Proceedings of the 4th International Conference of Control, Dynamic Systems, and Robotics (CDSR'17)
Toronto, Canada – August 21 – 23, 2017
Paper No. 114
DOI: 10.11159/cdsr17.114

Are Fractional PI$^\lambda$D$^\mu$ Controllers Good for All Processes?

Xin Shen, Jules Thibault
Department of Chemical and Biological Engineering, University of Ottawa
161 Louis Pasteur, Ottawa, ON, Canada K1N 6N5
Xshen063@uottawa.ca; Jules.Thibault@uottawa.ca

Abstract **-** Fractional PI$^\lambda$D$^\mu$ controllers are considered as a promising alternative of PID controllers for future industrial applications. In comparison to classical PI and PID controllers, improved performance of fractional controllers for a number of applications has been reported. However, it is still unclear for which type of systems the more computationally-demanding fractional controllers would be significantly better as a replacement for integer PI and PID controllers. In this investigation, fractional controllers and classical PI and PID controllers have been tested for different benchmark systems to determine which classes of systems would benefit the most of using a more complex control algorithm. Results show that, despite the added degrees of freedom, it is not always beneficial to use such a computationally expensive controller and, only for some types of systems, fractional controllers will enhance controller performances.

Keywords**:** PI$^\lambda$D$^\mu$ controllers, benchmark systems, multi-objective optimization, NSGA-III

1. Introduction

A generalized fractional controller PI$^\lambda$D$^\mu$, proposed by Podlubny in 1994 [1], has attracted significant interest in the last decade for its good performance and novelty. It was tested in different systems: automatic voltage regulator [2], path tracking control of tractors [3], hydraulic turbine regulating system [4], water level control [5] and many others. Although many applications have been reported, yet it is still unclear what type of systems could benefit the most from a more computationally expensive controller. In this investigation, a series of representative systems were selected as benchmark systems: linear and nonlinear first-order plus dead-time, linear higher-order and integrative systems. Performance of the fractional and classical controllers were compared to each system.

1.1. Fractional PI$^\lambda$D$^\mu$ Controller

Fractional PI$^\lambda$D$^\mu$ controller in the generalized Laplace domain is defined by Eq. (1).

$$G_C = K_C\left(1+\frac{1}{\tau_I s^{\lambda}}+\tau_D s^{\mu}\right) \tag{1}$$

where $s^{-\lambda}$ and s^{μ} are the generalized Laplace transform for the fractional integral and derivative, respectively.

1.2. Linear First Order Plus Dead Time System

A linear first order plus dead time (FOPDT) system, which is often used to approximate high order systems, is a very common model to represent chemical processes and used in control engineering. The FOPDT model used in this investigation is characterized by a process gain, a time constant and a dead time as shown in the Laplace domain in Eq. (2).

$$G_P = \frac{K_P e^{-\theta s}}{\tau_P s+1} \tag{2}$$

1.3. Controller Performance Criteria

To assess the performance of the various controllers, three performance criteria were used: the integral of the time weighted absolute error (ITAE), the time spent outside the ±5% zone (OZ), and the integral of the squares of the differences in the manipulated variable (ISDU).

$$ITAE = \int_0^t t\left|\varepsilon_t\right| dt \tag{3}$$

$$OZ = \int_0^t g\left(f(t)\right) dt \tag{4}$$

$$g\left(f(t)\right) = \begin{cases} 1 & \left|f(t) - f(\infty)\right| \geq 0.05\left|f(0) - f(\infty)\right| \\ 0 & \left|f(t) - f(\infty)\right| < 0.05\left|f(0) - f(\infty)\right| \end{cases} \tag{5}$$

$$ISDU = \int_0^t \Delta u_t^2 dt \tag{6}$$

where t is the time, ε_t the instantaneous error, $f(t)$ the system response, $f(0)$ and $f(\infty)$ the initial and final output of the system, respectively, and Δu_t represents the change of the manipulated variable at time t.

2. Methodology

2.1. Controller Design

For each system, the tuning of the $PI^{\lambda}D^{\mu}$ and classical linear controllers was formulated as a multi-objective optimization problem. The three controller performance criteria, ITAE, OZ, and ISDU, were selected as three optimization objectives to be minimized whereas the five controller parameters (K_c, τ_I, τ_D, λ, μ) were the decision variables. An evolutionary algorithm, the non-dominated sorting genetic algorithm III (NSGA-III) [6] [7], with a variety-preserving strategy was used to circumscribe the Pareto domain.

However, a true Pareto domain can sometimes be very difficult to obtain. According to experience, the Pareto domain was shown to be very sensitive to inappropriate operator settings. A small change in the settings of the optimization algorithm, may even lead to an instable, chaotic and false Pareto domain, different from the one of classical controllers in shape, density, and boundary. To obtain a good approximation of the Pareto domain, a real-time analysis (RTA) of the NSGA-III optimization procedure was performed to analyse the influence of the mutation operator, population size, decision space size, and crossover operator. The optimized algorithm settings were applied in the controller design.

2.2. Pareto Domain Analysis

The Pareto domain was obtained without bias and all sets of the three objective criteria contained in the Pareto domain are non-dominated solutions. Even though all solutions in the Pareto domain are non-dominated, they are not all equal in the eyes of the decision maker or expert. The next step in the optimization process is to rank all solutions of the Pareto domain using preferences of the decision maker. In this investigation, Net Flow algorithm [8] was used. The Net Flow algorithm resorts to four parameters to rank the entire Pareto domain: the relative weight and three thresholds (indifference, preference and veto) for each objective function. These four ranking parameters for each objective criterion are able to integrate the preferences of the decision maker and to determine the best zone of operation that will satisfy these preferences. The Net Flow Method attributes to each solution a score based a pairwise comparison of all solutions in the Pareto domain. Solutions are ranked according to these scores. For the final determination of the optimal operating zone, robustness needs to be considered to ensure the solution is far enough of the edge of the Pareto domain viewed in the decision space.

3. Results and Discussion

3.1. Linear First Order Plus Dead Time System

The decision variables and objective criteria of the Pareto domain, expressed in dimensionless form, of fractional $PI^{\lambda}D^{\mu}$ controller for the linear first order plus dead time system are presented in Figure 1. The FOPDT system had the following parameters: a process gain K_P of 1, a time constant τ_P of 2 and a dead time θ of 0.5. Solutions were categorized by grouping the top 5%, the next 20% and the remaining 75% of the solutions ranked by Net Flow. The solutions with the top scores are located in the middle portion of the Pareto domain which is an indication of their robustness. It is interesting to observe that for a large portion of Fig. 1A, it is required to change simultaneously the proportional gain and the integration constant of the controller to remain in the Pareto domain. Looking at Fig. 1C, the μ-λ plot, both 5% and 20% optimized solutions converge to $\lambda=1$, which suggests that for the FOPDT system the fractional integral component is not necessary and only the integer integral is required. Moreover, Fig. 1C also shows that the derivative component does not contribute significantly to the controller performance because the fractional order of the derivative μ is in the vicinity of 0. For a FOPDT system, it can be concluded that the fractional PIλDμ controller is not a better alternative than classical PI and PID controllers.

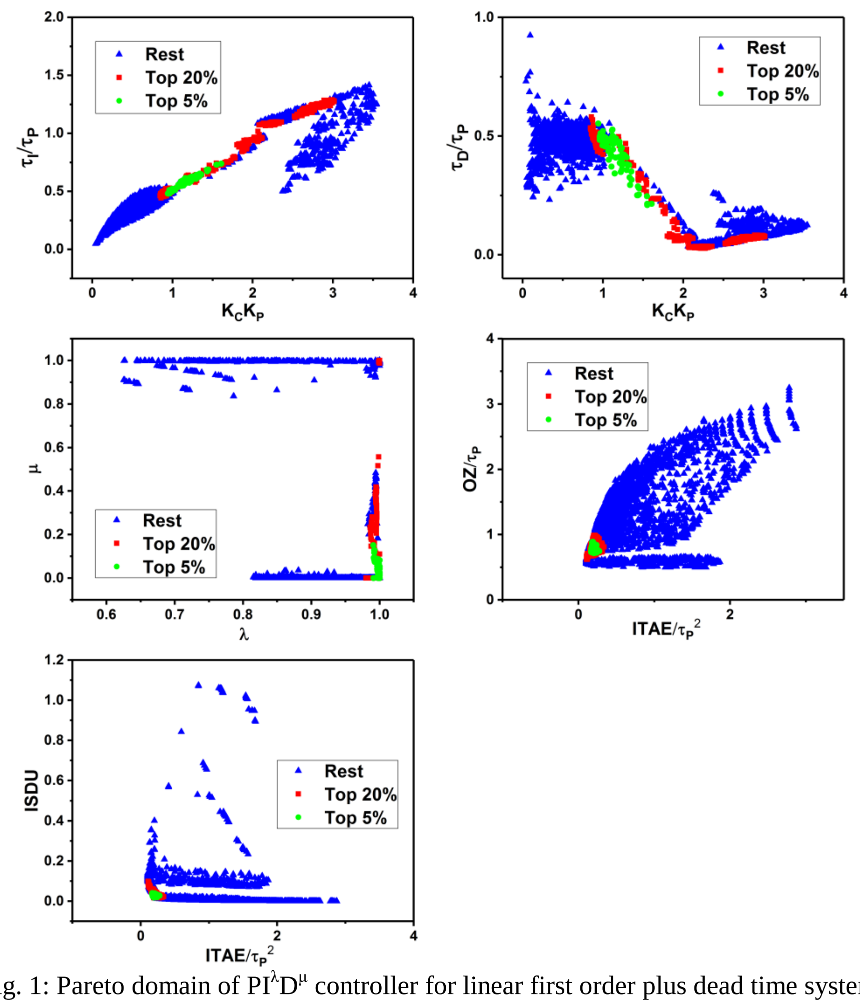

Fig. 1: Pareto domain of $PI^{\lambda}D^{\mu}$ controller for linear first order plus dead time system.

3.2. Comparison of Optimized Solutions

The comparison of the top 5% solutions of Pareto domain for both fractional and classical controllers is presented in Fig. 2. In the decision space, solutions of the PI controller have a high degree of overlap with the PI^{λ} controller. Similar overlaps are observed for the PID and $PI^{\lambda}D$ controllers, and PID^{μ} and $PI^{\lambda}D^{\mu}$ controllers. Moreover, solutions obtained with the $PI^{\lambda}D^{\mu}$ and PID^{μ} controllers converge to $\lambda = 1$ and $\mu = 0$, which in fact is a PI controller. Only PI^{λ} and $PI^{\lambda}D^{\mu}$ controllers have a small portion of the top 5% solutions not exactly at $\lambda = 1$ but still very close with the lowest at $\lambda = 0.98$. This clearly indicates that adding a fractional component has a very weak impact on the optimized solution in decision space.

In the objective space, similarly to the decision space, solutions obtained from fractional controllers are also highly overlapped with its corresponding integer controllers. The three objective criteria occupy slightly different optimal zones in the objective space. In Fig. 2E, it is possible to observe a trend where solutions associated with the PI, PI^{λ}, PID^{μ} and $PI^{\lambda}D^{\mu}$ controllers gradually decrease in both ITAE and ISDU in the lower left direction. To some extent, this shows that the performance marginally improves as larger fractional components are introduced. The optimal solutions of the $PI^{\lambda}D^{\mu}$ are found at the lower left boundary of the set of optimal solutions, which detach from the PID^{μ} solutions. Moreover, the solutions associated with this curve are solutions that slightly deviate from $\lambda = 1$. This implies that even if fractional components may potentially improve the controller performance, yet an important fractional component is still detrimental to a good controller performance and only a small deviation from the integer component is applicable. It can be concluded that linear FOPD cannot really benefit from the addition of fractional components.

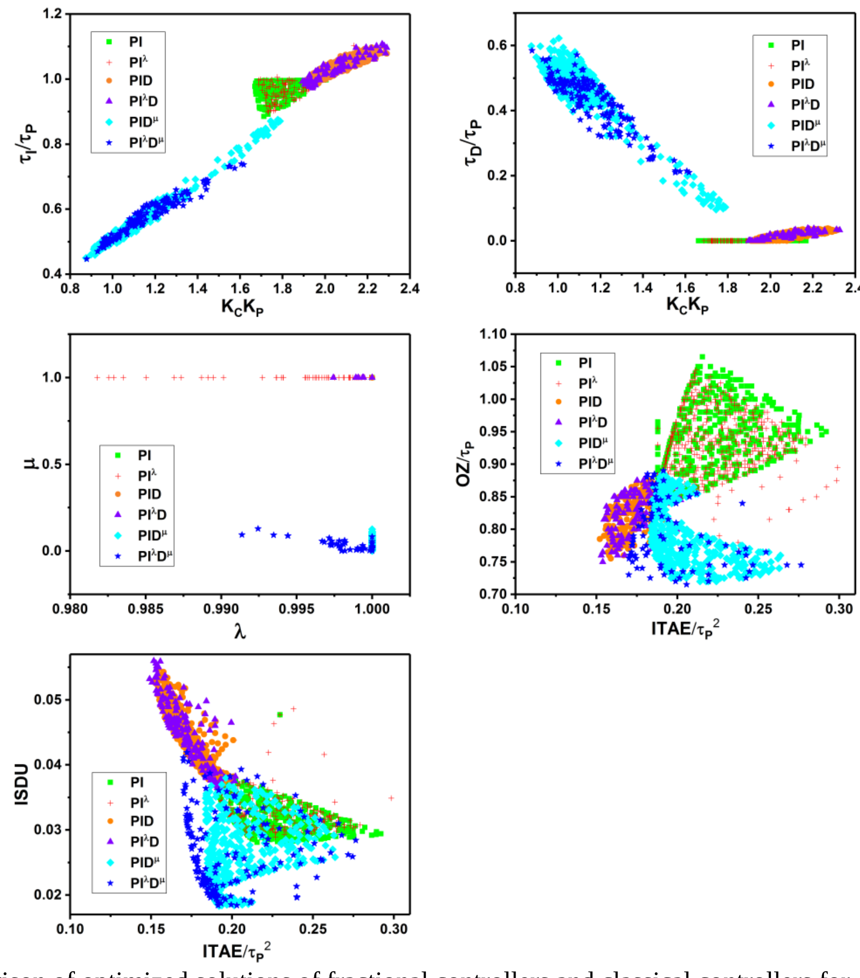

Fig. 2: Comparison of optimized solutions of fractional controllers and classical controllers for a linear FOPDT system.

4. Conclusion

The description of integer and fractional controllers have been presented and results were presented for a first order plus dead time system. It was shown that the use of a more computationally extensive controller, despite the additional degrees of freedom, is not required for FOPDT systems and other simple linear systems. Controller performance were evaluated for various benchmark systems to determine under which conditions fractional order PID will justify the additional controller complexity and computation time.

References

[1] I. Podlubny, D. Dorcak and I. Kostial, "On fractional derivatives, fractional-order dynamic systems and PI/sup /spl lambda//D/sup /spl mu//-controllers," *Proceedings of the 36th IEEE Conference on Decision and Control*, San Diego, pp. 4989-4990, 1997.

[2] I. Pan and S. Das, "Frequency domain design of fractional order PID controller for AVR system using chaotic multi-objective optimization," *International Journal of Electrical Power & Energy Systems*, vol. 51, pp. 106-118, 2013.

[3] M. Zhang, X. Lin and W. Yin, "An improved tuning method of fractional order proportional differentiation (FOPD) controller for the path tracking control of tractors," *Biosystems Engineering*, vol. 116, no. 4, pp. 478-486, 2013.

[4] Z. Chen, X. Yuan, B. Ji, P. Wang and H. Tian, "Design of a fractional order PID controller for hydraulic turbine regulating system using chaotic non-dominated sorting genetic algorithm II," *Energy Conversion and Management*, vol. 84, pp. 390-404, 2014.

[5] V. Bhambhani and Y. Chen, "Experimental study of fractional order proportional integral (FOPI) controller for water level control," *47th IEEE Conference on Decision and Control*, Cancun, pp. 1791-1796, 2008.

[6] K. Deb and H. Jain, "An Evolutionary Many-Objective Optimization Algorithm Using Reference-Point-Based Nondominated Sorting Approach, Part I: Solving Problems with Box Constraints," *IEEE Transactions on Evolutionary Computation*, vol. 18, no. 4, pp. 577-601, 2014.

[7] F. Fortin, F. Rainville, M. Gardner, M. Parizeau and C. Gagné, "EAP: Evolutionary Algorithms Made Easy," *Journal of Machine Learning Research*, vol. 13, no., pp. 2171−2175, 2012.

[8] A. Vandervoort, J. Thibault and Y. Gupta, "New PI Controller Tuning Methods Using Multi-Objective Optimization," in *Multi-objective optimization in chemical engineering: developments and applications*, 1st ed., R. Gade Pandu and A. Bonilla-Petriciolet, Ed. Chichester: John Wiley & Sons., 2013, pp. 479-501.