Rotor Angle Position Control with Tracking Error Constraint for Permanent Magnet Synchronous Motors

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Abstract - We reviewed the dynamic modeling of a permanent magnet synchronous motor (PMSM) and proposed a rotor angle position control method with an error constraint for the PMSM. To guarantee the tolerance of the position tracking error, a backstepping controller is proposed using the barrier Lyapunov function. In addition, closed-loop stability of the proposed method is analyzed using the Lyapunov theorem.

Keywords: PMSM, Barrier Lyapunov function, Backstepping control, Tracking error constraint

1. Introduction

Permanent magnet synchronous motors (PMSMs) are receiving increasing attention for electric drive applications such as computer peripherals, robotics, adjustable speed drives and electric vehicles. Based on the characteristics of the model of PMSMs, many modern and intelligent control methods have been applied to PMSMs. These include nonlinear and sliding mode controllers developed to address the speed and position control of PMSMs. Backstepping control is a newly developed technique for the control of uncertain nonlinear systems, particularly those systems that do not satisfy matching conditions [1], and its applications have been proposed [2]. Recently, the barrier Lyapunov function was applied to the backstepping control for output constraints [3] and its applications for various systems have been proposed [4, 5]. This method has the advantage that the control performance of the PMSM can be improved and the control gain can be easily tuned even in the presence of some load fluctuation or system uncertainty. In this study, a position control method with an error constraint is designed for the PMSM. In addition, closed-loop stability of the proposed method is analyzed using the Lyapunov theorem.

2. Mathematical Modeling

The mathematical model of a PMSM is described as follows: [6]

$$
\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} L_a & L_{ba} & L_{ca} \\ L_{ba} & L_b & L_{cb} \\ L_{ca} & L_{cb} & L_c \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix}
\tag{1}
$$

where v_a, v_b, v_c are voltages applied on stator phases a, b, and c, respectively, i_a, i_b, i_c are phase currents, e_a, e_b, e_c are back EMF voltages, $R_s = R_a = R_b = R_c$ are stator phase resistances, L_a, L_b, L_c are phase self inductances and L_{ba}, L_{ca}, L_{cb} are mutual inductances. The back EMF voltage waveforms are expressed by the equation:

$$
\begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} = \omega_r \frac{d}{dt} \begin{bmatrix} \Phi_a \\ \Phi_b \\ \Phi_c \end{bmatrix}
\tag{2}
$$

where Φ_a, Φ_b, Φ_c are the stator magnetic fluxes of the motor phases a, b, and c, respectively. As the motor windings are star connected, the next relationship is valid:

$$i_a + i_b + i_c = 0. \tag{3}$$

Assuming that there is no change in the rotor reluctance with the angular position,

$$L_a = L_b = L_c = L, \quad L_{ba} = L_{ca} = L_{cb} = M. \tag{4}$$

Taking into consideration (4), the equation (1) is rearranged as follows:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = R_s \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + L_s \frac{d}{dt} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} \tag{5}$$

where $L_s = L - M$ is the stator phase inductance. Hence, the mathematical model of a PMSM is similar to that of a wound rotor synchronous motor. The following assumptions are made in the derivation: 1) Saturation is neglected although it can be taken into account by parameter changes; 2) The back EMF is sinusoidal; 3) Eddy currents and hysteresis losses are negligible, and 4) There are no field current dynamics. With these assumptions the stator d, q equations in the rotor reference frame of the PMSM can be expressed as follows:

$$v_d = R_s i_d + \frac{d}{dt} \lambda_d - \omega_r \lambda_q,$$
$$v_q = R_s i_q + \frac{d}{dt} \lambda_q - \omega_r \lambda_d \tag{6}$$

where v_d and v_q are the d and q axes components of the voltages of the stator [V], respectively, i_d and i_q are the d and q axes components of the stator current [A], respectively, R_s is the resistance of the stator [W], $\lambda_d = L_d i_d + \lambda_m$ and $\lambda_q = L_q i_q$ are d and q axes components of the flux produced by the permanent magnet [Wb], respectively, λ_m is the magnet mutual flux linkage, and ω_r is the electrical rotor speed [rad/s]. In (6), v_d and v_q are obtained from voltages applied on stator phases v_a, v_b, v_c through the Park transform [7, 8].
The electric torque τ_e is given by

$$\tau_e = \frac{3 p_p}{2} (\lambda_m i_q + (L_d - L_q) i_d i_q). \tag{7}$$

where p_p is the number of pole pairs. The mechanical dynamics equations are as follows:

$$J \frac{d\omega_r}{dt} = \tau_e - \tau_L - B\omega_r, \quad \frac{d\theta}{dt} = \omega_r \tag{8}$$

where J is the total inertia referred to the respective motor shaft, B is the viscous damping coefficient and τ_L is the load torque of the mechanical system. For constant flux operation when i_d equals zero, the electrictorque $\tau_e = \frac{2}{3} \lambda_m i_q = K_t i_q$, where K_t is the motor torque constant. Note that this torque equation for the PMSM resembles that of the regular DC machine and hence provides ease of control. For a uniform airgap surfaced-mounted PMSM motor, the d- and q-axes inductances of the stator are equal, $L = L_d = L_q$. Thus the dynamic model of a surface-mounted PMSM can be described as follows:

$$\dot{\theta} = \omega_r$$

$$\dot{\omega}_r = \frac{3p_p\lambda_m}{2J}i_q - \frac{B}{J}\omega_r - \frac{1}{J}\tau_L$$

$$\dot{i}_q = -\frac{R_s}{L}i_q - \omega_r i_d - \frac{\lambda_m}{L}\omega_r + \frac{1}{L}u_q \qquad (9)$$

$$\dot{i}_d = -\frac{R_s}{L}i_d + \omega_r i_q + \frac{1}{L}u_d$$

where θ is the rotor angle [rad], J is the moment of inertia [kg-m^2], τ_L is the load torque [Nm], p_p is the number of pole pairs, B is the damping coefficient [Nm/(rad/s)], and $u_d = v_d$ and $u_q = v_q$ are the d and q axes components of the voltages of the stator serving as the control variables [V].

The electrical rotor speed ω_r is related to the mechanical rotor speed ω_m by $\omega_r = p_p\omega_m$.

3. Position Controller Design and Stability Analysis
3.1. Controller Design

We define the position tracking error z_1 as $z_1 = \theta - \theta_d$ where θ_d is the desired position. The states of the closed-loop system, z_{cl} are defined as $z_{cl} = [z_1, z_2, z_3, z_4]^T$, where z_2, z_3 and z_4 are defined in the following theorem.

Theorem: Consider the dynamic model (9) of the PMSM, and $|z_1(0)| < k_b$. Suppose that the control law is given by

$$z_1 = \theta - \theta_d,$$

$$\alpha_1 = -k_1 z_1 \left(k_b^2 - z_1^2\right),$$

$$z_2 = \omega_r - \omega_d,$$

$$\alpha_2 = \frac{2J}{3p_p\lambda_m}\left(-\frac{z_1}{k_b^2 - z_1^2} - k_2 z_2 + \frac{B}{J}\omega + \frac{\tau_L}{J} + \dot{\omega}_d + \dot{\alpha}_1\right) - i_q^d,$$

$$z_3 = i_q - i_q^d - \alpha_2, \qquad (10)$$

$$z_4 = i_d - i_d^d,$$

$$u_q = L\left(-z_2 - k_3 z_3 + \frac{R}{L}i_q + p\omega_r i_d + \frac{p\lambda_m}{L}\omega_r + i_q^d + \dot{\alpha}_2\right),$$

$$u_d = L\left(-k_4 z_4 + \frac{R}{L}i_d - p\omega_r i_q + i_d^d\right)$$

where k_1, k_2, k_3 and k_4 are the positive controller gains, and k_b is the tolerance of the position tracking error of the constraint, $|z_0| < k_b$. If $|z_1(0)| < k_b$, then $|z_1(t)| < k_b$, $\forall t > 0$ and $z_{cl}(t)$ asymptotically converges to zero.

3.2. Stability Analysis

We use the beackstepping technique to prove the closed-loop stability of the proposed method using the barrier Lyapunov function.

Step 1: Let us define the barrier Lyapunov candidate function V_1 as follows:

$$V_1 = \frac{1}{2}\log\left(\frac{k_b^2}{k_b^2 - z_1^2}\right) \qquad (11)$$

where $\log(\cdot)$ is the natural logarithm function. The derivative of V1 with respect to time is given by

$$\dot{V}_1 = \frac{z_1 \dot{z}_1}{k_b^2 - z_1^2} = \frac{z_1(z_2 + \alpha_1)}{k_b^2 - z_1^2}. \qquad (12)$$

Substituting α_1 with that in (10) results in

$$\dot{V}_1 = -k_1 z_1^2 + \frac{z_1 z_2}{k_b^2 - z_1^2}. \qquad (13)$$

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Step 2: Let us define V_2 as

$$V_2 = V_1 + \frac{1}{2}z_2^2 \tag{14}$$

Then

$$\dot{V}_2 = \dot{V}_1 + z_2\dot{z}_2 = \dot{V}_1 + z_2\left(\frac{3p_p\lambda_m}{2J}\left(z_3 + i_q^d + \alpha_2\right) - \frac{B}{J}\omega_r - \frac{\tau_L}{J} - \dot{\omega}_d - \dot{\alpha}_1\right). \tag{15}$$

With α_2, the time derivative of V_2 becomes

$$\dot{V}_2 = -k_1z_1^2 - k_2z_2^2 + z_2z_3. \tag{16}$$

Final step: We defined the overall Lyapunov candidate function V_{blf} as

$$V_{blf} = V_2 + \frac{1}{2}z_3^2 + \frac{1}{2}z_4^2. \tag{17}$$

We then obtain $\dot{V}_3$ as

$$\begin{aligned}
\dot{V}_{blf} &= \dot{V}_2 + z_3\dot{z}_3 + z_4\dot{z}_4 \\
&= -k_1z_1^2 - k_2z_2^2 + z_2z_3 \\
&\quad + z_3\left(-\frac{R}{L}i_q - p\omega_r i_d - \frac{p_p\lambda_m}{L}\omega_r + \frac{1}{L}u_q - i_q^d - \dot{\alpha}_2\right) + z_4\left(-\frac{R}{L}i_d + p_p\omega_r i_q + \frac{1}{L}u_d - i_d^d\right).
\end{aligned} \tag{18}$$

With the control input u_d and u_q, $\dot{V}_{blf}$ can be written as

$$\dot{V}_{blf} = -k_1z_1^2 - k_2z_2^2 - k_3z_3^2 - k_4z_4^2 \leq 0. \tag{19}$$

Thus, if $|z_1(0)| < k_b$, then $|z_1(t)| < k_b$, $\forall t > 0$ and $z_{cl}(t)$ asymptotically converges to zero.

4. Conclusion

We reviewed the dynamic modeling of the permanent magnet synchronous motor. The rotor angle position control method with error constraint is designed, and the closed-loop stability of the proposed method is studied using the Lyapunov theorem.

Acknowledgments

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