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Control Motion of A Human Arm: A Simulation Study

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Abstract - This article presents a simulation study to control a human arm motion using muscle excitations as inputs. Our simulation implements the musculoskeletal model Arm26 provided in OpenSim which has 2 DOF and 6 muscles as actuators. First, in order to drive the limbs' motion to track a desired trajectory, we propose an Adaptive Sliding Mode Controller (ASMC) to compute the necessary driving moments at each joint. Since the system is over actuated, the Generalized Reduced Gradient (GRG) method is implemented to optimally distribute forces to each muscle. Because the system has a cascade structure, another Sliding Mode Control (SMC) within a Back Stepping algorithm framework is used to drive the muscle excitation so that each muscle can produce the desired force.

Keywords: Sliding mode control, motion tracking, computed muscle control, FES, arm tremor.

1. Introduction

Functional electrical stimulation (FES), which uses small electrodes attached to the patient's skin to stimulate the muscles underneath, offers a potential solution to develop a soft, wearable rehabilitation device for parkinson patients. In fact, there are several on-going projects to develop a device of this kind. However, current research mainly use simple control strategies, such as open loop, PID, and fuzzy logic as in some recent publications (Gallego et al. (2011), Widjaja et al. (2008), Shariati et al. (2011)). These controllers are preferred because they do not require knowledge of the system model. However, their performance relies greatly on the tuning process, and they can not deal with large disturbances and great parametric variations. These drawbacks motivate the development of more robust control algorithms. The challenges in developing a feedback controller for tremor suppression include the complexity and highly non-linear properties of the human body. Moreover, many parameters of the model are not measurable and time variant. Therefore, this article proposes an adaptive controller, that uses the neural excitation to control the muscles in order to track a desired trajectory for the elbow and shoulder flexion. This controller can handle the mismatches between the mathematic model and physical body, and bounded disturbances. This will establish a foundation for future work, in which FES will be used to adjust the neural excitation in order to stabilize the arm tremors.

2. Body Dynamics and Adaptive Control

2.1. Dynamics Model

We limit to the task of controlling the joint flexion in the sagittal plane as illustrated in Fig. 1, where to the left is the right arm's skeleton model, and to the right is its free body diagram. Let m_1, P_1, I_1 and m_2, P_2, I_2 be the mass, the gravity force, and the inertial moment at the mass center points A and C of the lower arm and the upper arm, respectively. M_1 and M_2 are the elbow and shoulder moments. Let (α_1, α_2), $(\dot{\alpha}_1, \dot{\alpha}_2)$, and

$(\ddot{\alpha}_1, \ddot{\alpha}_2)$ be the elbow and shoulder flexion, and their angular rates and angular accelerations, respectively.

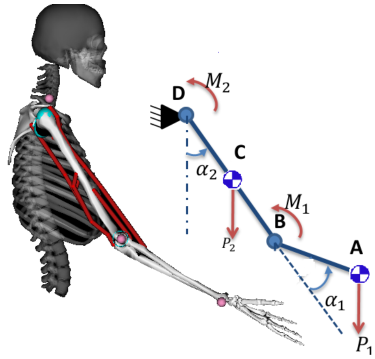

Fig. 1. Arm Model

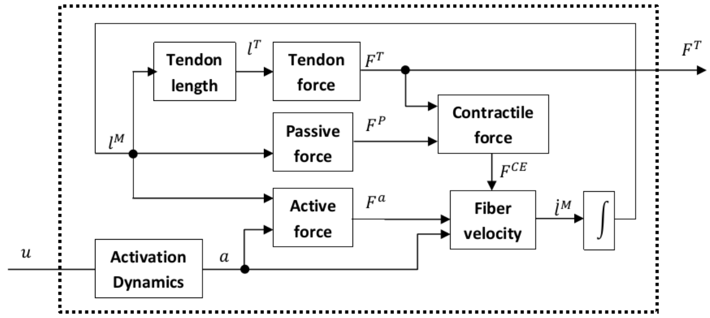

Fig. 2. Muscle dynamics model

Applying the Newton laws on the 2-link segments model depicted in Fig (1) yields the motion dynamics as

$$\begin{bmatrix} I_1 + m_1 l_{AB}^2 & m_1 l_{AB} l_{BD} \cos\alpha_1 + m_1 l_{AB}^2 \\ m_1 l_{AB} l_{BD} \cos\alpha_1 & I_2 + m_2 l_{CD}^2 + m_1 l_{BD}^2 + m_1 l_{AB} l_{BD} \cos\alpha_1 \end{bmatrix} \begin{bmatrix} \ddot{\alpha}_1 \\ \ddot{\alpha}_2 \end{bmatrix} = \begin{bmatrix} M_1 - (P_1 l_{AB} \sin(\alpha_1 + \alpha_2) + m_1 l_{AB} l_{BD} \sin\alpha_1 \dot{\alpha}_2^2) \\ M_2 - M_1 - (P_1 l_{BD} + P_2 l_{CD}) \sin\alpha_2 + m_1 l_{AB} l_{BD} \sin\alpha_1 (\dot{\alpha}_1 + \dot{\alpha}_2)^2 \end{bmatrix} \tag{1}$$

By introducing the following notation $z_1 \triangleq I_1 + m_1 l_{AB}^2$, $z_2(\alpha_1) \triangleq m_1 l_{AB} l_{BD} \cos\alpha_1 + m_1 l_{AB}^2$, $z_3(\alpha_1) \triangleq m_1 l_{AB} l_{BD} \cos\alpha_1$, $z_4(\alpha_1) \triangleq I_2 + m_2 l_{CD}^2 + m_1 l_{BD}^2 + m_1 l_{AB} l_{BD} \cos\alpha_1$, $Y_1(\alpha_1, \alpha_2, \dot{\alpha}_2) \triangleq P_1 l_{AB} \sin(\alpha_1 + \alpha_2) + m_1 l_{AB} l_{BD} \sin\alpha_1 \dot{\alpha}_2^2$, $Y_2(\alpha_1, \alpha_2, \dot{\alpha}_1, \dot{\alpha}_2) \triangleq (P_1 l_{BD} + P_2 l_{CD}) \sin\alpha_2 - m_1 l_{AB} l_{BD} \sin\alpha_1 (\dot{\alpha}_1 + \dot{\alpha}_2)^2$, Equ. (1) is rewritten as

$$\begin{bmatrix} \ddot{\alpha}_1 \\ \ddot{\alpha}_2 \end{bmatrix} = \frac{1}{z_1 z_4 - z_2 z_3} \begin{bmatrix} -z_4 Y_1 + z_2 Y_2 \\ z_3 Y_1 - z_1 Y_2 \end{bmatrix} + \frac{1}{z_1 z_4 - z_2 z_3} \begin{bmatrix} z_2 + z_4 & -z_2 \\ -(z_1 + z_3) & z_1 \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} \tag{2}$$

with the reduced notations $z_i \triangleq z_i(\alpha_1)$ and $Y_i \triangleq Y_i(\alpha_1, \alpha_2, \dot{\alpha}_1, \dot{\alpha}_2)$.

2.2. Adaptive Sliding Mode Controller (ASMC)

The dynamic model in Equ. (2) belongs to a general form :

$$\ddot{x}(t) = f(x(t), \dot{x}(t)) + g(x(t), \dot{x}(t)) u(t) + d(t), \tag{3}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^n$ is the control input, $f(x(t), \dot{x}(t))$, $g(x(t), \dot{x}(t))$ are function of $x(t)$ and $\dot{x}(t)$, and $d(t)$ is the unknown disturbance. The objective is to control the system to follow a desired trajectory $x_\mathrm{d}(t)$. We will use short notation i.e $x \triangleq x(t)$ and $f \triangleq f(x(t), \dot{x}(t))$ if it causes no confusion. Due to the errors between the mathematic model and the physical one, the plant dynamic can be rewritten as

$$\ddot{x} = f_\mathrm{n} + g_\mathrm{n} u + (\Delta f + \Delta g u + d) = f_\mathrm{n} + g_\mathrm{n} u + \overline{d}, \tag{4}$$

where $f_\mathrm{n}, g_\mathrm{n}$ describe the nominal system with known estimated parameters, $\Delta f, \Delta g$ are the unknown difference between the real and nominal systems and $\overline{d} \triangleq (\Delta f + \Delta g u + d)$ is the total uncertainty error and disturbance. Define the sliding mode variable $s \triangleq \dot{e} + Ce \in \mathbb{R}^n$, where $e \triangleq x - x_\mathrm{d}$ is the error, and $C \in \mathbb{R}^{n \times n}$ is a positive definite diagonal matrix. Differentiating the sliding mode variable yields

$$\dot{s} = \ddot{e} + C\dot{e} = f_\mathrm{n} + g_\mathrm{n} u + \overline{d} - \ddot{x}_\mathrm{d} + C\dot{e}. \tag{5}$$

The conventional SMC is

$$u = -g_{\mathrm{n}}^{-1}(f_{\mathrm{n}} - \ddot{x}_{\mathrm{d}} + c\dot{e} + sign(s)k) \tag{6}$$

where $sign(s) \triangleq diag([sign(s_1)...sign(s_n)])$ is the diagonal matrix and $k \triangleq [k_1...k_n]^{\mathrm{T}}$ with $k_i \geq |\overline{d}_{\max}|$, where $|\overline{d}_{\max}|$ is the maximum absolute value of $\overline{d}(t)$ (Huang et al. (2008)). Differentiating the Lyapunov function $V = \frac{1}{2}s^{\mathrm{T}}s$ along the trajectory (5) and (6) yields

$$\dot{V} = s^{\mathrm{T}}\dot{s} = s^{\mathrm{T}}(f_{\mathrm{n}} + g_{\mathrm{n}}u + \overline{d} - \ddot{x}_{\mathrm{d}} + c\dot{e}) = -|s|^{\mathrm{T}}k + s^{\mathrm{T}}\overline{d} \leq -|s|^{\mathrm{T}}k + |s|^{\mathrm{T}}|\overline{d}_{\max}| \leq 0. \tag{7}$$

which guarantees $s \to 0$. However, note that $\overline{d}$ depends on Δgu, which leads to the fact that $|\overline{d}_{\max}|$ can be magnified when u is large, and choosing a large constant $|\overline{d}_{\max}|$ can worsen the controller performance. Therefore, we modify the control law given above. Assume

$$\exists k^* \in R^n : k_i^* > |d_i + \Delta f_i(t)|_{\max}, \quad \exists K^* \in R^{n\times n} : K_{ij}^* > |\Delta g_{ij}(t)|_{\max} \quad \text{for} \quad i,j = 1,...,n. \tag{8}$$

and define the controller

$$u \triangleq u_{s1} + u_{s2}, \quad u_{s1} = -g_{\mathrm{n}}^{-1}(f_{\mathrm{n}} - \ddot{x}_{\mathrm{d}} + c\dot{e} + sign(s)k(t)), \quad u_{s2} = -g_{\mathrm{n}}^{-1}sign(s)K(t)|u|, \tag{9}$$

with the update law

$$\dot{k}_i = \mathrm{Proj}(k_i, \frac{1}{\alpha}|s_i|), \quad \dot{K}_{ij} = \mathrm{Proj}(K_{ij}, \frac{1}{\beta}|s_i u_j|), \tag{10}$$

where $\mathrm{Proj}(\theta, y)$ is the projection operator (Khalil (2002)). The effectiveness of the proposed controller is demonstrated by analyzing the Lyapunov function

$$V_{\mathrm{a}} = \frac{1}{2}s^{\mathrm{T}}s + \frac{1}{2}\alpha\sum_{i=1}^{n}(k_i - k_i^*)^2 + \frac{1}{2}\beta\sum_{i=1}^{n}\sum_{j=1}^{n}(K_{ij} - K_{ij}^*)^2. \tag{11}$$

Substituting the controller defined in Equ. (9) into Equ. (5), yields

$$\begin{aligned}\dot{s} &= -sign(s)k - sign(s)K|u| + (d + \Delta f + \Delta gu)\\ &= \begin{bmatrix} -sign(s_1)k_1 \\ \vdots \\ -sign(s_n)k_n \end{bmatrix} + \begin{bmatrix} -sign(s_1)(K_{11}|u_1| + \ldots + K_{1n}|u_n|) \\ \vdots \\ -sign(s_n)(K_{n1}|u_1| + \ldots + K_{nn}|u_n|) \end{bmatrix} + \begin{bmatrix} d_1 + \Delta f_1 \\ \vdots \\ d_n + \Delta f_n \end{bmatrix} + \begin{bmatrix} \Delta g_{11}u_1 + \ldots + \Delta g_{1n}u_n \\ \vdots \\ \Delta g_{n1}u_1 + \ldots + \Delta g_{nn}u_n \end{bmatrix}.\end{aligned} \tag{12}$$

Differentiating the Lyapunov function defined in Equ (11) along the trajectory (12) yields

$$\begin{aligned}\dot{V}_{\mathrm{a}} &= s^{\mathrm{T}}\dot{s} + \alpha\sum_{i=1}^{n}(k_i - k_i^*)\dot{k}_i + \beta\sum_{i=1}^{n}\sum_{j=1}^{n}(K_{ij} - K_{ij}^*)\dot{K}_{ij}\\ &= \sum_{i=1}^{n}(d_i + \Delta f_i)s_i - \sum_{i=1}^{n}\sum_{j=1}^{n}K_{ij}|s_i u_j| + \sum_{i=1}^{n}\sum_{j=1}^{n}\Delta g_{ij}s_i u_j - \sum_{i=1}^{n}k_i|s_i| + \sum_{i=1}^{n}(k_i - k_i^*)|s_i| + \sum_{i=1}^{n}\sum_{j=1}^{n}(K_{ij} - K_{ij}^*)|s_i u_j|\\ &= \sum_{i=1}^{n}(d_i + \Delta f_i)s_i - \sum_{i=1}^{n}k_i^*|s_i| + \sum_{i=1}^{n}\sum_{j=1}^{n}\Delta g_{ij}s_i u_j - \sum_{i=1}^{n}\sum_{j=1}^{n}K_{ij}^*|s_i u_j| \leq 0.\end{aligned} \tag{13}$$

due to the definition of k_i^*, K_{ij}^* in (8). Therefore, according to the Lasalle-Yoshizawa theorem, all signals remain bounded and the sliding variable s converges to zero. From Equ. (9), the controller is rewritten as

$$u = -(g_n + sign(s)K(t)sign(u))^{-1}(f_n - \ddot{x}_{\mathrm{d}} + c\dot{e} + sign(s)k(t)). \tag{14}$$

Comparing Equ.(14) to the conventional SMC, the proposed ASMC adds the gain K, to account for the uncertainty in estimating system control gain (Δgu) which can be magnified when the control u is large. Note that, when $u^{\mathrm{T}}s$ converges to zero, then the ASMC controller converges to the conventional SMC controller. Furthermore, K provides an easy way to handle the case of a singular g_{n}, which would cause the standard SMC to become unbounded. Also note that in Equ. (14), u depends on $sign(u)$. Therefore, in practical implementation, the previous value of u can be used to predict $sign(u)$, and compute new u. Finally, the $sign(s)$ can be replaced by the saturate function $sat(s, \varepsilon > 0)$ to avoid the chattering problem of SMC (Khalil (2002)).

2.3. Simulation Results

Table 1. Arm model simulation parameters

Model	m_1	I_1	l_{AB}	m_2	I_2	l_{BD}	l_{CD}
Physical	1.53	0.02	0.14	1.87	0.013	0.29	0.18
Nominal	1.8	0.022	0.16	2	0.015	0.31	0.20

Table 2. Muscle model parameters

$\varepsilon_0^{\mathrm{M}}$	k_{toe}	$\varepsilon_0^{\mathrm{T}}$	k_{lin}	$\varepsilon_{\mathrm{toe}}^{\mathrm{T}}$
0.6	3	0.033	$1.712/\varepsilon_0^{\mathrm{T}}$	$10.609\varepsilon_0^{\mathrm{T}}$
$\overline{F}_{\mathrm{toe}}^{\mathrm{T}}$	k_{P}	γ	A_f	$\overline{F}_{\mathrm{len}}^{\mathrm{M}}$
1/3	4	0.5	0.3	1.8

The physical parameters to simulate the arm model and the nominal parameters for the controller are given in Tab. 1. The controller parameters are chosen as $C = diag([7 \;\; 7])$, $\alpha = 0.01$, $\beta = 0.01$, $\varepsilon = 1°$. The system response and control inputs are shown in Fig. (3) and (4). The Fig.(3) shows that the system output converges to the desired output after $1.5s$ for the chosen control parameters.

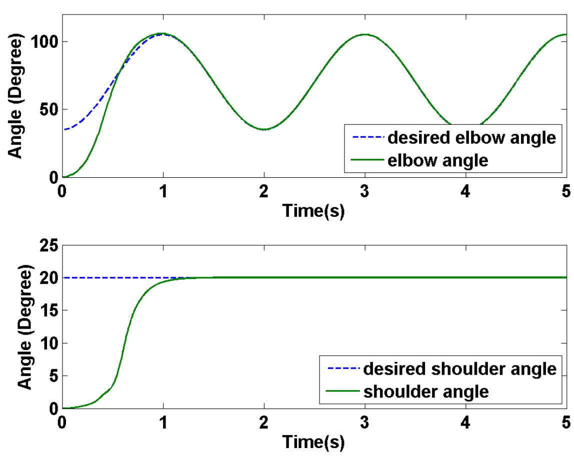

Fig. 3. Tracking performance

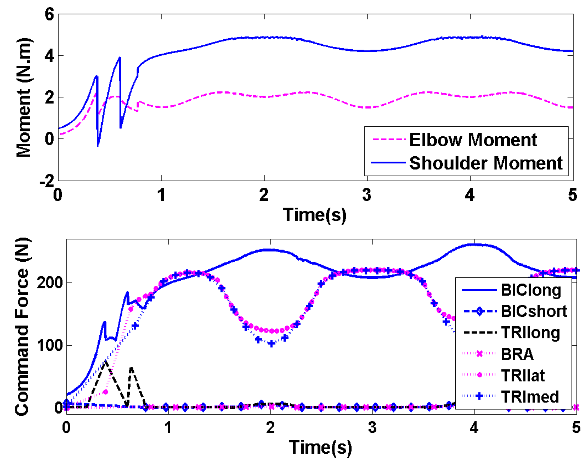

Fig. 4. Control moments and optimal force distribution

3. Optimal Force Distribution

In the Arm26 body model, 6 muscles *BIClong, BICshort, TRIlong, BRA, TRIlat, TRImed* (with respective index $i = 1, \ldots, 6$) create the moment M_1, and 3 of them (*BIClong, BICshort, TRIlong*) create the moment M_2 as well. Therefore, the criterion that the muscles contributes the least force to produce the motion is selected. The problem is formulated as minimizing the following cost function that satisfies the constraints

$$f(x) = \frac{1}{2}\sum_{i=1}^{6} F_i^2 + \frac{1}{2}\sum_{i=1}^{6}(F_i - F_i^{\mathrm{p}})^2, \;\text{s.t}\; \sum_{i=1}^{6} r_i^1 F_i = M_1, \quad \sum_{i=1}^{3} r_i^2 F_i = M_2, \;\; F_{i\min} \le F_i \le F_{i\max}, \;\; i = 1, \ldots, 6, \quad (15)$$

where F_i and F_i^{p} are the forces produced by muscle i in the current and in the previous step, r_i^1 and r_i^2 are the moment arms of the muscle i at the elbow and the shoulder respectively. Let $x \triangleq [F_1, ..., F_6]^{\mathrm{T}} \in \mathbb{R}^n$ is the state vector and m be the number of the equality constraints. The vector x can be partitioned as $x = [x_{\mathrm{B}}^{\mathrm{T}}, x_{\mathrm{N}}^{\mathrm{T}}]^{\mathrm{T}}$ where $x_{\mathrm{B}} \in \mathbb{R}^m$ is the basic vector and $x_{\mathrm{N}} \in \mathbb{R}^{n-m}$ is the nonbasic vector. The constraints in (15) can be rewritten as

$$\begin{bmatrix} r_1^1 & r_2^1 & r_3^1 & r_4^1 & r_5^1 & r_6^1 \\ r_1^2 & r_2^2 & r_3^2 & 0 & 0 & 0 \end{bmatrix} x = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} \Leftrightarrow Ax = b \Leftrightarrow [B|N] \begin{bmatrix} x_{\mathrm{B}} \\ x_{\mathrm{N}} \end{bmatrix} = b \Leftrightarrow Bx_{\mathrm{B}} + Nx_{\mathrm{N}} = b, \quad (16)$$

with obvious meaning of A and b, and B and N are partitions of matrix A corresponding to the vectors x_{B} and x_{N}. The basic vector x_{B} is chosen such that all its elements are not at the boundary $F_{i\min} < x_{\mathrm{B}i} < F_{i\max}$ and matrix B is nonsingular. Let x change by a small variation $\delta = [\delta_{\mathrm{B}}^{\mathrm{T}}, \delta_{\mathrm{N}}^{\mathrm{T}}]^{\mathrm{T}}$ and still satisfy the constraint (16)

$$B(x_{\mathrm{B}}+\delta_{\mathrm{B}})+N(x_{\mathrm{N}}+\delta_{\mathrm{N}})=b \Leftrightarrow \delta_{\mathrm{B}}=-B^{-1}N\delta_{\mathrm{N}}. \tag{17}$$

Let $\nabla f \triangleq [\nabla_{\mathrm{B}} f^{\mathrm{T}}, \nabla_{\mathrm{N}} f^{\mathrm{T}}]^{\mathrm{T}}$ be the gradient vector of function $f(x)$ with two components $\nabla_{\mathrm{B}} f^{\mathrm{T}}, \nabla_{\mathrm{N}} f^{\mathrm{T}}$ corresponding to the vector x_{B} and x_{N}. It follows from Equ.(17) that the change of f caused by the variation δ is

$$\Delta f=\nabla f^{\mathrm{T}}\delta=\nabla_{\mathrm{B}} f^{\mathrm{T}}\delta_{\mathrm{B}}+\nabla_{\mathrm{N}} f^{\mathrm{T}}\delta_{\mathrm{N}}=(\nabla_{\mathrm{N}} f^{\mathrm{T}}-\nabla_{\mathrm{B}} f^{\mathrm{T}}B^{-1}N)\delta_{\mathrm{N}}=\gamma_{\mathrm{N}}^{\mathrm{T}}\delta_{\mathrm{N}}, \tag{18}$$

where $\gamma_{\mathrm{N}}^{\mathrm{T}} \triangleq \nabla_{\mathrm{N}} f^{\mathrm{T}}-\nabla_{\mathrm{B}} f^{\mathrm{T}}B^{-1}N$. f is minimized if δ is chosen so that $\Delta f<0$ as $\delta=\alpha[\Gamma_{\mathrm{B}}^{\mathrm{T}}\ \ \Gamma_{\mathrm{N}}^{\mathrm{T}}]^{\mathrm{T}}$, where

$$\Gamma_{\mathrm{B}}=-B^{-1}N\Gamma_{\mathrm{N}}, \qquad \Gamma_{\mathrm{N}i}=\begin{cases}0 & \text{if } (x_{\mathrm{N}i}=F_{i\min},\ \gamma_{\mathrm{N}i}>0) \text{ or } (x_{\mathrm{N}i}=F_{i\max},\ \gamma_{\mathrm{N}i}<0),\\ -\gamma_{\mathrm{N}i} & \text{otherwise,}\end{cases} \tag{19}$$

Once $[\Gamma_{\mathrm{B}}^{\mathrm{T}}\ \ \Gamma_{\mathrm{N}}^{\mathrm{T}}]^{\mathrm{T}}$ is determined, the optimal α can be found as $\alpha=\mathit{argmin}\ f(x+\alpha\Gamma),\quad 0\le\alpha\le\alpha_{\max}$, where $\alpha_{\max}=\mathit{sup}\{\alpha|F_{i\min}\le x_i\le F_{i\max}\},\quad i=1,\ldots,6$. Suppose the optimum step size is α^*, the iteration will repeat with $x+\alpha^*\Gamma\mapsto x$ until $||\Gamma_{\mathrm{N}}||=0$ or $||x^{(k+1)}-x^{(k)}||<\varepsilon$, where ε is the chosen tolerance.

Fig. (4) illustrates the simulation results for the required moments simulated in the section (2). The algorithm converges in less than 5 iterations on average.

4. Control Muscle

4.1. Muscle Dynamics Model

This section summarizes the Thelen muscle model (Thelen et al. (2003)), whose block diagram is shown in Fig. 2. All muscles share the same block structures and formulas, but differ in four properties: the maximum isometric force F_0^{M}, the optimal fiber length l_0^{M}, the tendon slack length $l_{\mathrm{s}}^{\mathrm{T}}$, the pennation angle at the optimal fiber length α_0. Table 2 shows the same parameters across all muscles.

Tendon length The relationship between the tendon length $l^{\mathrm{T}}(t)$ and the muscle length l^{M} is given by $l^{\mathrm{T}}=l^{\mathrm{MT}}-l^{\mathrm{M}}\cos\alpha$, where $l^{\mathrm{MT}}\triangleq l^{\mathrm{MT}}(\alpha_1,\alpha_2)$ is the muscle tendon length and α is the pennation angle

$$\alpha=\begin{cases}0, & l^{\mathrm{M}}(t)=0 \quad\text{or}\quad w/l^{\mathrm{M}}(t)\le 0,\\ \sin^{-1}(w/l^{\mathrm{M}}(t)), & 0<w/l^{\mathrm{M}}(t)<1,\\ \pi/2, & w/l^{\mathrm{M}}(t)\ge 1,\end{cases} \qquad \text{with} \qquad w=l_0^{\mathrm{M}}\sin\alpha_0. \tag{20}$$

Tendon Force The tendon force $F^{\mathrm{T}}(t)$ is governed by the tendon length l^{T} as $F^{\mathrm{T}}(t)=F_0^{\mathrm{M}}\overline{F}^{\mathrm{T}}(\varepsilon^{\mathrm{T}})$, where

$$\overline{F}^{\mathrm{T}}(\varepsilon^{\mathrm{T}})=\frac{1+\varepsilon^{\mathrm{T}}}{1000}+\begin{cases}k_{\mathrm{lin}}(\varepsilon^{\mathrm{T}}-\varepsilon_{\mathrm{toe}}^{\mathrm{T}})+\overline{F}_{\mathrm{toe}}^{\mathrm{T}}, & \varepsilon^{\mathrm{T}}>\varepsilon_{\mathrm{toe}}^{\mathrm{T}},\\ \overline{F}_{\mathrm{toe}}^{\mathrm{T}}\dfrac{e^{k_{\mathrm{toe}}\varepsilon^{\mathrm{T}}/\varepsilon_{\mathrm{toe}}^{\mathrm{T}}}-1}{e^{k_{\mathrm{toe}}}-1}, & 0<\varepsilon^{\mathrm{T}}\le\varepsilon_{\mathrm{toe}}^{\mathrm{T}},\\ 0, & \varepsilon^{\mathrm{T}}\le 0,\end{cases} \qquad \text{with} \qquad \varepsilon^{\mathrm{T}}=l^{\mathrm{T}}/l_{\mathrm{s}}^{\mathrm{T}}-1. \tag{21}$$

Active Force The active force F_{a} is controlled by the muscle activation a and the normalized muscle length $\bar{l}^{\mathrm{M}}$

$$F_{\mathrm{a}}=aF_0^{\mathrm{M}}e^{-(\bar{l}^{\mathrm{M}}-1)^2}. \tag{22}$$

Passive Force The passive force F_{P} is controlled by the muscle length l^{M} as $F_{\mathrm{P}}=F_o^m\overline{F}_{\mathrm{P}}(\bar{l}^{\mathrm{M}})$, where $\bar{l}^{\mathrm{M}}=l^{\mathrm{M}}/l_0^{\mathrm{M}}$ is the normalized muscle length, and

$$\overline{F}_{\mathrm{P}}(\bar{l}^{\mathrm{M}})=\begin{cases}1+\frac{k_{\mathrm{P}}}{\varepsilon_0^{\mathrm{M}}}(\bar{l}^{\mathrm{M}}-(1+\varepsilon_0^{\mathrm{M}})), & \text{if}\quad \bar{l}^{\mathrm{M}}>(1+\varepsilon_0^{\mathrm{M}}),\\ \dfrac{e^{k_{\mathrm{P}}(\bar{l}^{\mathrm{M}}-1)/\varepsilon_0^{\mathrm{M}}}}{e^{k_{\mathrm{P}}}}, & \text{if}\quad \bar{l}^{\mathrm{M}}\le(1+\varepsilon_0^{\mathrm{M}}).\end{cases} \tag{23}$$

Contractile force The force in the contractile element F_{CE} is calculated from tendon force F_T and passive force F_P as follows

$$F_{\mathrm{CE}}=\frac{F^{\mathrm{T}}}{\cos\alpha}-F_{\mathrm{P}}. \tag{24}$$

Fiber Velocity The fiber velocity $\dot{l}^{\mathrm{M}}$ is calculated as $\dot{l}^{\mathrm{M}} = (5+5a)l_0^{\mathrm{M}}\bar{\dot{l}}^{\mathrm{M}}$, where $\bar{\dot{l}}^{\mathrm{M}}$ is the normalized contraction velocity

$$\bar{\dot{l}}^{\mathrm{M}} = \Psi_1(l^{\mathrm{M}},a) = \begin{cases} \dfrac{F_{\mathrm{CE}}}{\varepsilon}\left(\dfrac{\varepsilon - F_{\mathrm{a}}}{F_{\mathrm{a}} + \frac{\varepsilon}{A_f} + \xi} + \dfrac{F_{\mathrm{a}}}{F_{\mathrm{a}} + \xi}\right) - \dfrac{F_{\mathrm{a}}}{F_{\mathrm{a}} + \xi}, & \text{if} \quad F_{\mathrm{CE}} < 0, \\ \dfrac{F_{\mathrm{CE}} - F_{\mathrm{a}}}{F_{\mathrm{a}} + \frac{F_{\mathrm{CE}}}{A_f} + \xi}, & \text{if} \quad 0 \le F_{\mathrm{CE}} < F_{\mathrm{a}}, \\ \dfrac{F_{\mathrm{CE}} - F_{\mathrm{a}}}{\dfrac{1}{\overline{F}^{\mathrm{M}}_{\mathrm{len}} - 1}(2 + \dfrac{2}{A_f})(F_{\mathrm{a}}\overline{F}^{\mathrm{M}}_{\mathrm{len}} - F_{\mathrm{CE}}) + \xi}, & \text{if} \quad F_{\mathrm{a}} \le F_{\mathrm{CE}} < 0.95F_{\mathrm{a}}\overline{F}^{\mathrm{M}}_{\mathrm{len}}, \\ f_{v0} + \dfrac{F_{\mathrm{CE}} - 0.95F_{\mathrm{a}}\overline{F}^{\mathrm{M}}_{\mathrm{len}}}{F_{\mathrm{a}}\overline{F}^{\mathrm{M}}_{\mathrm{len}}}(f_{v1} - f_{v0}), & \text{if} \quad 0.95F_{\mathrm{a}}\overline{F}^{\mathrm{M}}_{\mathrm{len}} \le F_{\mathrm{CE}}, \end{cases} \tag{25}$$

where

$$f_{v0} = \frac{0.95F_{\mathrm{a}}\overline{F}^{\mathrm{M}}_{\mathrm{len}} - F_{\mathrm{a}}}{\dfrac{1}{\overline{F}^{\mathrm{M}}_{\mathrm{len}} - 1}(2 + \dfrac{2}{A_f})0.05(F_{\mathrm{a}}\overline{F}^{\mathrm{M}}_{\mathrm{len}}) + \xi}, \qquad f_{v1} = \frac{(0.95+\varepsilon)F_{\mathrm{a}}\overline{F}^{\mathrm{M}}_{\mathrm{len}} - F_{\mathrm{a}}}{\dfrac{1}{\overline{F}^{\mathrm{M}}_{\mathrm{len}} - 1}(2 + \dfrac{2}{A_f})(0.05-\varepsilon)(F_{\mathrm{a}}\overline{F}^{\mathrm{M}}_{\mathrm{len}}) + \xi}. \tag{26}$$

Activation dynamics The activation dynamics a is modeled as the first order lowpass filter of the excitation signal u, with $T_{\mathrm{act}} = 0.01$ and $T_{\mathrm{dact}} = 0.04$ are the activate time and the deactivate time

$$\dot{a} = \Psi_2(a,u) = \begin{cases} (u-a)/T_{\mathrm{act}} & \text{if} \quad u > a, \\ (u-a)/T_{\mathrm{dact}} & \text{if} \quad u \le a. \end{cases} \tag{27}$$

4.2. Muscle Control

The entire system dynamics has a cascade form and can be summarized as follows

$$\ddot{x}(t) = f(x(t),\dot{x}(t)) + g(x(t),\dot{x}(t))u(t), \tag{28a}$$

$$\dot{l}_i^{\mathrm{M}} = \Psi_1(l^{\mathrm{M}}{}_i, a_i), \tag{28b}$$

$$\dot{a}_i = \Psi_2(a_i,u_i), \qquad i = 1,\dots,6 \tag{28c}$$

where Equ. (28a) is the arm dynamics defined in Section (2) and (3) with $x = [\alpha_1(t) \;\; \alpha_2(t)]^{\mathrm{T}}$ are the joint angle, $u(t) = [\sum_{i=1}^{6} r_i^1 F_i(l_i^{\mathrm{M}}) \;\; \sum_{i=1}^{6} r_i^2 F_i(l_i^{\mathrm{M}})]^{\mathrm{T}}$ is the control moments. The back-stepping algorithm can be applied to control the system (Khalil (2002)). First, suppose that the set of desired forces $\{F_{\mathrm{d}i} = F_i(l_{\mathrm{d}i}^{\mathrm{M}}), \quad i = 1,\dots,6\}$ is available after solving the optimal force distribution step as described in section (2) and (3), and also suppose that their time derivative $\dot{F}_{\mathrm{d}}$ is available. Then, the desired muscle length $l_{\mathrm{d}}^{\mathrm{M}}$ and its time derivative $\dot{l}_{\mathrm{d}}^{\mathrm{M}}$ can be computed using the Equ. (20) and (21) (we skip presenting due to the length constraint).

Back-Stepping Level 1 Let $u_{\mathrm{d}} = \left[\; \sum_{i=1}^{6} r_i^1 F_{\mathrm{d}i} \quad \sum_{i=1}^{6} r_i^2 F_{\mathrm{d}i} \;\right]^{\mathrm{T}}$ be the desired joint moment that satisfy

$$V_1 = \frac{1}{2}s^{\mathrm{T}}s \Rightarrow \dot{V}_1 = s^{\mathrm{T}}(f_{\mathrm{n}} + g_{\mathrm{n}}u_{\mathrm{d}} + \bar{d} - \ddot{x}_{\mathrm{d}} + c\dot{e}) \le 0. \tag{29}$$

Back-Stepping Level 2 Let $e_{l^{\mathrm{M}}i} \triangleq l_i^{\mathrm{M}} - l_{\mathrm{d}i}^{\mathrm{M}}$ be the error between the actual muscle length l_i^{M} and the desired muscle length $l_{\mathrm{d}i}^{\mathrm{M}}$. The control moment u can be rewritten in term of the desired u_d and the error $e_{l^{\mathrm{M}}i}$ as

$$u = u_{\mathrm{d}} + \begin{bmatrix} \sum_{i=1}^{6} r_i^1 \nabla F_i(l_{\mathrm{d}i}^{\mathrm{M}}) e_{l^{\mathrm{M}}i} \\ \sum_{i=1}^{6} r_i^2 \nabla F_i(l_{\mathrm{d}i}^{\mathrm{M}}) e_{l^{\mathrm{M}}i} \end{bmatrix}, \quad \text{where} \quad \nabla F_i(l_{\mathrm{d}}^{\mathrm{M}}) \triangleq \frac{F_i(l_i^{\mathrm{M}}) - F_i(l_{\mathrm{d}i}^{\mathrm{M}})}{e_{l^{\mathrm{M}}i}}. \tag{30}$$

Differentiating the Lyapunov function $V_2 = \frac{1}{2}s^{\mathrm{T}}s + \frac{1}{2}\sum_{i=1}^{6} {e_{l_i^{\mathrm{M}}}}^2$ along the trajectories (29) and (30) yields

$$\dot{V}_2 = s^{\mathrm{T}}(f_{\mathrm{n}} + g_{\mathrm{n}}u + \bar{d} - \ddot{x}_{\mathrm{d}} + c\dot{e}) + \sum_{i=1}^{6} e_{l_i^{\mathrm{M}}}(\Psi_1(a_i) - \dot{l}_{\mathrm{d}}^{\mathrm{M}}) = \dot{V}_1 + \sum_{i=1}^{6} e_{l_i^{\mathrm{M}}}(\Psi_1(a_i) - \dot{l}_{\mathrm{d}}^{\mathrm{M}} + s^{\mathrm{T}} g_{\mathrm{n}} r_i \nabla F_i(l_{\mathrm{d}i}^{\mathrm{M}})), \quad (31)$$

where $r_i \triangleq [r_i^1 \ \ r_i^2]^{\mathrm{T}}$. Therefore, the control activation $a_{\mathrm{d}i}$ is chosen such that $\dot{V}_2 \leq 0$, as

$$a_{\mathrm{d}i} = \Psi_1^{-1}(-k_{a_i} sign(e_{l_i^{\mathrm{M}}}) + \dot{l}_{\mathrm{d}i}^{\mathrm{M}} - s^{\mathrm{T}} g_{\mathrm{n}} r_i \nabla F_i(l_{\mathrm{d}i}^{\mathrm{M}})) \quad (32)$$

where k_{a_i} is the chosen control gain. Equ. (32) can be approximated by a quadratic equation in term of $a_{\mathrm{d}i}$ and solved analytically or by using the NewtonRaphson method.

Back-Stepping Level 3 Suppose $a_{\mathrm{d}i}$ is available after solving the Equ. (32), let $e_{\mathrm{a}i} \triangleq a_i - a_{\mathrm{d}i}$ be the error between the actual activation a_i and the desired activation $a_{\mathrm{d}i}$. The muscle contraction velocity are rewritten

$$\Psi_1(a_i) = \Psi_1(a_{\mathrm{d}i}) + \nabla\Psi_1(a_{\mathrm{d}i})e_{\mathrm{a}i}, \quad \text{where} \quad \nabla\Psi_1(a_{\mathrm{d}i}) = \frac{\Psi_1(a_i) - \Psi_1(a_{\mathrm{d}i})}{e_{\mathrm{a}i}}. \quad (33)$$

Differentiating the Lyapunov function $V_3 = \frac{1}{2}s^{\mathrm{T}}s + \frac{1}{2}\sum_{i=1}^{6} e_{l_i^{\mathrm{M}}}^2 + \frac{1}{2}\sum_{i=1}^{6} e_{a_i}^2$ along the trajectory (31),(33) yields

$$\dot{V}_3 = \dot{V}_1 + \sum_{i=1}^{6} e_{l_i^{\mathrm{M}}}(\Psi_1(a_i) - \dot{l}_{\mathrm{d}i}^{\mathrm{M}} + s^{\mathrm{T}} g_{\mathrm{n}} r_i \nabla F_i(l_{\mathrm{d}i}^{\mathrm{M}})) + \sum_{i=1}^{6} e_{ai}(\Psi_2(u_i) - \dot{a}_{\mathrm{d_i}}) = \dot{V}_2 + \sum_{i=1}^{6} e_{ai}(\Psi_2(u_i) - \dot{a}_{\mathrm{d}i} + e_{l^{\mathrm{M}}i}\nabla\Psi_1(a_{\mathrm{d}i})) \quad (34)$$

Define $z_i \triangleq -k_{u_i} sign(e_{a_i}) + \dot{a}_{\mathrm{d_i}} - e_{l_i^{\mathrm{M}}}\nabla\Psi_1(a_{\mathrm{d}i})$, the excitation control signal u_i is chosen such that $\dot{V}_3 \leq 0$, as

$$u_i = \Psi_2^{-1}(-k_{u_i} sign(e_{a_i}) + \dot{a}_{\mathrm{d_i}} - e_{l_i^{\mathrm{M}}}\nabla\Psi_1(a_{\mathrm{d}i})) = \begin{cases} T_{\mathrm{act}}(a_i + z_i) & \text{if} \quad z_i > 0, \\ T_{\mathrm{dact}}(a_i + z_i) & \text{if} \quad z_i \leq 0, \end{cases} \quad (35)$$

5. Simulation Results

We ran the experiment using the interface between OpenSim and Simulink (Mansouri & Reinbolt (2012)). The simulation parameters are given in Tab.1 and Tab.2, with initial states $l^{\mathrm{M}}(0) = [0.1138\ 0.1138\ 0.0858\ 0.134\ 0.1321\ 0.1157]$ and $a_i(0) = 0,\ i = 1, \ldots, 6,\ \alpha_j(0) = \dot{\alpha}_i(0) = \ddot{\alpha}_i(0) = 0,\ j = 1, 2$. Fig. 5 shows that the responses converge to the reference trajectories after 2s with the chosen parameters $C = [3 \ \ 11],\ \alpha = 0.5,\ \beta = 0.02$. Fig. 7 shows the required moments and muscle forces, respectively. Fig. 8 shows the required muscle length and muscle excitation, respectively. The tracking error depends on the chosen boundary of saturate function. In this example, the error boundary is 1°, and the maximum error reported is 1.5° at the shoulder.This is because the muscles BIClong, BICshort, TRIlong contributes to both the shoulder and elbow moments. This leads to the slight vibration at the shoulder angle to achieve the small error at elbow angle. Moreover, the gain can be adjusted to reduce the error. Figure 6 shows the system response when the control parameters are chosen as $C = diag([4 \ \ 40]),\ \alpha = 0.5,\ \beta = 0.02$ and the saturate boundary is 0.1°. The maximum settling error is 0.9° at the shoulder and 1° at the elbow.

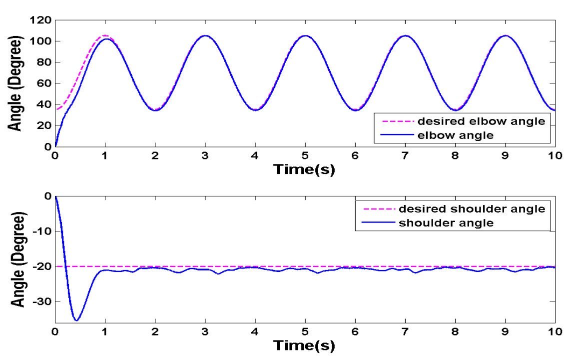

Fig. 5. System output using the control gain $C = [3 \ \ 11]$, $\alpha = 0.5,\ \beta = 0.02$.

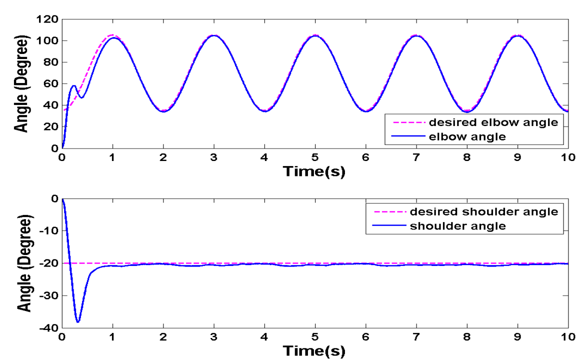

Fig. 6. System output using the control gain $C = [4 \ \ 40]$, $\alpha = 0.5,\ \beta = 0.02$.

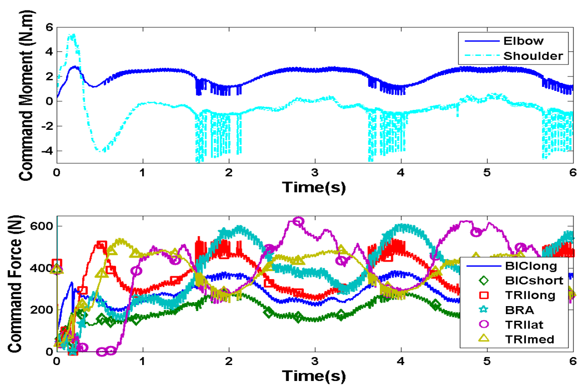

Fig. 7. Requires moments and optimal forces

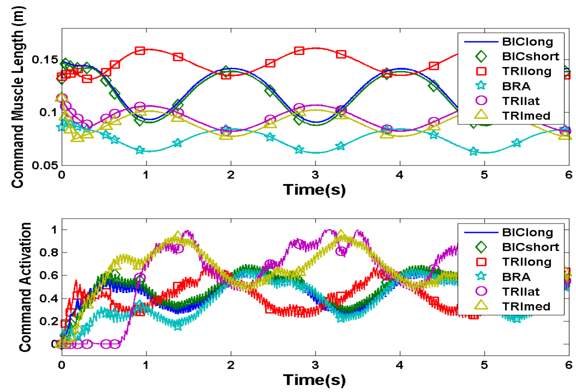

Fig. 8. Muscle length and activation response

6. Conclusion

In this paper, we proposed an adaptive controller to control the arm movement through a simulation study. First, we proposed an ASMC to derive the driving moments. Secondly, we implemented the Generalized Reduced Gradient method to optimally distribute forces to each muscle. Finally, we used another SMC to drive the activation and excitation. Because the model dynamics had a cascade form, the backstepping technique was implemented to compute the muscle excitations. The simulation study showed that our controller can handle the parametric uncertainties. Comparing to the Computed Muscle Control toolbox provided in OpenSim which uses the PID controller, the proposed method does not require tuning process. Therefore, the controller can provide an accurate and robust method to compute muscle control for the task human body motion tracking in OpenSim. Future work could include incorporating into the controller an observer to estimate the muscle length and activation. The ultimate goal is to use FES to stabilize arm tremors.

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