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High Resolution Time-Frequency Analysis of Non-stationary Signals

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Abstract - Time-frequency representations of signals are very important analysis tools which are required in various fields of science and engineering. Observations of most dynamic systems provide non-stationary signals where frequency properties vary in time. There are vast amounts of studies on the development of new time-frequency representations in the literature. Some methods provide better resolutions in time or frequency, or some providing improvements on both resolutions but with the cost of complexity and observation of cross-terms. Currently all methods provide somewhat balanced solutions when compared to their trade-off observables; computation cost, complexity, and performances. Further,their performances are greatly limited by the Heisenberg-Gabor limit. In this work, three-dimensional time-frequency distribution (3D-TFD), three dimensional short-time Fourier transform (3D-STFT) are proposed, and the resolution improvement metric vector is defined. They are used to fuse the independent measurements of high resolution time and frequency. A heuristic practical approach is proposed as a proof-of-concept, and tested on a short duration pulse of constant frequency. It is observed that only two layers of 3D-STFT could be sufficient to obtain both high time and high frequency resolutions. Higher resolutions in both axes are obtained simultaneously. This novel method can be used in feature extraction, detection and other analysis applications. A more detailed study is planned as a future work.

***Keywords*:** Time-frequency distributions, time-frequency analysis, non-stationary signals, high time-frequency resolution, three-dimensional short-time Fourier transform (3D-STFT)

1. Introduction

A physical observation which produces some form of output signal carries information about the observed dynamic system. Signals can originate from different types of sources including electromagnetics, acoustics and mechanics. Signals have been analyzed in either the time or the frequency domain. Each of these forms is widely used for data analysis and processing. Time domain domain analyses are based on some form of models including, the autoregressive and moving average where for the frequency domain, the Fourier transform is at the heart of a wide range of techniques. The Fourier transform-based techniques are effective as long as the frequency contents of the signal do not change with time. However, many signals of interest have a distribution of energy that varies in time and frequency. Thus, most signals belong to the class of non-stationary signals. Time-frequency representations are commonly used to analyze or characterize such signals. Time-Frequency Distributions (TFDs) describe signals in term of their joint time-frequency content.They map the one-dimensional time-domain signal into a two-dimensional function of time and frequency. These distributions are useful for analyzing signals with both time and frequency variations. Therefore, for signals with time-varying frequency contents, TFDs offer a powerful analysis tool [1]. The spectrogram is the most commonly-used time-frequency representation, probably because it is well-understood, and immune to so-called cross-terms that sometimes make other time-frequency representations difficult to interpret. But the windowing operation required in spectrogram computation introduces an unsavory tradeoff between time resolution and frequency resolution, so spectrograms provide a time-frequency representation that is blurred in time, in frequency, or in both dimensions. The need for super-resolution algorithms are not published very recently [2-8], but significant improvements over any of the previous time-frequency analysis methods are still necessary.

The most widely known time-frequency analysis techniques belong to the Cohen class with the leading transform being the Wigner-Ville distribution, which is based on a bilinear model [9, 10]. This model is able to analyze time-varying signals with relatively high resolution. However, being a bilinear model it introduces cross-term artifacts[9]. Hence, filtering techniques have been proposed to reduce the cross-term artifacts. Among them, the most well known filtering method uses the Choi-Williams filter. Higher than bilinear order models have also been proposed for time-frequency analysis, e.g., Wigner bispectrum and trispectrum. Choi-Williams filters have also been incorporated into these models. These higher-order models work well with isolated signals (time-varying or stationary).

Wavelet-based transforms have also been proposed for time-frequency analysis [11]. Wavelet transforms break the signal into a set of bases with a shape that is based on affine transformations (i.e., translations and dilations) of a basic wavelet called the mother wavelet. Among the most simple but also effective time-frequency wavelets is the Gabor based time-frequency expansion, where the mother wavelet is a Gaussian function. The Gabor transform originally used rectangles to designate each of the time-frequency elementary signals. Wavelet-based time-frequency analysis belongs to the Weyl-Hiesenberg generalized class. The introduction of chirplets for time-frequency analysis also belongs to the same class and has been used to better describe the second and higher-order signal nonlinearity [12, 13]. Another transform that is also considered as a generalization of the Gabor transform is the s-transform [10]. All these transforms do not introduce cross-term artifacts as those of the Cohen class time-frequency analysis methods. However, they often result in inferior resolution when compared to the Cohen class transforms. Other techniques in time-frequency analysis include the Adaptive Joint Time-Frequency (AJTF) transform. It is based on the series analysis of a time-varying signal into a high-order polynomial of time. The estimation of parameters assumes the presence of very strong (i.e., high signal-to-noise ratio) signal components.

An early definition of the fractional Fourier transform (FrFT) was introduced in 1937 [14], and in recent years the FrFT has been reintroduced [15]. The FRFT is able to find linear changes in the frequency over time. However, for higher-order nonlinearity, extensions of this transform to time-frequency domain must be incorporated. Enhancements and filtering techniques on the results of the above mentioned transforms have been proposed where most promising methods are the Choi-Williams filters which are applied in the Wigner-Ville class of time-frequency transforms and the reassignment methods, which are applied to any time-frequency distribution [12, 16]. The chirp function is one of the most fundamental functions in nature. Many natural events can be modelled as a superposition of short-lived chirp functions. Hence, the chirp-based signal representation, such as the Gaussian chirplet decomposition, has been an active research area in the field of signal processing. Since real signals have different shapes in the ambiguity domain, no single kernel can give adequate performance for a large class of signals. Hence, there has been increasing interest in signal-dependent or adaptive TFDs, in which the kernel function varies with the signal. Adaptation of the kernel over time is beneficial because it permits the kernel to match the local signal characteristics [10].

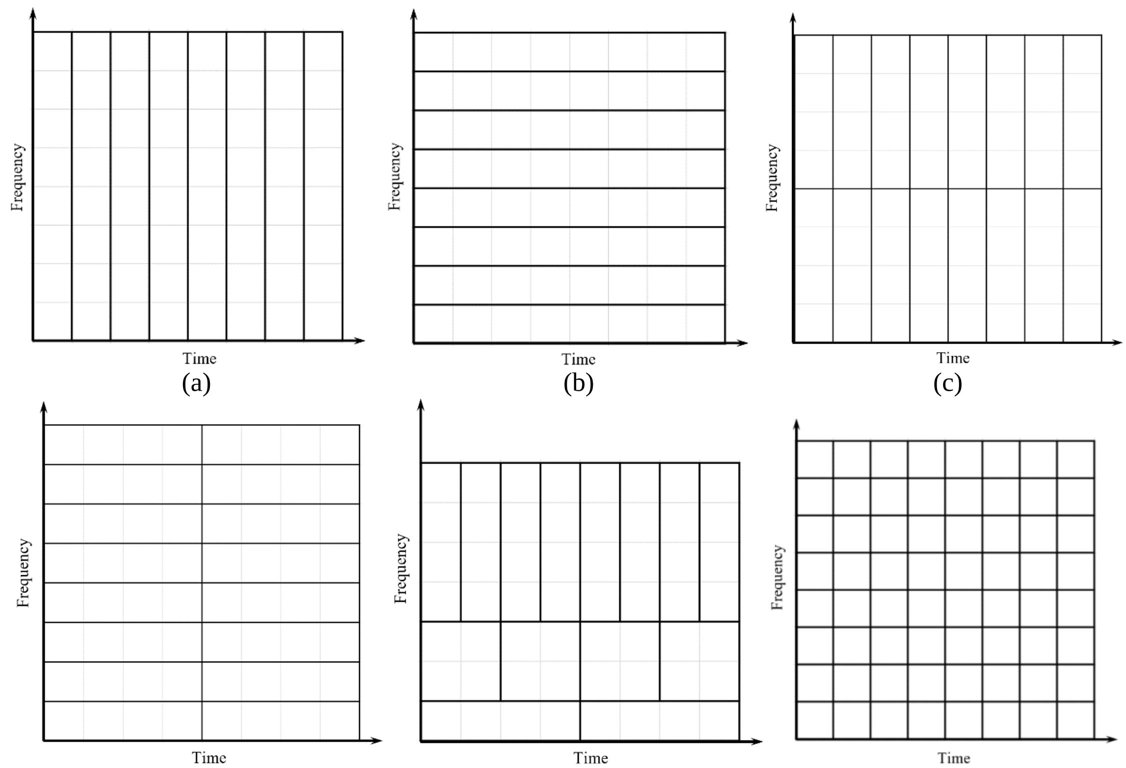
It was found that TFDs provide additional insight into the analysis, interpretation, and processing of radar signals that are sometimes superior to what is achievable in the traditional time or frequency domain alone [16]. The specific applications where TFDs have been used to obtain both time and frequency variations of signal components.

Multiresolution analysis provides ways to analyze signals at different frequencies with different resolutions [12]. In this work, a new class of time-frequency representation is proposed to obtain improved time-frequency resolution.

2. Time-Frequency Analysis of Non-Stationary Signals

In signal processing, a form of stationarity is employed which is known as weak-sense stationarity, wide-sense stationarity, covariance stationarity, or second-order stationarity. Most often used weak-sense stationarity require the mean and the autocovariance not to vary in time. In contrast, nonstationarity can simply be defined as processes that are not stationary and that have statistical properties that are deterministic functions of time. Demonstrating nonstationarity is more complex than stationarity [12]. The frequency based techniques have been widely used for stationary signal analysis. For non-stationary signals, the time-frequency techniques are used, such as short-time Fourier transform (STFT), wavelet transform (WT) and Wigner-Ville distribution (WVD), etc. Those methods provide alternative solutions for extracting the transient features of the signals. They produce some type of time-frequency representation for a signal.

Fast Fourier implementation of Discrete Fourier Transform (DFT) for a sequence of N complex samples (time) can be viewed as the DFT of each short-time section corresponding to a different shift in the finite-length analysis window $w[n]$ [12, 17]. Additional references can be found in [18, 19].



(a) (b) (c)
(d) (e) (f)

Fig. 1: Various approaches to extract time and frequency information about a signal activity. (a) Time samples of a signal. (b) Frequency components. Time-frequency analysis; (c) high time - low frequency resolution, (d) low time - high frequency resolution. (e) Multi-resolution analysis. (f) Desired high time – high frequency resolution.

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn} \quad (1)$$

where $x[n]$ are the samples of the input signal sampled at intervals of $1/f_s$, and $X[k]$ are the frequency samples obtained at $f = (k/N)f_s/2$ for $k = 0, 1, \ldots, N - 1$. The time representation of a signal lacks frequency information, and similarly, the frequency representation given by (1) shows the frequency components of the signal, but lacks time information as illustrated in Figs. 1(a, b).

Discrete-time short-time Fourier Transform provides both time and frequency information [11],

$$X[n,k] = \sum_{m=-\infty}^{\infty} w[n-m] x[m] e^{-j\frac{2\pi}{N}km}. \quad (2)$$

where $w[n]$ is the window function. The magnitude squared of the STFT yields the spectrogram of $x[n]$.

$$spectrogram\{x[n]\}[n,k] = |X[n,k]|^2 \quad (3)$$

Eqns. (2) and (3) is very helpful for extracting the transient features of a non-stationary signal, i.e. time-frequency band-limited activites could be detected. However, there is an unfortunate trade-off between time and frequency resolutions as illustrated in Figs. 1 (c, d). Generally, better time resolution can be obtained when frequency resolution is low, and better frequency resolutions when time resolution is low. There are various methods to provide the optimum time-frequency resolutions, including wavelet, Wigner-Ville, multitapers and other numerous analysis tools (Fig. 1(e)) [3, 9, 10]. The regorous comparisons of different time-frequency representations are outside the scope of this work. In this work, better resolutions are desired as illustrated in Fig. 1(f).

3. Estimation of the Average and Instantaneous Frequency of a Signal

Treating the time-frequency distribution of a function of $g(t)$ as a probability distribution allows the average frequency at any time to be expressed as [9]

$$f_{aver}(t) = \frac{\int_{-\infty}^{\infty} f\, C_g(t,f)dt}{\int_{-\infty}^{\infty} C_g(t,f)dt} \tag{4}$$

where $C_g(t,f)$ is the time-frequency distribution (TFD) satisfying

$$|g(t)|^2 = \int_{-\infty}^{\infty} C_g(t,f)df \tag{5}$$

$$|G(f)|^2 = \int_{-\infty}^{\infty} C_g(t,f)dt \tag{6}$$

where $|G(f)|^2$ is the power spectral density, and the instantaneous frequency

$$f_i(t) = \frac{1}{2\pi}\frac{\partial\theta(t)}{\partial t} \tag{7}$$

where $\theta(t)$ is the signal phase at time t. These results are accurate when $g(t)$ can be assumed to be a simple amplitude-modulated signal, and do not provide any information about the time variation of the signal. For the general case, Heisenberg-Gabor uncertainty principle states that both time and frequency resolutions cannot be simultaneously improved [11, 12],

$$\Delta t.\Delta f \geq \frac{1}{4\pi}. \tag{8}$$

In the next section, a practical method which provides simultaneous improvements both in time and frequency is proposed.

4. Three Dimensional Time-Frequency Distribution (3D-TFD)

In Section 2 and 3, it is shown that time and frequency resolution can not be improved simultaneously since both requirements bring conflicting constraits. For non-stationary signals designing an optimum kernel for the TFDs could become costly and tricky, especially when signal is the superposition of components with different time-frequency bandwidths. Thus, better methods providing solutions with practical implementations are necessary. An interesting solution to the conflicting time-frequency resolution constraints could be found if we can go back to Heisenberg's original experiment.

It is generally assumed that the Heisenberg-Gabor limit cannot be improved. This limit is correct for single-run real-time experiments where the experiment cannot be repeated under the exact same conditions. For offline conditions however, any number of analysis can be done when analyzing a recorded signal samples iteratively, and exact experiment can be repeated any number of times under different measurement conditions. This critical difference makes it possible to obtain time and frequency information separately, where the accuracy of both results becomes

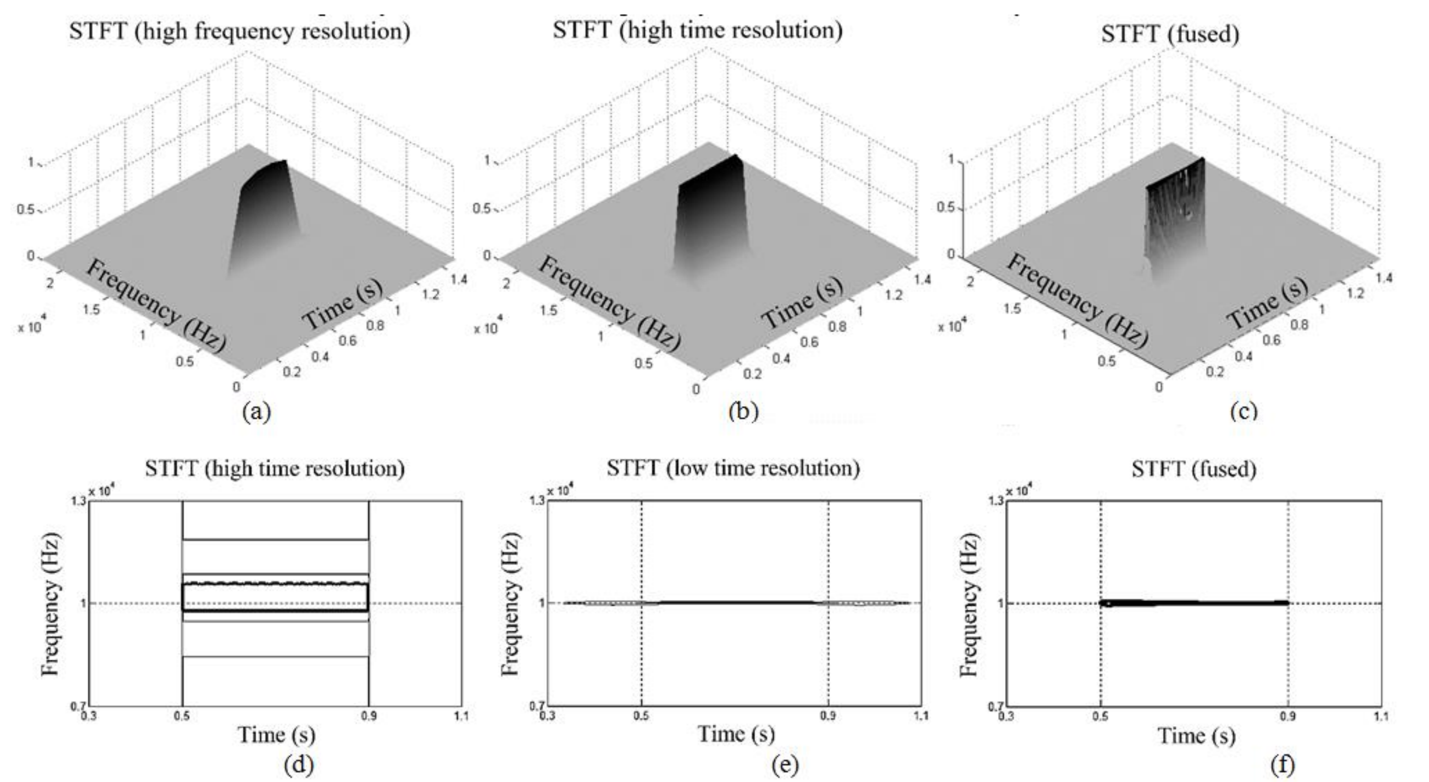


Fig. 2: Short-time Fourier transform (STFT) results where time-frequency resolution compromise is illustrated on a pulse waveform. (a,d) Low time – high frequency resolution, (b,e) high time – low frequency resolution, (c,f) Novel three-dimensional STFT analysis for both high time and high frequency resolution below the Heisenberg-Gabor limit. (Bottom row) Contour plots for the 3D-STFT results where the outermost contour lines corresponding for the smallest value are selected to be 0.005 for better comparison.

independent. All independent experiments provide a multi-dimensional time-frequency information giving a three dimensional representations,

$$X(t,f,\varphi) = \int_{-\infty}^{\infty} w_{\varphi}(t-\sigma)x(\sigma)e^{-j2\pi f\sigma}d\sigma \tag{10}$$

where X is the the three dimensional time-frequency distribution (3D-TFD) of signal $x(t)$, $w_{\varphi}(t)$ is the window function which is nonzero in $-\varphi < t < \varphi$. For digital signal processing applications, the dicrete-time analysis can be based on the dicrete-time short-time Fourier transform (STFT) where the novel three-dimensional discrete-time STFT (3D-STFT) can be defined as

$$X[n,k,p] = \sum_{m=-\infty}^{\infty} w_p[n-m]\, x[m]\, e^{-j\frac{2\pi}{L_p}km} \tag{11}$$

where L_p is the length of the window function, $w_p[n]$, and it is nonzero only for $-p < n < p$, $p = 1, 2, \ldots$, and for $L_p = 2p + 1$. For the FFT applications even lengths of window functions are necessary, and $w_p[n]$ should be selected to be nonzero only in $-p + 1 < n < p$ where $L_p = 2p$. Note that frequency components of the input signal, $x[n]$, are observed at discrete locations given by $f_k = (f_s/p)k$ where f_s is the sampling frequency. Note also that $X[n, k, p]$ provides infinite number of discrete time-frequency representation layers of $x[n]$. 3D-TFD and its one implementation 3D-STFT can be analyzed for gathering more information. This novel approach provides extra degrees of freedom to estimate the locations of signal's energy on the time-frequency plane, accurately. One can examine the behaviour of 3D-STFT around non-stationary signal activities. Finally, three dimensional spectrogram, and periodogram functions can be defined similarly.

Estimation of the time and frequency properties of a single pulse:
Let us define $x(t)$ as

$$x(t) = w(t)\sin(\phi + 2\pi f_0 t) \quad (12)$$

where $w(t)$ is the envelope function which is nonzero only for $t_A < t < t_B$. If the frequency of the signal is to be accurately estimated using the digital samples of $x(t)$, $x[n]$, then largest size of time-window should be selected to fit in $t_A < t < t_B$. This will provide the most accurate frequency information. However, wider time-window degrades the time resolution as shown in Fig. 2(a). According to (8) time resolution can be improved if shorter windows are selected, thus degrading the frequency resolution as shown in Fig. 2(b) as predicted in (8).

A metric G can be defined to localize the presence of signal components accurately which are band-limited both in frequency,

$$G([n,k]) = F\{|\nabla(X[n,k,p])|\} \quad (13)$$

where F is some function monitoring the variations of the 3D-FFT as a function of p, $|\quad|$ is magnitude operator,

$$\nabla(X[n,k,p]) = \left\{\hat{a}_n \frac{\partial}{\partial n} + \hat{a}_k \frac{\partial}{\partial k}\right\} X[n,k,p], \quad (14)$$

where ∇ is the gradient operator, $\hat{a}_n$ and $\hat{a}_k$ are the unit vectors in the direction of time and frequency axis, respectively, and where $\partial/\partial n$, and $\partial/\partial k$ are the corresponding partial derivative operators. Note that (11) provides different STFT results for each p which accurate information about the time-frequency concentrations can be obtained. These 3D-STFT results are expected to change considerably around the edges of time-frequency band-limited signals where those changes are monitored by some monitoring function F.

In this work, a heuristic solution is proposed for the practical implementation of (13), and a more detailed discussion is planned as a future work. The variations of the 3D-STFT can be examined at two planes defined by p = 64, and p = 8.192 (= 128 x 64) which are illustrated in Fig. 2(a, b). By comparing both STFT results, it is observed that when accuracy degrades in any direction, signal energy smooshes around the edge. The result for a a simple proof-of-concept approach is shown in Fig. 2(c). The two results of Fig. 2(a) and (b) are compared and the smaller one is selected where the gradient value around an edge is expected to be larger which shows the rapid drop of energy concentration. It is shown that only two layers of p is sufficient to improve the time-frequency resolution. This shows that multi-layers of the 3D-STFT can be fused to challenge the Heisenberg-Gabor limit. Thus, the novel method provides improvement of resolution in time and frequency simultaneously. Proof-of-concept 2D-STFT implementation is illustrated using contour plots in Figs. 2(d-f). The contour lines are set for values (0.005, 0.5, 0.75, 0.99). It is observed that the input signal parameters; center frequency, start and end times are estimated very accurately to be 10 KHz, 0.5 s. and 0.9 s. respectively as shown in Fig. 2(f). This illustrates that the proof-of-concept works, and both time and frequency resolutions can be improved simultaneously.

4. Conclusion

The problem of resolution optimization for time-frequency analysis is examined. The conflicting time and frequency resolution constraints are studied. Heisenberg-Capon resolution limit is illustrated on a simple example. A novel time-frequency representation; the three-dimensional time-frequency distribution (3D-TFD) is proposed. Its discrete time version, three-dimensional short-time Fourier transform (3D-STFT) is also proposed. A practical proof-of-concept method based on a metric utilizing the gradient of the signal as a function of time and frequency is developed. It is observed that simultaneous improvement of both time and frequency resolutions is possible. Inial results suggest that super resolution time-frequency analysis is possible.

Misaligned layers of 3D-STFT are observed to introduce some error at the fusion step which is described in (13). The perfect overlapping of the multi-layers of 3D-STFT is observed to require some attention since different window lengths correspond to different time shifts. Numerically higher order implementations of the gradient function is expected to improve accuracy even more. Finally, the performance of the novel method is planned to be tested in noisy environments in the future. More detailed derivations of 3D-TFD, 3D-STFT and numerical implementations of the metric G are planned as a future work.

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