Models for a Family of Parallel Manipulators

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***Abstract* -** This paper investigates the difference between several stiffness models for a family of parallel manipulators that has n-DOF with an n-DOF central passive leg and n identical actuated legs. For demonstration purpose, the 3UPS-PU parallel manipulator is employed as an example. The Jacobian matrix of the mechanism is first analysed for the purpose of subsequent analysis; then the traditional stiffness matrix of the mechanism is derived based on calculation of the Jacobian matrix. Thirdly, based on the kinetostatic and the principle of virtual work, one derives the kinetostatic compliance matrix of the 3UPS-PU mechanism by considering the central passive limb as rigid. Furthermore, the dexterous stiffness model is proposed to evaluate the transferring ability of the manipulator. Through comparison among compliance in each direction under different stiffness models, it comes to the fact that the kinetostatic compliance model is the closest one to the traditional stiffness model, which verifies the accuracy of the kinetostatic stiffness/compliance model.

***Keywords*:** Parallel manipulator, traditional stiffness model, kinetostatic stiffness model, dexterous stiffness model, stiffness comparison.

1. Introduction

Parallel mechanisms have been widely used recently, especially in parallel robotic machine tools (Zhang, 2009), automotive realm (Yu, 2010), medical devices (Pan, 2011, Castelli, Ottaviano, 2010), etc. This is largely due to the fact that the parallel mechanisms possess high stiffness and high payload capacity, etc., comparing to their serial counterparts. These attributes all attribute to the structure of parallel mechanisms, i.e. the moving platform is connected to the base by several parallel limbs and therefore, the force and loads acting on the moving platform can be distributed by those limbs.

Stiffness can be viewed as one of the most important characteristics for parallel manipulators, since high stiffness can lead to high precision when manufacturing pieces in the machine tools arena, and until now there are several stiffness models being proposed. The basic one is the traditional stiffness model (TSM), which is derived based on calculation of Jacobian matrix of the mechanism, and it has been widely used in the area of parallel manipulators. The traditional stiffness model didn't consider the central passive leg, i.e. it assumes that the central passive leg is rigid if applicable, it only accounts for the actuated leg, and it normally assumes the joint stiffness of each actuator is 1000. Scholar Zhang (Zhang, 2000) employed the kinetostatics and the principle of virtual work to derive the kinetostatic stiffness model (KSM) when considering the central passive limb as compliant and the kinetostatic compliance model (KCM) when considering the central passive limb as rigid, respectively, for a family of parallel manipulators that has n-DOF with a n-DOF central passive leg and n identical actuated legs. By default all the actuated links are assumed rigid, and the compliance of the mechanism is solely induced by the compliance of the actuator. Note that in the traditional stiffness model, the joint stiffness of each actuator is 1000. For the comparison purpose, we assume the central leg is rigid for the kinetostatic stiffness/compliance model, and the compliance of the actuator is set to 0.001, which is equivalent to the value 1000 for the stiffness of each actuator. Furthermore, the dexterous stiffness model (DSM) was proposed to account for the singular position of the mechanism in order to evaluate the transferring ability of the manipulator (Zhang, 2012).

In (Wei, et al., 2012), the conservative congruence transformation (CCT) stiffness model and the traditional stiffness model of a 4UPS-PU parallel manipulator are compared, and it is shown that the stiffness in each direction under CCT model is larger than that of the traditional stiffness model. Similarly, in (Hong, 2002, Li, Gosselin, 2007), the CCT stiffness model and the traditional stiffness model of parallel manipulators are compared and analysed. To the best of the authors' knowledge, no one has ever compared kinetostatic stiffness model and dexterous stiffness model with the traditional stiffness model. The importance of comparing these stiffness models is that it can verify the accuracy and correspondence of these stiffness models, which can pave solid foundation for future usage, especially for the kinetostatic stiffness/compliance model. In order to compare and verify the correspondence and accuracy of these different stiffness models, the 3UPS-PU parallel manipulator is employed as an example for demonstration, and the compliance in each direction of 3UPS-PU parallel manipulator is compared under the above three stiffness models. The reason why we didn't compare the stiffness in each direction here is that the compliance matrix derived by the kinetostatic and principle of virtual work cannot be inverted.

2. Inverse Kinematic and Jacobian Matrix

Figure 1(a) is the 3UPS-PU mechanism. The moving platform is connected to the base by three identical actuated limbs U-P-S and one central passive limb P-U. Due to there is a middle passive limb, this parallel mechanism has three degrees of freedom, i.e. two rotations about X and Y axes and one translation along Z axis. For the purpose of analysis, two Cartesian coordinate systems $O(X,Y,Z)$ and $P(x,y,z)$ are attached to the centre of the base and moving platform, respectively, as shown in figure 1(b).

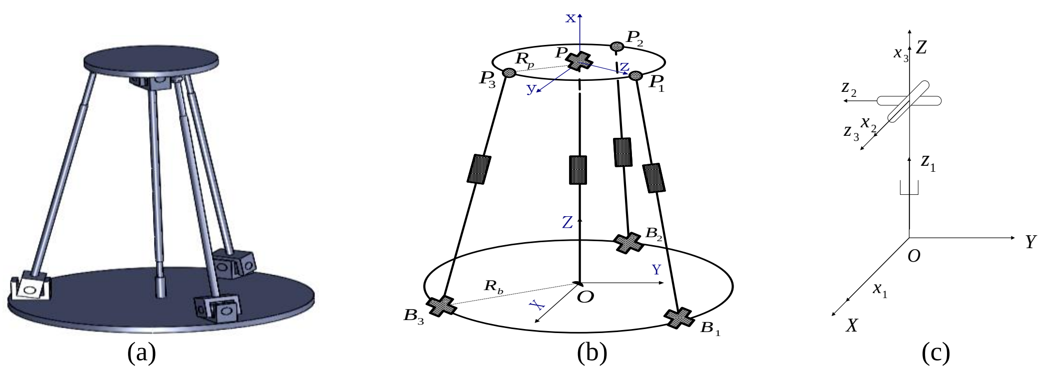

Fig.1. (a) 3UPS-PU mechanism (b) schematic representation (c) kinematic structure of passive constraining leg

2. 1. Inverse Kinematic

Based on the vector loop equation, the following can be obtained,

$$P_i - b_i = {}^{O}P + Qr_i^{'} - b_i \qquad (i = 1,2,3) \tag{1}$$

where b_i are the position vectors of point B_i with respect to the fixed frame, $r_i^{'}$ and P_i are position vectors of attachment points on the moving platform with respect to moving frame and fixed frame, respectively, ${}^{O}P$ is the coordinate of point P with respect to fixed frame, and Q is rotation matrix of the moving frame with respect to the fixed frame. The solution of the inverse kinematic can be written as follows,

$$\rho_i^{\,2} = (P_i - b_i)^T (P_i - b_i) \tag{2}$$

2. 2. Jacobian Matrix

Considering the parallel component of the mechanism, the parallel Jacobian matrices A and B can be computed by differentiating equation (2) with respect to time (Zhang, 2000), one obtains,

$$\rho_i \dot{\rho}_i = (P_i - b_i)^T \dot{P} + [(Qr_i^{'}) \times (P_i - b_i)]^T \omega \tag{3}$$

One can write the velocity equation as

$$At = B\dot{\rho} \tag{4}$$

where $A = \begin{bmatrix} m_1^T & m_2^T & m_3^T \end{bmatrix}^T$, $B = diag\begin{bmatrix} \rho_1 & \rho_2 & \rho_3 \end{bmatrix}$, $m_i = \begin{bmatrix} (Qr_i^{'}) \times (P_i - b_i) & (P_i - b_i) \end{bmatrix}^T$

Then the Jacobian matrix of the pure parallel component can be expressed as:

$$J_{parallel} = B^{-1}A \tag{5}$$

The central passive limb can be viewed as a serial component, the kinematic structure of it is shown in figure 1(c), and the D-H parameters are subsequently obtained as listed in table 1.

Table. 1. The D-H parameters for the central passive leg.

i	a_i	d_i	α_i	θ_i
0	0	0	0	0
1	0	z	90°	0
2	0	0	90°	θ_2
3	0	0	0	θ_3

where θ_1 and θ_2 are the joint angles of the universal joint in the central passive leg. For the central passive leg, one has,

$$t = J_{serial}\dot{\theta} \tag{6}$$

where t is the twist of the platform , $\dot{\theta}$ is the joint velocity vector associated with the central passive leg, and J_{serial} is the Jacobian matrix of the central passive leg of the mechanism (Zhang, 2000). The Jacobian matrix for the whole mechanism can therefore be expressed as (Zhang, 2011):

$$J_{rigid} = J_{parallel} J_{serial} \tag{7}$$

3. Different Stiffness Models of the Mechanism

3.1. Traditional Stiffness Model

The traditional stiffness model can be determined based on the calculation of the Jacobian matrix, and it is already well known so we just directly give its form as follows:

$$K = J_{rigid}{}^{T} K_q J_{rigid} \tag{8}$$

where K_q is the joint stiffness matrix of the parallel mechanism, with $K_q = diag[k_1, k_2, k_3]$. The scalar $k_i (i = 1, 2, 3)$ represent the joint stiffness of each actuator. Normally the actuated limbs are the same for the purpose of easy controlling, and then the above equation can be rewritten as:

$$K = kJ_{rigid}{}^{T} J_{rigid} = 1000 J_{rigid}{}^{T} J_{rigid} \tag{9}$$

where $k = k_1 = k_2 = k_3 = 1000$. The leading diagonal element of stiffness matrix K represents stiffness in each direction. The traditional stiffness model didn't consider the central passive leg, i.e. it assumes that the passive leg is rigid if applicable. It only accounts for the actuated leg, and it normally assumes the joint stiffness of each actuator is 1000 as shown in above equation.

3.2. Kinetostatic Compliance Model

Scholar D. Zhang derived the kinetostatic compliance model (KCM) when considering the central passive limb as rigid for a family of parallel manipulators that has n-DOF with a n-DOF central passive leg and n identical actuated legs by employing the kinetostatics and the principle of virtual work (Zhang, 2000), and by default all the actuated links are rigid, and the compliance of the mechanism is only induced by the compliance of the actuator. Here for the purpose of comparison with the traditional stiffness model, it is assumed that the central leg is rigid, and the compliance of the actuator is set to 0.001, which is equivalent to the stiffness of each actuator 1000 in the traditional stiffness model. One can use the principle of virtual work to derive the compliance matrix of the 3UPS-PU mechanism. Based to the principle of virtual work, one has the following equation,

$$\tau^T \dot{\rho} = w^T t \tag{10}$$

where τ is the vector of actuator forces and w is the wrench applied to the platform. Rearranging equation (4) and substituting it into equation (10), one obtains the following,

$$(AJ_{serial})^T B^{-T} \tau = J_{serial}{}^{T} w \tag{11}$$

An actuator compliance matrix C is defined as $C\tau = \Delta\rho$, and C is a (3×3) diagonal matrix whose ith diagonal element is the compliance of the ith actuator. Like stated above, here we assume the compliance of each actuator is 0.001, which corresponds to the joint stiffness of each actuator in the traditional stiffness model. Make use of equation (6) and (11), and rearrange the equation finally yields the following,

$$\Delta c = J_{serial} (AJ_{serial})^{-1} BCB^T (AJ_{serial})^{-T} J_{serial}{}^{T} w = C_c w \tag{12}$$

where C_c is 6 by 6 Cartesian compliance matrix and it cannot be inverted.

3.3. Dexterous Stiffness Model

For the working environment of the manipulator, when the robotic system works in the singular points or near-singular region, its accuracy, rigidity and other performances will be worse (Zhang, 2012). Thus the leading diagonal elements cannot truly reflect the stiffness of the mechanism. Hence the dexterous stiffness model was proposed to evaluate the transferring ability of the manipulator. The dexterous stiffness matrix can be written as:

$$K_L^{'} = k' J_{rigid}{}^T J_{rigid} \tag{13}$$

where $k' = \dfrac{1000}{(svd(J_{rigid}))^T \cdot svd(J_{rigid})}$, $svd(J_{rigid})$ means the singular value decomposition of Jacobian matrix J_{rigid}.

4. Compliance Comparison among Three Models

4.1 Traditional Stiffness Model

For the purpose of analysis, we assume $R_p = 0.07$, $R_b = 0.16$, z=0.66, $\theta_2 = 100^\circ$, $\theta_3 = 0^\circ$, then the stiffness matrix K of the 3UPS-PU parallel manipulator can be determined:

$$K = \begin{bmatrix} 2872.6 & 0.4 & -3.7 \\ 0.4 & 6.9 & 0.3 \\ -3.7 & 0.3 & 6.8 \end{bmatrix}$$

The compliance matrix is the inverse of the stiffness matrix, and it has

$$C = \begin{bmatrix} 0.0003 & 0 & 0.0002 \\ 0 & 0.1461 & -0.0055 \\ 0.0002 & -0.0055 & 0.1472 \end{bmatrix}$$

The compliance in each direction is:

$$[c_z \quad c_{\theta_x} \quad c_{\theta_y}] = [0.0003, 0.1461, 0.1472]$$

The sum of the compliance is: 0.0003+0.1461+0.1472=0.2936.

4.2. Kinetostatic Compliance Model

The same parameters are used for comparison purpose, and one has the following compliance matrix:

$$C = \begin{bmatrix} 0.1428 & 0.0054 & 0.0252 & 0 & 0 & 0.0002 \\ 0.0054 & 0.1461 & 0.0010 & 0 & 0 & 0 \\ 0.0252 & 0.0010 & 0.0044 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0002 & 0 & 0 & 0 & 0 & 0.0003 \end{bmatrix}$$

The compliance in each direction is:

$$C_c = [c_{\theta_x} \quad c_{\theta_y} \quad c_{\theta_z} \quad c_x \quad c_y \quad c_z] = [0.1428\ , 0.1461, 0.0044, 0, 0,\ 0.0003]$$

The sum of the compliance is: 0.1428+0.1461+0.0044+0+0+0.0003=0.2936.

4.3. Dexterous Stiffness Model

Similarly, the same parameters are used and the dexterous stiffness matrix is as follows,

$$K_L^{'} = \begin{bmatrix} 995.2671 & 0.1525 & -1.2921 \\ 0.1525 & 2.3743 & 0.0885 \\ -1.2921 & 0.0885 & 2.3586 \end{bmatrix}$$

And its corresponding compliance matrix is as follows,

$$C = \begin{bmatrix} 0.0010 & -0.0001 & 0.0006 \\ -0.0001 & 0.4218 & -0.0159 \\ 0.0006 & -0.0159 & 0.4249 \end{bmatrix}$$

The compliance in each direction is:

$$Compliance = [c_z \quad c_{\theta_x} \quad c_{\theta_y}] = [0.0010, 0.4218, 0.4249]$$

The sum of the compliance is: 0.001+0.4218+0.4249=0.8477.

4.4. Compliance Comparison

The compliance in each direction of the 3UPS-PU parallel manipulator under different stiffness models are listed in table 2.

Table 2. Comparisons between three stiffness models.

	Traditional stiffness model	Kinetostatic compliance model	Dexterity stiffness model
Compliance in Z direction	0.0003	0.0003	0.0010
Compliance in θ_x direction	0.1461	0.1428	0.4218
Compliance in θ_y direction	0.1472	0.1461	0.4249
Compliance sum	0.2936	0.2936	0.8477

One can see that the compliance in Z direction under the traditional stiffness model and kinetostatic compliance model are the same, which means the stiffness in Z direction under these two models are the same.

The difference of compliance in θ_x direction is: 0.1461-0.1428=0.0033;

The difference of compliance in θ_y direction is: 0.1472-0.1461=0.0011;

The sum of the compliance is the same 0.2936.

One can see that the compliance difference in θ_x and θ_y are very small, therefore the accuracy of the kinetostatic compliance model is verified. The difference between traditional stiffness model and dexterous stiffness model is as follows:

The difference of compliance in Z direction is: 0.001-0.0003=0.0007;
The difference of compliance in θ_x direction is: 0.4218-0.1461=0.2757;
The difference of compliance in θ_y direction is: 0.4249-0.1472=0.2777;
The difference of the sum of the compliance is: 0.8477-0.2936=0.5541.

There is difference between these two stiffness models, which is quite expectable and reasonable, and these can be seen from stiffness equations (9) and (13). From figures 2-5, one can see that the kinetostatic compliance model is the closest one to the traditional stiffness model, which can also verify the accuracy of the kinetostatic stiffness/compliance model.

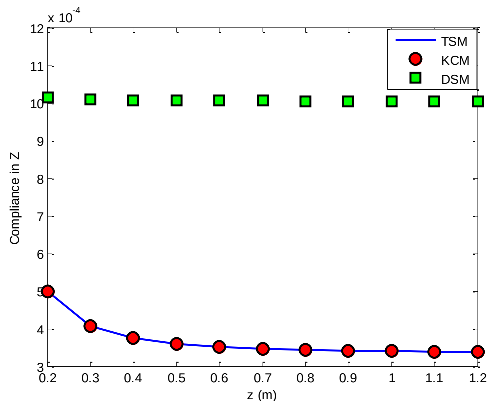

Fig. 2. Compliance in Z.

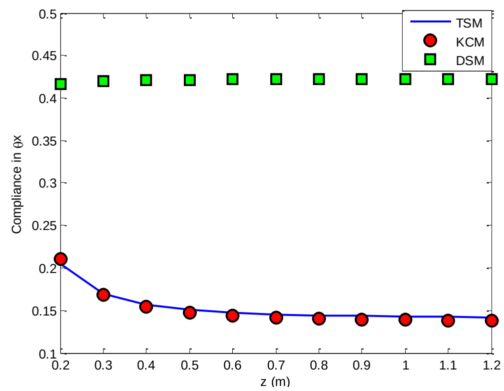

Fig. 3. Compliance in θ_x

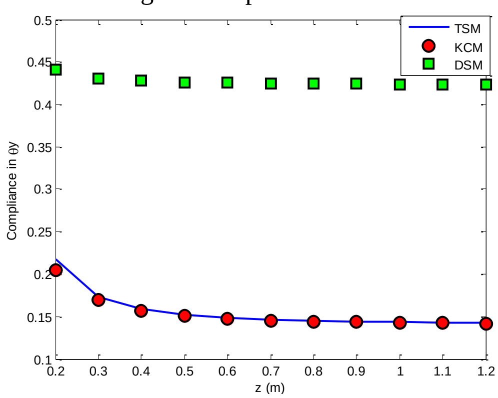

Fig. 4. Compliance in θ_y.

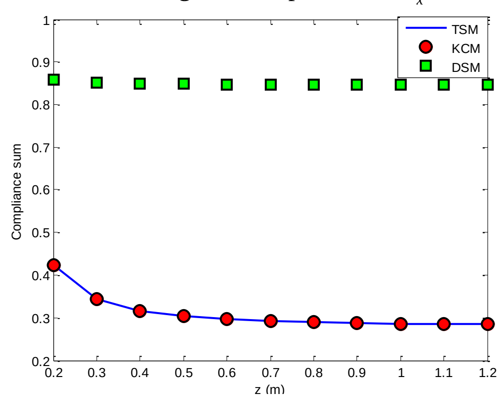

Fig. 5. Global Compliance.

In the traditional stiffness model, if the joint stiffness of each actuator is set to k=2000, then one has the following results as shown in figures 6-8. One can see that there is difference for compliance in Z direction, θ_x direction and θ_y direction between the TSM and KCM, which is quite expectable and reasonable, because the joint stiffness of each actuator (k=2000) does not correspond to the compliance of the actuator 0.001. Overall, from figures 2-8, it verifies the correspondence and accuracy of the kinetostatic compliance/stiffness model. For parallel manipulators that has 4-DOF with a 4-DOF central passive leg and four identical actuated legs and 5-DOF with a 5-DOF central passive leg and five identical actuated legs, the same results can be obtained.

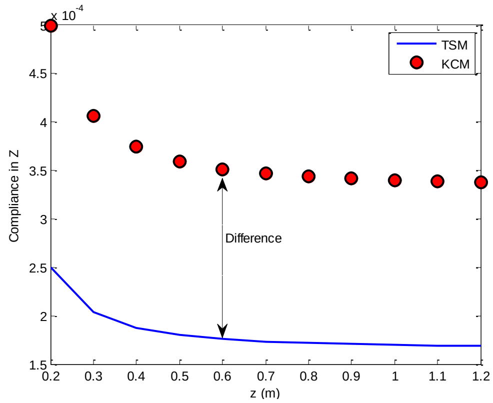

Fig. 6. Compliance in Z.

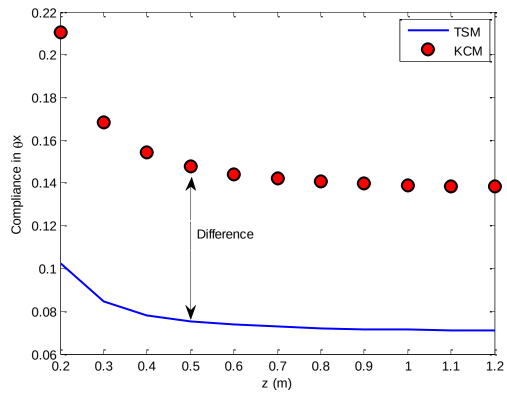

Fig. 7. Compliance in θ_x.

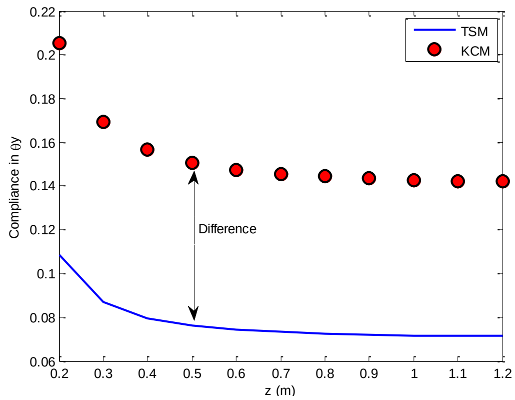

Fig. 8. Compliance in θ_y.

5. Conclusion

This paper studied the difference between three stiffness models for a family of parallel manipulators. The traditional stiffness matrix was first derived based on calculation of the Jacobian matrix. Secondly, one derived the compliance matrix of the 3UPS-PU mechanism by considering the central passive limb as rigid for the purpose of comparison based on the kinetostatic and the principle of virtual work; furthermore the dexterous stiffness model was proposed. Through comparison among compliance in each direction under different stiffness models, it was found that the compliance in each direction of different stiffness models corresponds to each other, and most importantly, the accuracy of the kinetostatic stiffness/compliance model was verified.

Acknowledgement

The authors would like to thank the financial support from the Natural Sciences and Engineering Research Council of Canada (NSERC). The corresponding author gratefully acknowledges the financial support from Canada Research Chairs program.

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