Proceedings of the 5th International Conference of Control, Dynamic Systems, and Robotics (CDSR'18)
Niagara Falls, Canada – June 7 – 9, 2018
Paper No. 139
DOI: 10.11159/cdsr18.139

Robust Control of Robot Manipulators Using Difference Equations as Universal Approximator

Morteza Rezazadeh Mehrjou
Department of Azad University of Damghan
Semnan Province, Damghan, Iran
mortezamehrjo56@gmail.com

Abstract - This paper presents a simple and robust non inversion-based perfect tracking control strategy for robot manipulators. The proposed approach is capable to eliminate the environmental problems arising from classic feedforward control design and so guarantees an appropriate level of robustness of control system to uncertainties including external disturbances, un- modeled dynamics, and parametric uncertainty. Extensive simulation results performed using a two degree-of-freedom actuated elbow robot prove the effectiveness of the proposed approach. Using free model of system in control law design is a considerable point in the field of robot manipulator control.

Keywords: Model Free, Robust Control, Robot Manipulator.

1. Introduction

Conventional model-based feed forward control (FFC) fails to produce good trajectory tracking performance, in presence of uncertainties such as system parameter variations, external disturbance, friction force and unmodeled dynamics. Some of inherent weaknesses of this approach have been mentioned in reference [1]. More ever, application of this control methodology as a standard approach for linear systems has made it unsuitable for nonlinear systems. The main reason for this sensitivity refers to necessity for solving partial differential equations for obtaining the feed forward path signal [2]. On the other hand, using of this approach for digital control systems encounters with a few difficulties. Because discretization process by zero-order holds usually leads to a discrete-time system with at least one notorious unstable zero in out of unit circle [3], thus the feed forward branch becomes unstable and consequently, its realization will become impossible. Existence of this zero, also leads to many other lateral problems. As a sample we can mention significant phase errors over a broad range of frequencies, which cause problems in adaptive controller design [4]. Therefore, many attempts have been made by researches to overcome on the mentioned problems over the last decade [3-15]. However, all of these studies are based-model and need solving the complicated equations. This work attempts to address a unified digital feed forward control scheme for a 2 degree of freedom robotic manipulator using linear state feedback and without needing any additional control method. An analytical consideration for tracking problem is presented including free model of plant for controller design. The paper is organized as follows: The motion equations of the system and model-free digital control design are constructed in section 2. Stability analysis and simulation results are presented in section 3 and 4 respectively and conclusions are drawn in section 5.

2. Model-Free Digital Control

Consider motion equations of an integrated actuator-robot system described in the joint space as below [16]

$$\bar{D}(q)\ddot{q}+\bar{C}(q,\dot{q})\dot{q}+\bar{G}(q)=\alpha \mathrm{u} \tag{1}$$

where, q is the n×1 vector of generalized joint coordinates, $\bar{D}(q)$ is the inertia matrix, $\bar{C}(q,\dot{q})\dot{q}$ is the vector of centripetal and Coriolios terms, $\bar{G}(q)$ is the vector of gravitational torques, α is a constant matrix and u is the control input vector. In addition, $\bar{D}(q)$ and α are nonsingular matrices. It can easily be shown that, by introducing appropriate state variables and simple manipulations Eq. 1, a non-linear, time-variant, continuous-time control system, can be described by the following model

$$\dot{x} = \mathrm{A}_x x + \mathrm{B}_x \mathrm{u} \tag{2}$$

$$\mathrm{y} = C\,x \tag{3}$$

where

$$\boldsymbol{x} = \begin{bmatrix} q \\ \dot{q} \end{bmatrix},\ \mathrm{A}_x = \begin{bmatrix} 0 & I \\ A_{x,0} & A_{x,1} \end{bmatrix},\ \mathrm{B}_x = \begin{bmatrix} 0 \\ B_{x,0} \end{bmatrix},\ C = \begin{bmatrix} I & 0 \end{bmatrix} \tag{4}$$

where x is a $2\mathrm{n} \times 1$ state vector, **y** is the $n\times 1$ output vector, and 0 and I are the $n\times n$ zero and identity matrices, respectively. Also, $\mathrm{A}_{x,0}$, $\mathrm{A}_{x,1}$ and $\mathrm{B}_{x,0}$ are defined as follows:

$$\begin{aligned} \mathrm{A}_{x,0}.x &= -D^{-1}(q)G(q) \\ \mathrm{A}_{x,1} &= -D^{-1}(q)C(q,\dot{q}) \\ \mathrm{B}_{x,0} &= D^{-1}(q)\alpha \end{aligned} \tag{5}$$

Easily can be shown that, Eq. 2 can be rewritten as

$$\dot{x} = \mathrm{A}x + \mathrm{B}\mathrm{u} + \delta \tag{6}$$

where δ is the vector of uncertainties including external disturbance, and unmodeled dynamics, and A, B and C matrices are given by

$$A = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix},\quad B = \begin{bmatrix} 0 \\ I \end{bmatrix} \tag{7}$$

Discritization process by zero order hold leads to a discrete-time form of Eq. 3 and Eq. 6 as below

$$x(k+1) = Gx(k) + Hu(k) + \wp(k) \tag{8}$$

$$y(k) = Cx(k) \tag{9}$$

Now, we design a linear control law of the form

$$u(k) = -kx(k) + k_0 r(k) \tag{10}$$

where k and k_0 are constant. It must be noted that $r(k)$ is the robustifying control input, such that it leads to minimization of the tracking error. By substituting Eq. 10 into Eq. 8 we will have:

$$x(k+1) = \left(G - Hk\right)x(k) + Hk_0 r(k) + \wp(k) \tag{11}$$

Now we develop an algorithm to adjust $r(k)$. Toward this end, suppose that the desired closed loop state equations are given by:

$$x_d(k+1) = \left(G - Hk\right)x_d(k) + Hk_0 r_d(k) \tag{12}$$

$$y_{\mathrm{d}}(k) = Cx_{\mathrm{d}}(k) \tag{13}$$

where $r_d(k)$ and $y_{\mathrm{d}}(k)$ are the desired trajectory and desired output in joint space, respectively. It must be noted that, the coefficient vector k is designed so that, $y_{\mathrm{d}}(k)$ agreeably follows $r_d(k)$. For continuation of this subject, let us introduce the following transformation:

$$\xi(k) \triangleq y(k) - y_{\mathrm{d}}(k) \tag{14}$$

$$e(k) \triangleq x(k) - x_d(k) \tag{15}$$

$$\mathrm{v}(k) \triangleq r(k) - r_d(k) \tag{16}$$

By these assumptions, Eq. 11, 12 in new coordinates becomes

$$e(k+1) = \left(G - Hk\right)e(k) + Hk_0\mathrm{v}(k) + \wp(k) \tag{17}$$

$$\xi(k) = \mathrm{C}e(k) \tag{18}$$

Now, by considering linear system properties, we arranged a difference equation as follows

$$\begin{aligned} \Psi(k+p+1) &= \left(G - Hk\right)\Psi(k+p) + Hk_0\Lambda(k+p) \\ &\quad + \left\{ \wp(k+p) - \sum_{j=1}^{p} b_j \wp(k+p-j) \right\} \end{aligned} \tag{19}$$

Where

$$\begin{aligned} \Psi(k) &= e(k) - \sum_{j=1}^{p} b_j e(k-j) \\ \Lambda(k) &= \mathrm{v}(k) - \sum_{j=1}^{p} b_j \mathrm{v}(k-j) \end{aligned} \tag{20}$$

Here, if we assume, $\wp(k)$ can be modeled by a p-order difference equation as below, where order p reflects the dynamic structure of $\wp(k)$, we will have

$$\wp(k+p) = \sum_{j=1}^{p} b_j \wp(k+p-j) \tag{21}$$

The continuous-time form of this assumption has been addressed in [17]. In the next step, we define a digital control law as follows

$$\Lambda(k+p) = -\sum_{\mathrm{j}=1}^{\mathrm{p}} \mu_{\mathrm{j}} \zeta(k+p-j) - \mu_0 \Psi(k+p) \tag{22}$$

Substituting the last equation in Eq. 19 we obtain

$$
\begin{aligned}
\Psi(k+p+1) = &\left(G - Hk - Hk_0\mu_0\right)\Psi(k+p) \\
&- Hk_0 C \sum_{j=1}^{p} \mu_j e(k+p-j)
\end{aligned} \tag{23}
$$

whereas $\Lambda(k)$, $\zeta(k)$ are functions of tracking error $e(k)$, therefore by proper selection of closed-loop system poles we can guarantee that $e(k)$ converges to zero asymptotically. Finally, we adjust r(k) in Eq. 11, from Eq. 16.

Therefore, we used a 2-stage approach for digital feed forward control design. First, we designed an inner state feedback control for tracking of reference input $r(k)$ by output $y(k)$ (based on desired state design). Then we utilized an outer state feedback control to suppress effects of uncertainties. It is useful to note that, good tracking accuracy can be achieved with order p=1 or 2 [19]. The block diagram of the proposed scheme is depicted in Fig. 1.

It is its turn now that, we show the proposed approach above, is cancelling the disturbances in a feed forward combination and does not need any lateral control scheme. In other word, by completing the proposed approach instead of classic feedforward form, we will have more tranquillity. Toward this end, the z transform of equation (11) is obtained as follows

$$
X(z) = \left(ZI - G + Hk\right)^{-1}\left(Z\mathrm{x}(0) + H\,k_0 r(z) + \wp(z)\right) \tag{24}
$$

where $X(z)$ is z transform of $x(k)$. Furthermore the z transform of equation (12) is defined as

$$
H\,k_0 r_d(z) = \left(ZI - G + Hk\right)X_d(z) - Z\mathbf{x}_d(0) \tag{25}
$$

With multiplication extremes of equation (16) in $H\,k_0$ we will have:

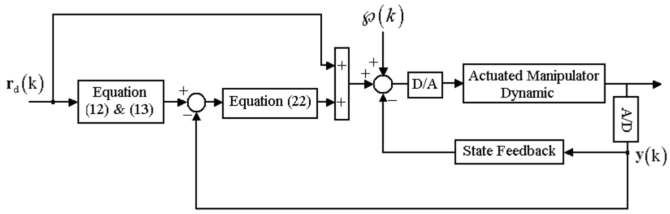

Fig. 1: Model free digital feed forward control scheme.

$$
H\,k_0 r(z) = H\,k_0 \mathrm{v}(z) + H\,k_0 r_d(z) \tag{26}
$$

Also, z transform of equation (22) and (20) under initial condition e(0)=0 is given by

$$
\begin{aligned}
\left(z^p - \sum_{j=1}^{p} b_j z^{p-j}\right) v(z) = &- C\sum_{j=1}^{p} \mu_j z^{p-j} e(z) \\
&- \mu_0\left(z^p - \sum_{j=1}^{p} b_j z^{p-j}\right) e(z) + \Xi\left(v(z), e(z)\right)
\end{aligned} \tag{27}
$$

where $e(z)$ and $v(z)$ are z transform of $e(k)$ and $v(k)$, respectively. Hence

$$v(z)=\frac{-\left(C\sum_{j=1}^{p}\mu_j z^{p-j}+\mu_0\left(z^p-\sum_{j=1}^{p}b_j z^{p-j}\right)\right)}{\left(z^p-\sum_{j=1}^{p}b_j z^{p-j}\right)}e(z)+\frac{\Xi\left(v(k),e(k)\right)}{\left(z^p-\sum_{j=1}^{p}b_j z^{p-j}\right)} \tag{28}$$

where $e(z)$ is defined as:

$$e(z)=X(z)-X_{\mathrm{d}}(z) \tag{29}$$

In addition, equation (28) can be rewritten in the form

$$v(z)=-\Upsilon(z)e(z)+\Phi(k,z) \tag{30}$$

where

$$\Upsilon(z)=\frac{\left(C\sum_{j=1}^{p}\mu_j z^{p-j}+\mu_0\left(z^p-\sum_{j=1}^{p}b_j z^{p-j}\right)\right)}{\left(z^p-\sum_{j=1}^{p}b_j z^{p-j}\right)}$$
$$\Phi\left(k,z\right)=\frac{\Xi\left(v(k),e(k)\right)}{\left(z^p-\sum_{j=1}^{p}b_j z^{p-j}\right)} \tag{31}$$

Multiplication extremes of equation (30) in HK_0 yields

$$H\,k_0\mathrm{v}(z)=-H\,k_0\Upsilon(z)e(z)+H\,k_0\Phi(k,z) \tag{32}$$

Finally, using Eq. 25, Eq. 26 and Eq. 32, the feed forward scheme becomes complete. In proposed approach, the feed forward branch is inversion of controlled process by state feedback theory, (*ZI-G+HK*). The equivalent block diagram of the proposed scheme is shown in Fig 2.

3. Stability Analysis

Here we will show the proposed approach above leads to a stable scheme in presence of disturbances. Toward this end, Substituting Eq. 25, Eq. 26, Eq. 32 into Eq. 24 leads to:

$$e(z)=(ZI-G+H\,k+H\,k_0\Upsilon(z))^{-1}(\wp(z)+H\,k_0\Phi(k,z)) \tag{33}$$

By proper selection of eigenvalues for outer control loop, $H\,k_0\Phi(k,z)$ term for rejecting of uncertainties, final theorem, and also notification of this point of view that, good tracking accuracy can be achieved with low uncertainty model error (p=1 or 2), thus the proposed approach is stable and tracking error tends to zero asymptotically[18].

$$e_{ss}(t=\infty)=\lim_{z\to 1}(1-Z^{-1}).e(z) \tag{34}$$

4. Simulation Results

In order to demonstrate usefulness of the proposed controller, we used a 2-link elbow robot manipulator for digital simulation, under 10 ms sampling simulation time. The major steps of the proposed algorithm can be summarized as bellow:

- *The desired trajectory is specified as follows*

$$\theta=-a\cos(\frac{\pi}{T}t)+a,\quad \mathrm{t}\geq 0 \tag{35}$$

where we set $a=0.5rad$, and $T=2\sec$

- Calculating the state feedback vector k as Table-1
- Modeling of uncertainty by a pth-order difference equation, set the uncertainty equation to zero and finally obtain bj. In this step, if we choose p=1 for the uncertainty, we will have

$$\wp(k+1)=b_1\wp(k)=0 \qquad \mathrm{k}\geq 0 \tag{36}$$

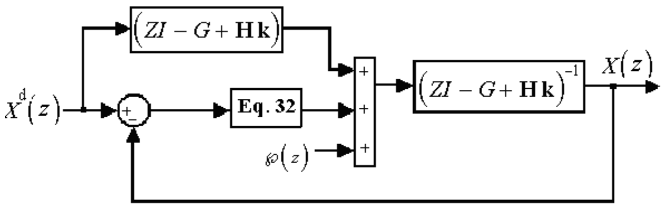

Fig. 2: Equivalent form of Model free digital feed forward control scheme.

Table 1: Gains of the Controllers.

Joint	*k*
1, 2	[22100 189.5]

By these assumptions, b_1 is set to zero and consequently, $\wp(k)$ is obtained as an arbitrary constant at the time zero.

- *Calculation of state feedback vector μ as Table-2*

In the simulation, we set the masses and lengths of link 1, 2 as m_1 = 17.4kg, m_2 = 4.8kg, l_1 = 0.4318m, l_2 =0.4318m, respectively. Also, the true actuator dynamic coefficients are defined as: R=1.086Ω, L=.01216H, k_m=0.189, k_{b1}=0.189, b_m=0.02, j_m=0.05 and r=0.02. Based on aforementioned expressions, Fig. 3 depicts tracking error of all joints for assumed model-free system with Eq. 12. In this manner the motors voltage obtained as Fig. 4. To show the ability of this approach in presence of external disturbances (external load torques on the motors shaft) and model uncertainties, we obtained the technical limits such as, torque limit, tracking error , voltage limit and control signal as Fig. 5 to Fig. 8, respectively. As can be seen, tracking error, shown by Fig. 6, is bounded and so the proposed approach leads to asymptotic stability. Simulation results show that the robot can be effectively controlled and robustified subject to uncertainties based on using a free model of plant as seen in Fig.9.

Table 2: Gains of the Controllers.

Joint	μ
1,2	[0.0065 0.0148 0.0003]

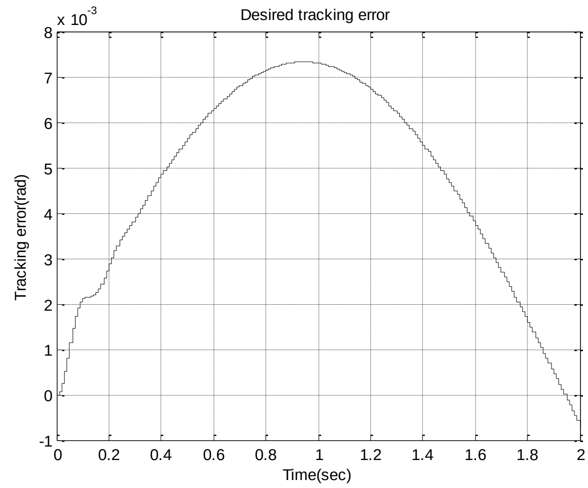

Fig. 3: Tracking error.

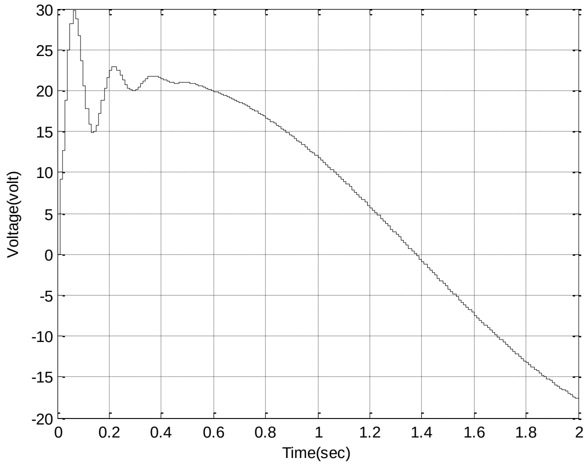

Fig. 4: Voltages of motors.

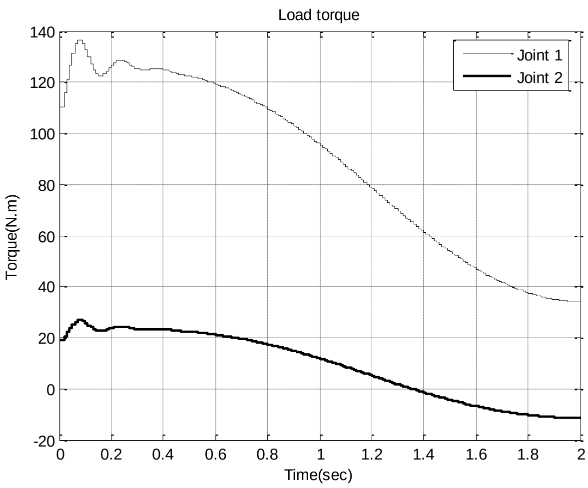

Fig. 5: load torques.

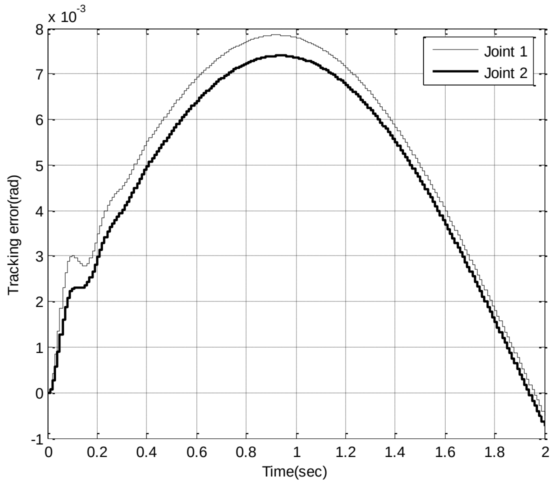

Fig. 6: Tracking error subject to disturbances.

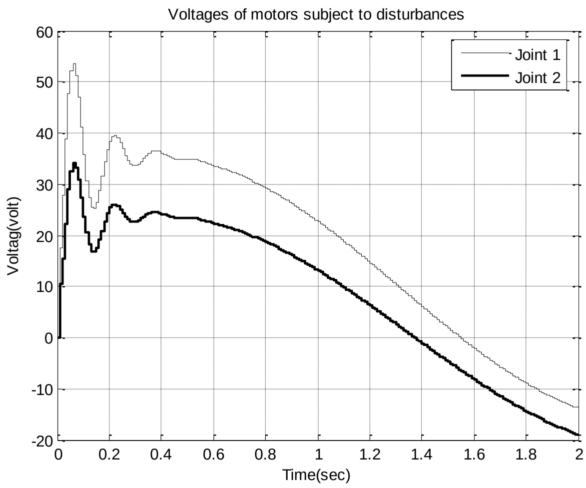

Fig. 7: Voltages of motor subject to disturbances

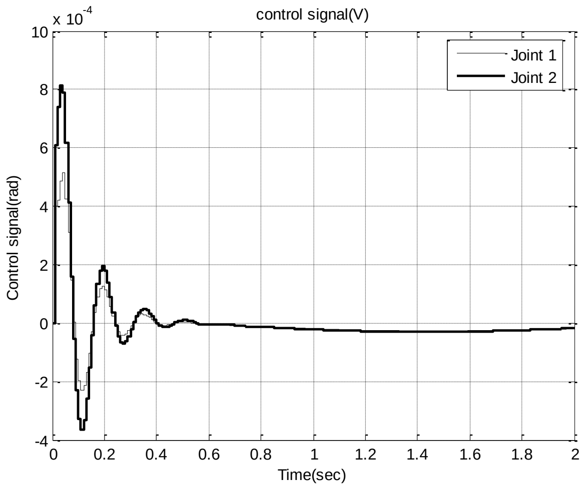

Fig. 8: Control signal

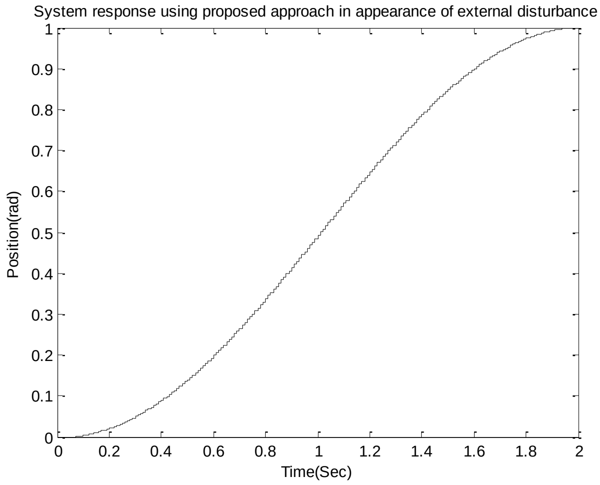

Fig. 9: System response

5. Conclusions

A model-free digital control scheme proposed for motion tracking control of robotic manipulator with unstructured uncertainty. This controller design is extended form of our previous wok in continuous-time systems. The main advantages of the proposed approach are simplicity, practicably, and low computation burden of this method to control robotic manipulator systems.

References

[1] E. J. Adam, J. L. Marchetti, ”Designing and tuning robust feed forward controllers,” ELSEVIER, Computers and Chemical Engineering, vol. 28, pp. 1899-1911, 2004.

[2] A. Isidori, and C. I. Byrnes, “Output regulation of nonlinear systems,” *IEEE Transactions on Automatic Control*, vol. 35, pp. 131-140, 1990.

[3] K. J. Åström, P. Hangander, and J. Sternby, “Zeros of sampled system,” *Automatica*, vol. 20, no. 1, pp. 31-38, 1984.

[4] M. Tomizuka, “Adaptive Zero phase error tracking algorithm for digital control,” *Trans. ASME, J. Dyn. Syst. Meas. Control*, vol. 109, pp. 65-68, 1987.

[5] S. Devasia, D. Chen and B. Paden, ”Nonlinear inversion-based output tracking,” *IEEE Transactions on Automatic Control*, vol. 41, pp. 930-942, 1996.

[6] A. Piazzi and A. Visioli, “Optimal inversion-based control for the set-point regulation of non minimum-phase uncertain scalar systems,” *IEEE Transactions on Automatic Control*, vol. 46, pp. 1654-1659, 2001.

[7] C. G. L. Bianco and A. Piazzi, “A servo control system design using dynamic inversion,” *Control Engineering Practice*, vol. 10, pp. 847-855, 2002.

[8] A. Izadbakhsh, M. Masoumi, “FAT-based Robust Adaptive Control of Flexible-Joint Robots: Singular Perturbation Approach,” *Industrial Technology (ICIT) IEEE International conference*, 2017.

[9] M. Benosman and G. Le Vey, “Stable inversion of SISO non minimum phase linear systems through output planning: an experimental application to the one link flexible manipulator,” *IEEE Transactions on Control Systems Technology*, vol. 11, pp. 588-597, 2003.

[10] E. Gross, M. Tomizuka, and W. Messner, “Cancellation of discrete time unstable zeros by feedforward control,” *Trans. ASME, J. Dyn. Syst. Meas. Control*, vol. 116, pp. 33-38, 1994.

[11] V. Feliu, E. Pereira, I. M. Dýaz, P. Roncero, “Feedforward control of multimode single-link flexible manipulators based on an optimal mechanical design,” *Robotics and Autonomous Systems*, vol. 54, pp. 651-666, 2006.

[12] C. S. Chiu, “Mixed Feedforward/Feedback Based Adaptive Fuzzy Control for a Class of MIMO Nonlinear Systems,” *IEEE Transactions On Fuzzy Systems*, vol. 14, no. 6, pp. 716-727, 2006.

[13] H. Fujimoto, Y. Hori, and A. Kawamura, “Perfect Tracking Control Based on Multirate Feed forward Control with Generalized Sampling Periods,” *IEEE Transactions on Industrial Electronics*, vol. 48, no. 3, pp. 633-644, 2001.

[14] V. Santibañez and R. l Kelly, ”PD control with feed forward compensation for robot manipulators: analysis and experimentation,” *Robotica*, vol. 19, pp. 11-19, 2001.

[15] K. Graichen, V. Hagenmeyer, M. Zeitz, “A new approach to inversion-based feed forward control design for nonlinear systems,” Elsevier, pp. 2033-2041, 2005.
R. P. Srivastava, “Use of genetic algorithms for optimization in digital control of dynamic systems,” *ACM,* 1992, pp. 219-224.

[16] C. D. Doyle and G. Stein, “Multivariable feedback design: Concepts for a classical/modern synthesis,” *IEEE Trans. Automat. Contr.*, vol. AC-26, no. 1, pp. 4-16, 1981.