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Linear Control Design for a Plastic Extruder

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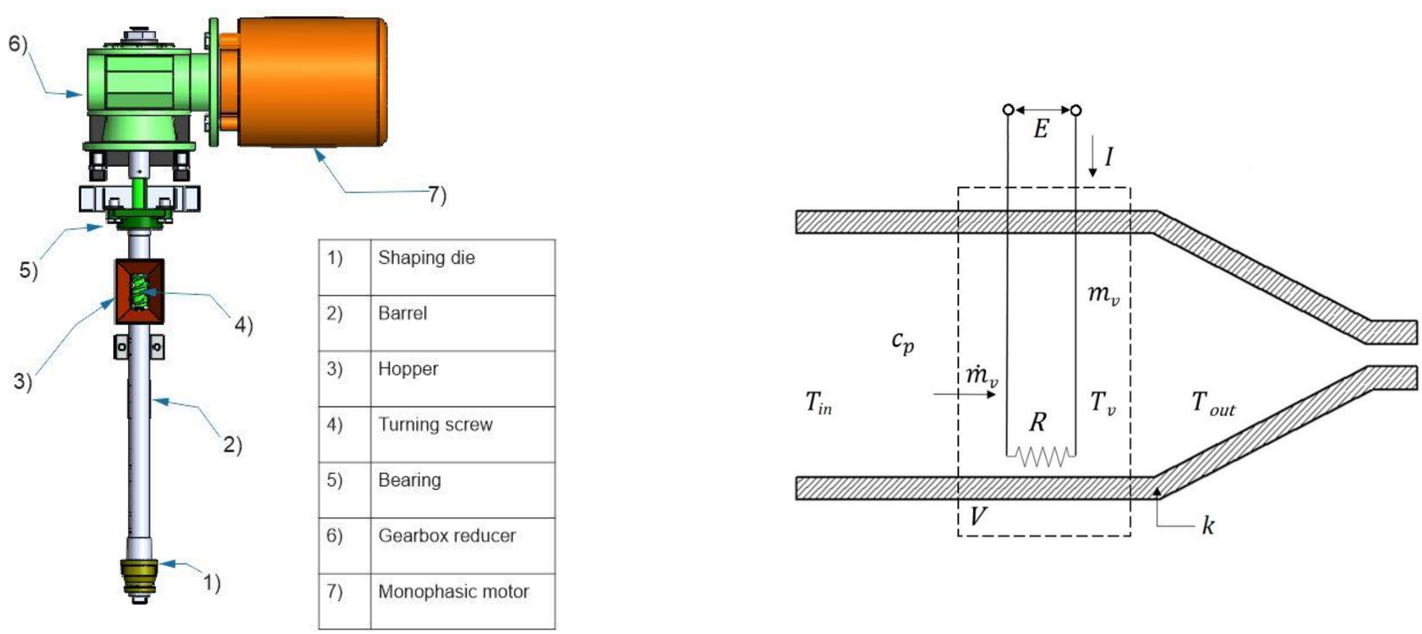
***Abstract* -** In this paper, we describe the process of design, simulation and measuring of a PID control used on a plastic extruder built up to work with polypropylene pellets. Even though there are standard values given by controller makers, we have used pole placement techniques based on a model obtained for the extruder with specific values that characterize it. The resulted constants are then introduced in a commercial device in order to regulate temperature to fuse the plastic while flowing through the output. Such value of temperature is measured to feedback the control loop, but it is also recorded in order to compare it with the values calculated by simulation. Both sets of data show a good match for the setpoint, confirming that our model as well as our PID design are in agreement with those real devices and processes that are being emulated.

***Keywords*:** Thermoplastic Production, Thermal Model, PID Control.

1. Introduction

Many manufactured products are made with plastic and a large amount of them are produced with thermoplastics [1]. The specific material is fed to the manufacturing process in the form of pellets, granules, flakes or powders from a hopper into a barrel of the extruder machine. Mechanical motion and heat exchange (see Fig. 1a) melt this material while it is transported towards a die, where it is shaped and then it is left to cool [2].



(a) (b)

Fig. 1: Plastic extruder.

One of the main variables to watch for a proper quality and control of the process is temperature. Too low or too high values of such a variable can affect negatively the final product or the process itself [3]. In a more punctual form, temperature in a portion near of the exit from the barrel can fluctuate due to the environment in the outside and the temperature at what the material is being entered [1, 2, 4].

A correct model of the phenomenon at this region of the extruder is important not only to understand what is happening, but because it can lead to the design of accurate controls which regulate and optimize the energy needed to heat and melt the plastic [5, 6, 7]. This implies to know the physics of the thermal exchange, as well as the values of the parameters involved. A proper control design is then easily developed once that information is achieved [8]. Such a design not only permits that the correct temperature is reached, but it avoids unnecessary transient fluctuations, undesirable long preheat times and it can be capable to diminish the impact of possible disturbances.

This paper is ordered as follows. In the next section, the model of the extreme of the extruder where polypropylene (PP) plastic is poured out is presented, together with those values of the parameters needed to calibrate it. Section 3 is devoted to the design of the PID controller, a pole placement technique in this case. Section 4 shows some simulations of the expected behaviour of the extruder being controlled by our PID controller, and Section 5 shows some experimental results with the real extruder working with a commercial PID device. Simulated and measured data are then compared, with a very good matching, confirming that our model is precise enough and that our control design works satisfactorily. At the end, a set of conclusions are stated.

2. Extruder Model

2.1. Extruder Thermal Scheme

The span of this communication is focused on the thermal behaviour of the plastic at the extreme of the extruder where this material is heated and flows out. Fig. 1b describes the boarders of the system under interest, as well as the variables to consider. It is noticeable that the main heat source is an electric resistor. Table 1 enumerates and describes all those quantities.

Table 1: List of variables and parameters of the thermodynamic system.

V:	Control volume	m_v:	Mass of the plastic material (inside V)
A_v:	Perimeter area of the wall of V	c_p:	Specific thermal capacity of the polypropylene
T_v:	Temperature in the control volume	k:	Heat transfer coefficient of the walls
T_{env}:	Temperature of the environment	E:	Voltage of the electric resistor
T_{in}:	Temperature of the entering material	R:	Resistance of the electric resistor
T_{out}:	Temperature of the coming-out material	I:	Current flowing through the electric resistor

2.2. Extruder Thermodynamic Model

Thermal energy balance is classically described [9] as in Eq. (1)

$$\begin{matrix}\text{Accumulated}\\ \text{Heat}\end{matrix} = \begin{matrix}\text{Input}\\ \text{Heat}\end{matrix} - \begin{matrix}\text{Output}\\ \text{Heat}\end{matrix} + \begin{matrix}\text{Generated}\\ \text{Heat}\end{matrix} \tag{1}$$

Each one of the terms included in (1) is detailed as in Eqs. (2)

$$\begin{aligned} \text{Accumulated Heat:} \quad & Q_v = m_v c_p \dot{T}_v \\ \text{Input Heat:} \quad & Q_e = \dot{m}_v c_p (T_{in} - T_{out}) \\ \text{Output Heat:} \quad & Q_s = k A_v (T_{out} - T_{env}) \\ \text{Generated Heat:} \quad & Q_g = E^2/R \end{aligned} \tag{2}$$

Through substitution of Eqs. (2) into Eq. (1) it is possible to obtain

$$m_v c_p \frac{dT_v}{dt} = \dot{m}_v c_p (T_{in} - T_{out}) - kA_v (T_{out} - T_{env}) + \frac{E^2}{R} \tag{3}$$

Where mass flow rate $\dot{m}_v$ can be considered as a constant M_v, implying that Eq. (3) modifies as in Eq. (4)

$$\frac{dT_v}{dt} = \frac{M_v}{m_v}(T_{in} - T_{out}) - \frac{kA_v}{m_v c_p}(T_{out} - T_{env}) + \frac{1}{m_v c_p} W \tag{4}$$

With W as the electric power consumed in the electric resistor.
Consider that $T_{out} \approx T_v$, then Eq. (4) can be rewritten and rearranged as in Eq. (5)

$$\frac{dT_v}{dt} = -\left(\frac{M_v c_p + kA_v}{m_v c_p}\right) T_v + \left(\frac{M_v}{m_v} T_{in} + \frac{kA_v}{m_v c_p} T_{env}\right) + \frac{1}{m_v c_p} W \tag{5}$$

The second term in the right side of Eq. (5) and identified in Eq. (6) will be considered as a disturbance to this system.

$$\dot{T}_p = \frac{M_v}{m_v} T_{in} + \frac{kA_v}{m_v c_p} T_{env} \tag{6}$$

Fig. 2 is a block diagram representing the system in Eq. (5).

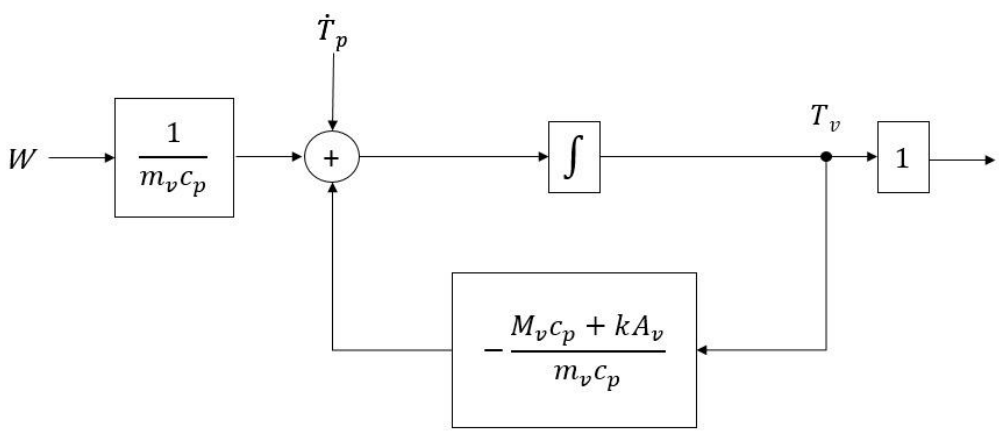


Fig. 2: Block diagram representing the thermal system.

3. Control Design

3.1. PID Control

By examining Eq. (5), this extruder's model has only one pole

$$s = -\frac{M_v c_p + kA_v}{m_v c_p} \tag{7}$$

Taking this as a reference pole, a PID design is proposed.
Let a pole in the origin be placed, next a very near zero, which it is a tenth of the pole shown by Eq. (7).

$$G_{PI} = \frac{s + 0.1\,\frac{M_v c_p + kA_v}{m_v c_p}}{s} \tag{8}$$

With this, a PI scheme is achieved.

To conveniently accelerate the response of this regulator, an additional zero is placed in a location that has a multiplied value of the pole indicated by Eq. (7).

$$G_{PI} = \left(\frac{s + 0.1\,\frac{M_v c_p + kA_v}{m_v c_p}}{s}\right)\left(s + K\,\frac{M_v c_p + kA_v}{m_v c_p}\right) \tag{9}$$

With this, a complete PID scheme is obtained. Eq. (8) shows the resulting control. By proper algebraic manipulation, Eq. (9) can be better expressed as in Eq. (10).

$$G_{PI} = (K + 0.1)\,\frac{M_v c_p + kA_v}{m_v c_p} + \left(\frac{M_v c_p + kA_v}{m_v c_p}\right)^2 \frac{1}{s} + s \tag{10}$$

Where it is possible to identify the constants of the PID control

$$K_P = (K + 0.1)\,\frac{M_v c_p + kA_v}{m_v c_p}$$

$$K_I = \left(\frac{M_v c_p + kA_v}{m_v c_p}\right)^2 \tag{11}$$

$$K_D = 1$$

Fig. 3 depicts a block diagram representing this control scheme

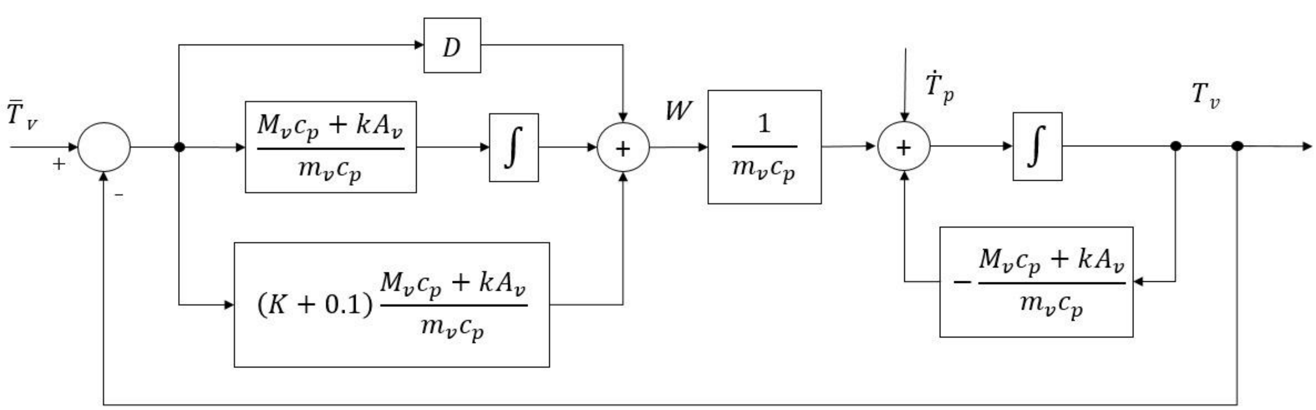


Fig. 3: Block diagram of the proposed PID.

3.2. Analysis of the Steady State Error

Tolerance to disturbances is one of the main advantages of an output feedback control like a PID regulator. It has been mentioned that a term like that identified in Eq. (6) is considered a disturbance input to the model represented by Eq. (5).

Input terms related to power received by the system and those related to disturbances are represented in Eq. (12)

$$\frac{1}{m_v c_p}W + \dot{T}_p = \frac{1}{m_v c_p}W + \frac{M_v}{m_v}T_{in} + \frac{kA_v}{m_v c_p}T_{env} = \\ = \frac{1}{m_v c_p}\left(W + M_v c_p T_{in} + kA_v T_{env}\right) \tag{12}$$

In this case, the block diagram of the whole control system has the form shown in Fig. 4, where it is possible to identify

$$T_v = [eG_c + d]G_s = eG_cG_s + dG_s \tag{13}$$

Besides

$$e = \bar{T}_v - T_v \tag{14}$$

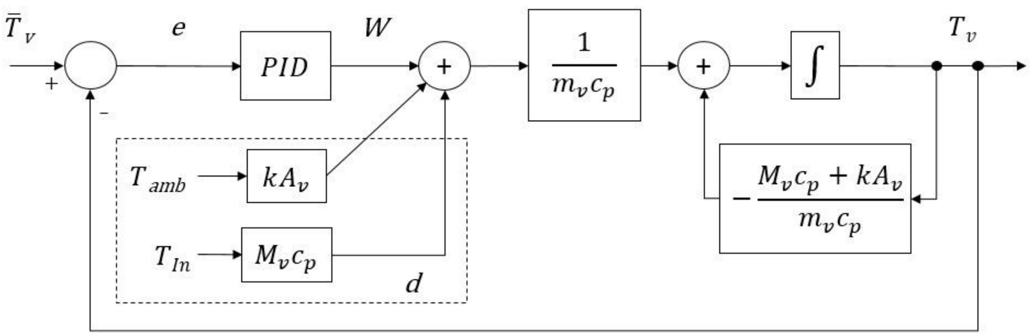


Fig. 4: Block diagram of the control system.

Then, combining Eqs. (13) and (14)

$$T_v = \bar{T}_v - e \; = \; eG_cG_s + dG_s \tag{15}$$

Therefore

$$e = \frac{1}{1 + G_cG_s}\bar{T}_v \; - \; \frac{G_s}{1 + G_cG_s}d \tag{16}$$

The first term in the right side of Eq. (16) relates $\bar{T}_v$ with the error signal e, while the second term relates that signal with the disturbance d.

Applying the final value theorem to Eq. (16)

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} se(s) = \\ = \lim_{s\to 0}\frac{s}{1 + G_c(s)G_s(s)}\bar{T}_v(s) \; - \; \lim_{s\to 0}\frac{s(s)}{1 + G_c(s)G_s(s)}d(s) \tag{17}$$

Quantity $d(s)$ contains input temperature T_{in} and environment temperature T_{env}. If it is considered that both of change very slowly with respect to the dynamics of the system, then $d(s)$ can be considered as a constant quantity. In the complex variable domain, this quantity can be modelled as a step function C/s with an amplitude C.

Then, for the term containing $d(s)$ in Eq. (17) it is possible to write

$$\begin{aligned} \lim_{t\to\infty} e_d(t) &= \lim_{s\to 0} \frac{sG_s(s)}{1+G_c(s)G_s(s)} \frac{C}{s} \frac{\frac{1}{G_s(s)}}{\frac{1}{G_s(s)}} = \\ &= \lim_{s\to 0} \frac{C}{\frac{1}{G_s(s)} + G_c(s)} = \\ &= \frac{C}{\lim\limits_{s\to 0} \frac{1}{G_s(s)} + \lim\limits_{s\to 0} G_c(s)} = \end{aligned} \tag{18}$$

From Eq. (18) it is possible to deduce that the error due to disturbances can be reduced if the system gain $G_s(s)$ is reduced or if the control gain $G_c(s)$ increases. For the proposed PID

$$\lim_{s\to 0} G_c(s) = \lim_{s\to 0} \frac{1}{s} \left[s^2 + (K+0.1)\frac{M_v c_p + kA_v}{m_v c_p} s + \left(\frac{M_v c_p + kA_v}{m_v c_p}\right)^2 \right] = \tag{19}$$

That is to say:

$$\lim_{s\to 0} G_c(s) = \lim_{s\to 0} \left[s + (K+0.1)\frac{M_v c_p + kA_v}{m_v c_p} + \left(\frac{M_v c_p + kA_v}{m_v c_p}\right)^2 \frac{1}{s} \right] \to \infty \tag{20}$$

Then, with such a PID's gain, this control scheme will be able to reduce to zero the effect of disturbances.

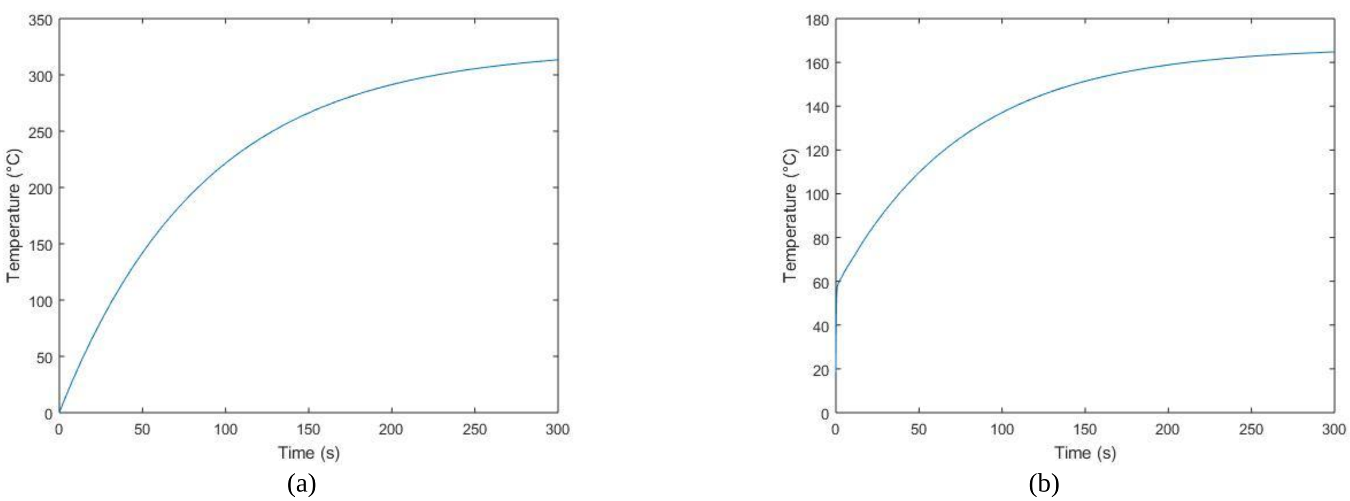


Fig. 5: Temperature behaviour of the plastic in the extruder.

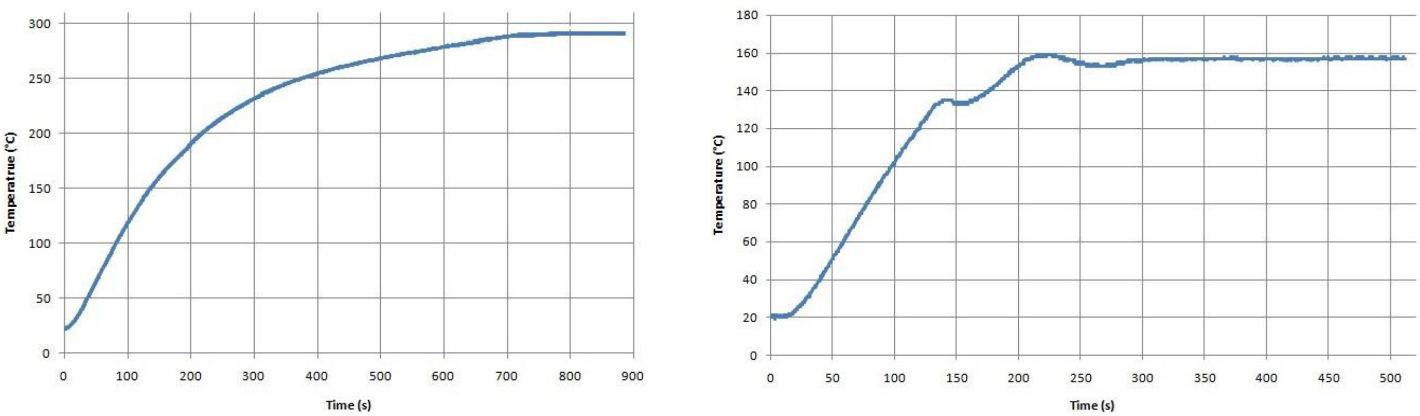
4. Simulations

The extruder model has been simulated by using Simulink™ software. As previously mentioned, a first order model was obtained as for Eqs. (5) and (6), or the block diagram in Fig. 2. A typical plot of such a system is depicted in Fig. 5a, where that simulation shows a growing exponential function reaching a temperature of more than 300°C for the first 5 minutes.

When the model of the extruder is affected by the PID controller, Eq. (10) or the block diagram in Fig, 3, a plot like that in Fig. 5b is obtained, where the transient during the first seconds goes very fast from the environment temperature, but then turns in a more slowly trajectory, achieving a stabilization at a temperature a little over 160°C, where the melting temperature of the propylene is located. These results seem to lead to acceptable real performances.

5. Experimental Results

A set of experiments were conducted over the real system. Temperature of the polypropylene into the corresponding control volume in the extreme of the extruder was measured by means of a thermocouple, which was wired to a data acquisition card. Leaving without any control the rise of the temperature due to the resistor gives a behaviour in such a variable as depicted in Fig. 6a. It can be seen that the melting point of the polypropylene (PP) is surpassed [10]. In fact, Fig. 6a shows almost a temperature of 300°C.



(a) (b)

Fig. 6: Temperature behaviour of the plastic in the extruder.

The values of the parameters K_P, K_I and K_D were entered to a PID controller electronic card which was then connected to the resistor to regulate it. Fig. 6b shows the new behaviour for such a device. It is noticeable that the temperature does not rise to extreme values as in the case of the system with no control. In fact, it achieves $\bar{T}_v = 160°C$ as expected, which is the nominal melting value of this plastic. This value is reached in approximately 5 minutes and then it is stabilized, without any hazardous condition to the material or the operator.

When Fig. 6 is compared to Fig. 5, it is evident that trajectories in each one of such figures have observable differences, like overshoots or oscillations of the real measured quantities that are not present on those results from the simulations. This is an indication that the thermal model that should represent to the extruder would be of a second o higher order instead of one of first order. Besides, it is necessary to mention that there are important simplifications and assumptions which affect directly the model in its conception, as those concerning the type of transfer heat among the present elements and the heat leaks, due to the joint among the heaters and the surfaces, for example, or the geometry of the pieces of the extruder where the polypropylene is being fused.

However, even though those inaccuracies are very important and should be taken into account in an improved model, there are important advantages in this first approach, and many similarities for both sets of acquired values, simulated and measured, achieve a successful match and a good approach to the real practical control system.

6. Conclusion

A thermodynamic model of a real extruder is presented in this paper. By proper physics knowledge about the involved variables and laws a first order model has been obtained. Such a model has then used to design a PID controller by means of fundamental control theory.

As an additional but important result, a steady-state error analysis is included, which sows that the PID is not only capable of take over the task to regulate the temperature of the polypropylene, but it is also capable to diminish those input disturbances that are present in the environment.

Some simulations have been then performed for the thermodynamic model and for the PID control system, giving reasonable and expected results.

Lastly, a set of experiments were conducted, for the extruder alone, without any control, and then connecting it to a PID electronic card. The results have been very similar to those obtained in simulations, even though some dissimilarities are evident. It is noticeable that those differences among the behaviour of the simulated calculations and the real quantities are mainly due to the order of the model. By the observed features in the behaviour of the measured data, it is expected that a more precise model would be one of second or higher order. This is a motivation to look for a better and more sophisticated model which can lead to a more precise control design.

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