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A Model-Free Control Algorithm Derived Using the Sliding Mode Control Method

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Abstract- In this paper, a model-free Sliding Mode Control scheme is derived and applied to linear and nonlinear systems that is solely based on observable measurements and therefore does not require a theoretical system model for developing the controller form. The general Sliding Mode controller form is first derived for a general n^{th}-order system and is limited to a single-input case for this work. The controller form is based only on system measurements assuming the order of the system is known. The switching gain form is derived so that stability of the closed-loop Sliding Mode controller system is proven using Lyapunov's direct method. The controller form is reformulated using a smoothing boundary layer to eliminate chattering of the control effort. A simulation study is presented for a single-input case applied to both a linear and nonlinear system. The measurement based controller form is shown to be identical regardless of the system's kinematics to be controlled assuming the order is known. Results of simulation effort show that good tracking performance is achieved with stable convergence for the tracking convergence regardless of the system to be controlled.

***Keywords*:** Model-Free, Sliding Mode Control, Lyapunov

1. Introduction

Lyapunov based controllers have received much attention recently due to their robustness and ability to control nonlinear systems directly with stability guaranteed in the Lyapunov sense. In particular, the Sliding Mode Control method is a Lyapunov based control scheme allowing for asymptote stable tracking of state trajectories in the presence of modelling uncertainties and can be applied to nonlinear system models directly. However, in most cases a system model must be developed (that can be either linear or nonlinear) to derive the form of the Sliding Mode Controller.

Various methods have been discussed by researchers to develop Sliding Mode Controllers. Bandyopadhyay et al. (2007) proposed designing a sliding mode controller using a reduced order model of a system. Using a reduced model allows for quicker and easier computing of complex, high order systems. Chang and Wang (1998) used the invariance property of Sliding Mode Control on the error state covariance assignment such that the result could ignore the model reference input and plant error. Cunha et al. (2003) used adaptive control formulation and a unit vector control approach to develop an output-feedback model-reference sliding mode controller for multivariable linear systems. Laghrouche et al. (2007) described a controller that uses integral Sliding Mode concept to control uncertain nonlinear systems. Nizar el al. (2013) proposed a discrete predictive Sliding Mode Control to control time delay systems. Pai (2009) proposed a technique to accurately track uncertain linear systems using a discrete-time integral Sliding Mode Control. The method described in this paper of developing a model-free Sliding Mode Control scheme has not been discussed.

The purpose of this paper is to show that a model-free Sliding Mode Control scheme can achieve accurate tracking performance for both linear and nonlinear systems along with stable convergence for the tracking convergence. The paper is outlined as follows. Sections 2 defines the sliding surface and control law of the described system. Section 3 proves the controller form described is accurate and defines the switching gain and boundary layer of the controller. Section 4 provides illustrative examples of the derived Sliding Mode Control scheme. Conclusions are described in Section 5.

2. System Description

Consider control of an n^{th}-order single-input-single-output system where n is the highest order of the system. The following approximation can be made:

$$x^{(n)} \approx x^{(n)} + u - u_{k-1} \tag{1}$$

Where $x^{(n)}$ represents the system to be controlled, u is the controller input, and u_{k-1} is the previous value of the controller input.

2. 1. Define Sliding Surface

The sliding surface for an n^{th}-order single-input-single-output system can be defined as:

$$s(t) = \left(\frac{d}{dt} + \lambda\right)^{n-1} \tilde{x}(t) \tag{2}$$

Where λ is the strictly a positive constant and is slope of the sliding surface or the bandwidth of the closed-loop system, $\tilde{x}(t) = x(t) - x_d(t)$ where $x(t)$ is the state measurement and $x_d(t)$ is the desired state to be tracked and followed.

2. 2. Define Control Law

The derivative of the sliding surface shown in Eq. (1) is set to zero to ensure no movement is allowed of the state trajectories in the state-plane once the trajectories reach the sliding surface. Substituting the approximation of the system model shown in Eq. (1) into the derivative of the sliding surface shown in Eq. (2) and solving for the current value of the update control effort ensures of the state trajectories remain on the sliding surface once the trajectories reach the surface, so that:

$$\hat{u} = -\left(\frac{d}{dt} + \lambda\right)^{n} \tilde{x}(t) + u_{k-1} \tag{3}$$

The control effort shown above represents the best approximation of the effort since the system model is imperfect and represents only an approximation as shown in Eq. (1). Since the system contains uncertainties, a discontinuous term is added to the control law in order to drive the system trajectories onto the sliding surface in the presence of modeling error. The control law is then defined as follows:

$$u = -\left(\frac{d}{dt} + \lambda\right)^{n} \tilde{x}(t) + u_{k-1} - \eta \operatorname{sgn}(s) \tag{4}$$

Where η is a small strictly positive constant and sgn(s) is a signum function of the sliding surface s.

3. The Controller Form

3. 1. Theorem

The underlying premise of the method is assuming that the system model is not known and only state measurements are known and assumed available. The controller input will have little change for each time step, resulting in the controller input and previous value of the controller input to cancel out to zero as the time step tends to zero. A robust controller resulting in a stable closed-loop system in the Lyapunov sense can be derived assuming the time step is finite using knowledge of the previous control law time step value and shown in Eq. (4). By describing a system model in this fashion, a controller form can be developed based on solely system measurements and does not rely on a system model.

3. 2. Proof of the Controller Form

Lyapunov's direct method is used to ensure the system states trajectories are asymptotically stable during the reaching phase when the trajectories are not on the sliding surface. Lyapunov's direct method states that a sufficient condition for stability is there exist a continuously differentiable function $V(x)$ that is strictly positive definite resulting in $\dot{V}(x)$ being strictly negative definite then the equilibrium point is *asymptotically* stable. A candidate Lyapunov function that is strictly positive definite is defined by:

$$V(x) = \frac{1}{2}s^2 > 0 \tag{5}$$

Which satisfies the first criteria for ensuring *asymptotically* stability. By taking the derivative of the candidate Lyapunov function and substituting the derivative of the sliding surface:

$$\dot{V}(x) = s\left[\left(\frac{d}{dt} + \lambda\right)^n \tilde{x}(t)\right] \tag{6}$$

Substituting the assumed model form shown in Eq. (1) and the control effort shown in Eq. (4) in the resulting equation yields:

$$\dot{V}(x) = -\eta|s| < 0 \tag{7}$$

The Lyapunov function is always negative definite for positive values of η, thus Lyapunov's stability criterion is satisfied and the form of the controller effort, *u* shown in Eq. (4) is realized.

3. 3. Definition of the Control Effort and Switching Gain

The control law described in Eq. (4) is now rewritten as follows:

$$u = -\left(\frac{d}{dt} + \lambda\right)^n \tilde{x}(t) + u_{k-1} - K\text{sgn}(s) \tag{8}$$

Where K is a to-be-determined switching gain and is derived to ensure closed-loop stability in the Lyapunov sense of the system during the reaching phase. The sliding condition, as described by Eq. (7) and by taking the derivative of Eq. (5), is used to find the minimum value of K:

$$s\dot{s} = \frac{1}{2}\frac{d}{dt}s^2 \leq -\eta|s| \tag{9}$$

Using the definition of the sliding surface, the system model, and the control law described by Eq. (8) we derive the following after canceling out like terms:

$$-K|s| \leq -\eta|s| \tag{10}$$

From the above proof, the switching gain, K, must be greater than or equal to η to ensure stability during the reaching phase, i.e.:

$$K \geq \eta \tag{11}$$

3. 4. Boundary Layer

In order to eliminate chattering of the control effort, a time varying smoothing boundary layer should be applied to the controller form. The sliding condition needs to be maintained to guarantee the distance to the boundary layer is always decreasing. The sliding condition can be updated to show:

$$|s| \geq \phi \rightarrow \frac{1}{2}\frac{d}{dt}s^2 \leq \left(\dot{\phi} - \eta\right)|s| \tag{12}$$

In order to satisfy the new sliding condition, the term $K\mathrm{sgn}(s)$ is replaced by $\left(K - \dot{\phi}\right)\mathrm{sat}(s/\phi)$ where "sat"is a saturation function defined as:

$$\mathrm{sat}(y) = y \text{ if } |y| \leq 1 \tag{13a}$$

$$\mathrm{sat}(y) = \mathrm{sgn}(y) \text{ otherwise} \tag{13b}$$

The controller then becomes:

$$u = -\left(\frac{d}{dt} + \lambda\right)^n \tilde{x}(t) + u_{k-1} - \left(K - \dot{\phi}\right)\mathrm{sat}(s/\phi) \tag{14}$$

Where:

$$\dot{\phi} = -\lambda\phi + \eta \tag{15}$$

With:

$$\phi(0) = \eta\ /\lambda \tag{16}$$

4. Illustrative Examples

A mass, spring, damper system was used to test the validity of this proposed method. Both a linear and nonlinear system was considered.

4. 1. Linear Example

The linear system to be controlled was chosen to be:

$$m\ddot{x} + c\dot{x} + kx = u \tag{17}$$

Where *m* is the mass of the system, *c* is the damping coefficient of the system, *k* is the spring constant of the system, *u* is the input to the system, and x, $\dot{x}$, and $\ddot{x}$ are the state measurement variables of the system. The values used for the mass, damping coefficient, and spring constant were 2 kg, 0.8, and 2 respectively. The sampling time used was 0.0001 seconds and the system was simulated for 30 seconds. The value of λ used was 20 and the value of η used was 0.1.

Using the derived control law, sliding condition, and switching gain a control system was developed in Simulink and MATLAB for the mass, spring, damper system above. The desired tracking of the system as $x_d(t) = \sin(\pi/2)$. The results are as follows.

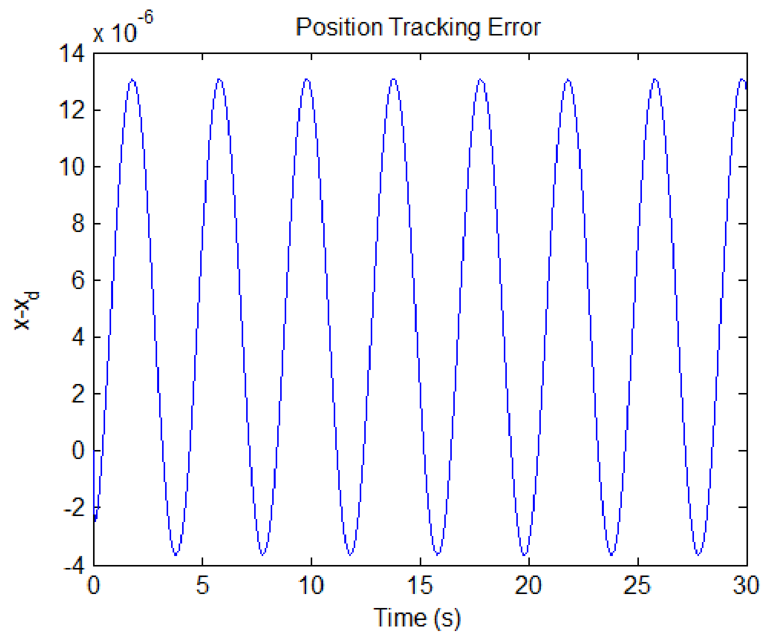

Fig. 1. Position Tracking Error for Linear System

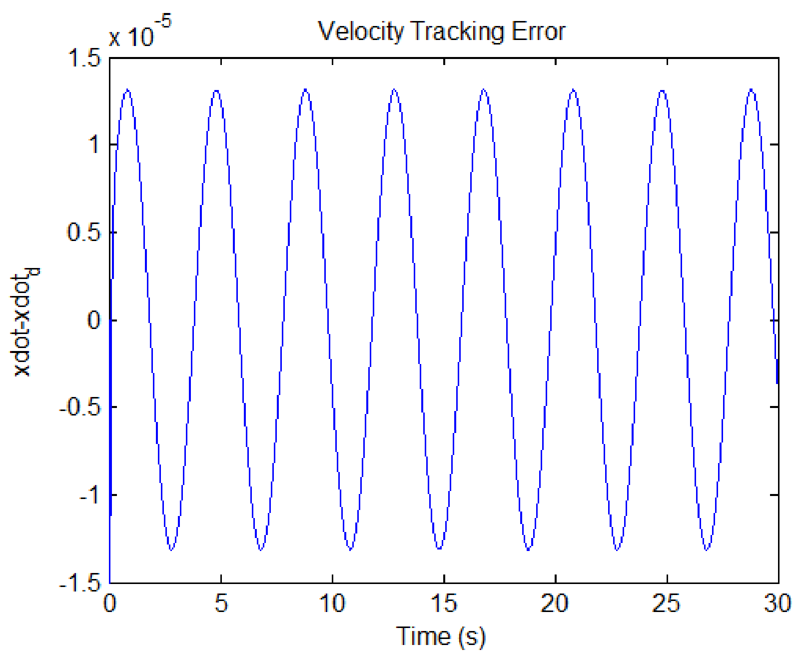

Fig. 2. Velocity Tracking Error for Linear System

Figures 1 and 2 show the control system is robust and produces minimal error with the error on position tracking being less than 14e-06 and the error on velocity tracking being less than 1.5e-05.

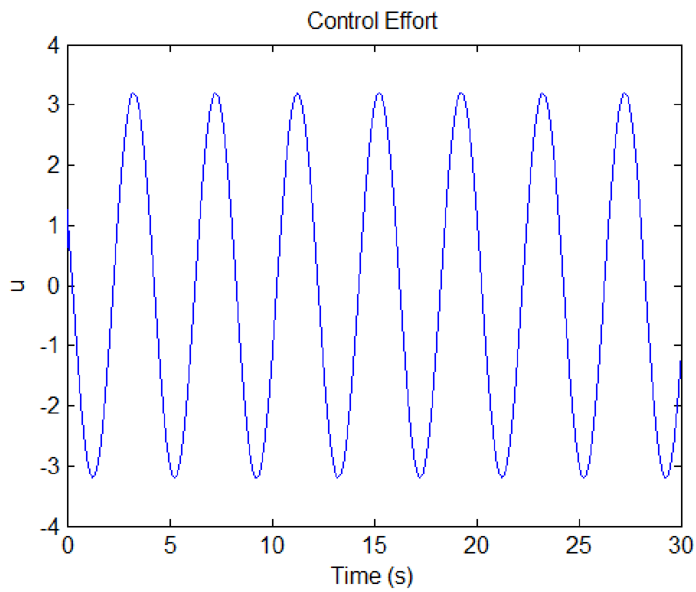

Fig. 3. Control Effort for Linear System

Figure 3 represents the control effort for the system and shows the time varying boundary layer resulted in a smooth control effort.

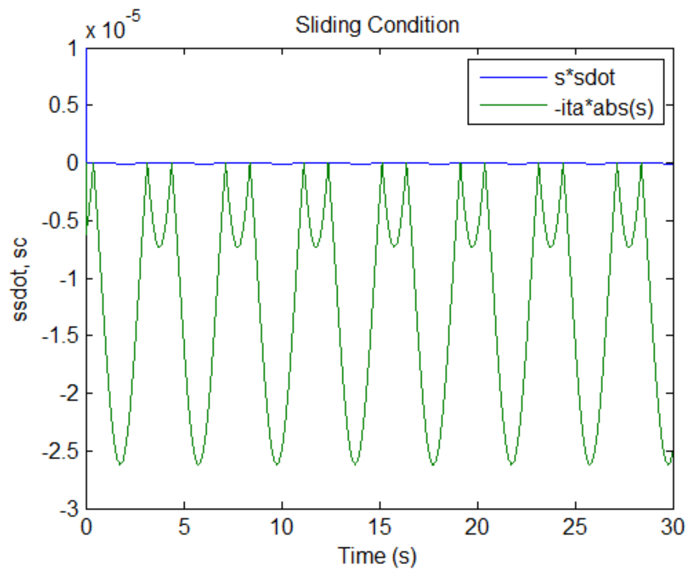

Fig. 4. Verifying the Sliding Condition is satisfied for Linear System

Figure 4 shows the sliding condition from Eq. (12) was satisfied at all times.

4. 2. Nonlinear Example

The nonlinear system to be controlled was chosen to be:

$$m\ddot{x} + c\dot{x} + kx^2 = u \tag{18}$$

Where m is the mass of the system, c is the damping coefficient of the system, k is the spring constant of the system, u is the input to the system, and x, $\dot{x}$, and $\ddot{x}$ are the state variables of the system. The values used for the mass, damping coefficient, and spring constant were 2 kg, 0.8, and 2 respectively. The sampling time used was 0.0001 seconds and the system was simulated for 30 seconds. The value of λ used was 20 and the value of η used was 0.1.

Using the derived control law, sliding condition, and switching gain a control system was developed in Simulink and MATLAB for the mass, spring, damper system above. The desired tracking of the system as $x_d(t) = \sin(\pi/2)$. The results are as follows.

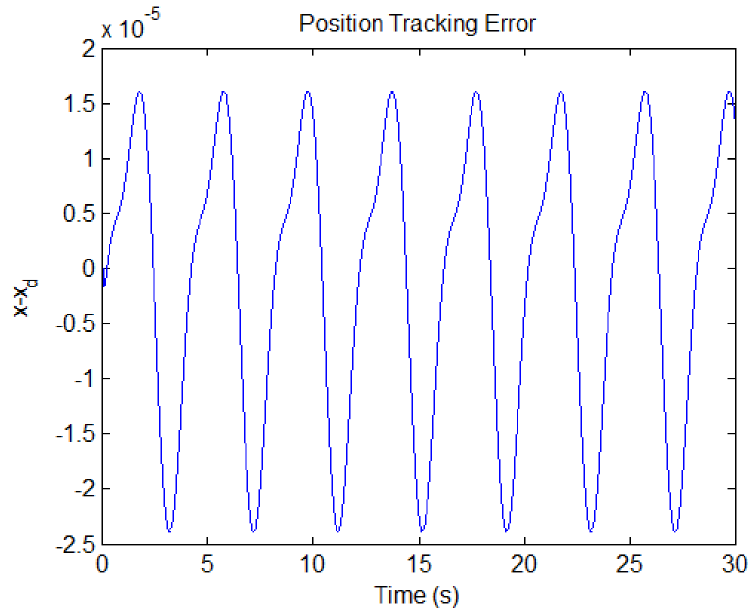

Fig. 5. Position Tracking Error for Nonlinear System

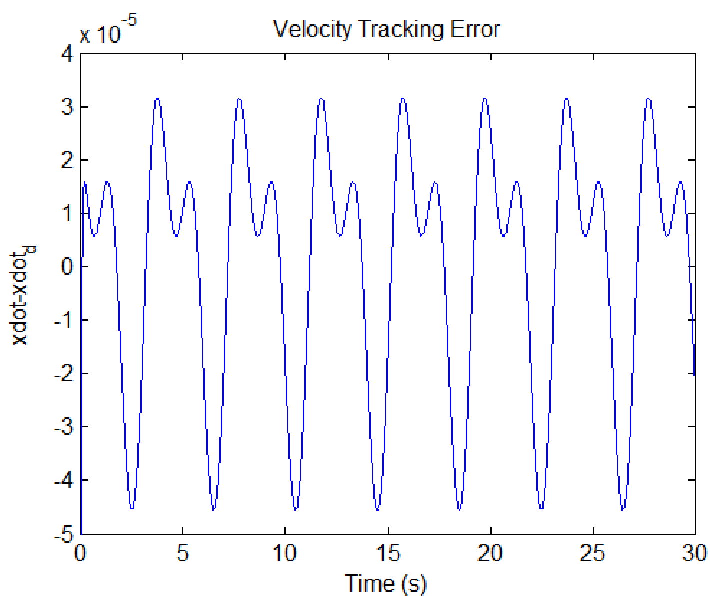

Fig. 6. Velocity Tracking Error for Nonlinear System

Figures 5 and 6 show the control system is robust and produces minimal error with the error on position tracking being less than 2e-05 and the error on velocity tracking being less than 4-05.

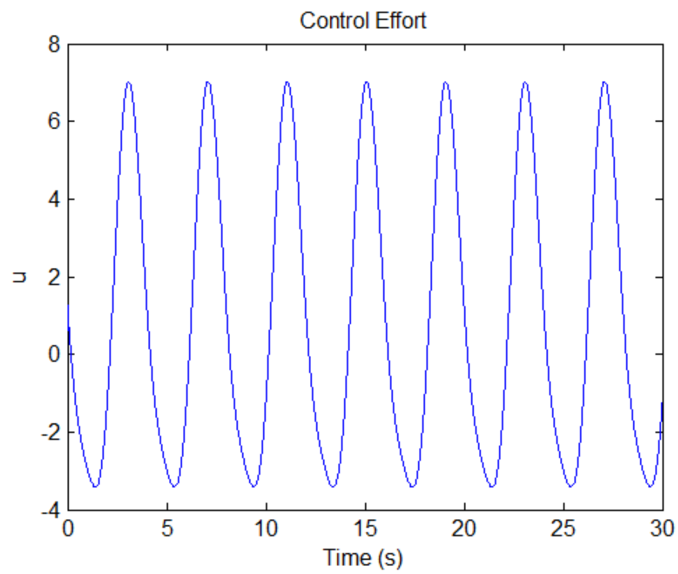

Fig. 7. Control Effort for Nonlinear System

Figure 7 represents the control effort for the system and shows the time varying boundary layer resulted in a smooth control effort.

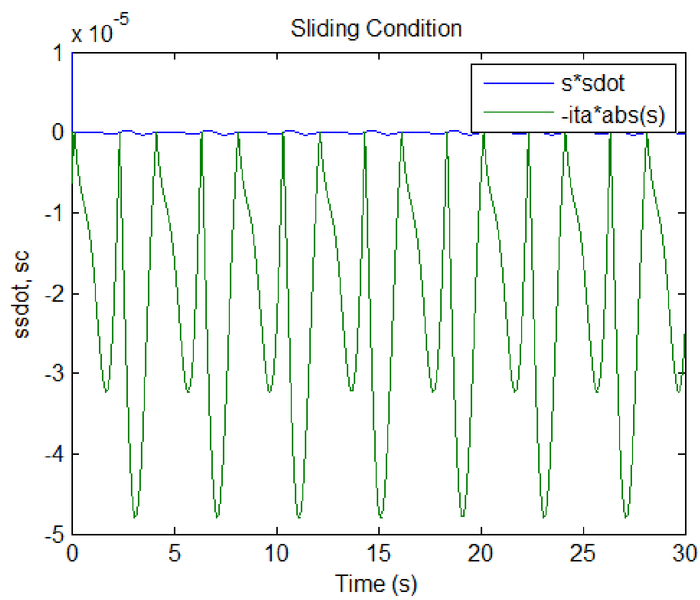

Fig. 8. Verifying the Sliding Condition is satisfied for Nonlinear System

Figure 8 shows the sliding condition from Eq. (12) was satisfied at all times.

5. Conclusions

A model-free Sliding Mode Control scheme based solely on observable measurements was presented. Lyapunov's direct method proved the system is asymptotically stable during the reaching phase. A time varying smoothing boundary layer was applied to the controller form to remove chattering on the control effort. The method was applied to a linear and nonlinear mass, spring, damper system. The results proved the method produces a robust control system with precise tracking, smooth controller effort, and satisfies the sliding condition.

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