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Sensor Fusion INS/GNSS based on Fuzzy Logic Weighted Kalman Filter

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Abstract **-** A Fuzzy Logic Adaptive Control (FLAC) is used to adjust the exponential weighting parameter of a weighted Error-State Kalman Filter (ESKF) in an INS/GNSS system. The FLAC is used to prevent the Kalman Filter (KF) from diverging or to reach to a high bound when the IMU produces colored noise. Furthermore, a matrix notation for the weighting parameter alpha is introduced and compared against the single alpha value. First, the results show the influence of a colored noise in the system, which makes the ESKF reaching a large error bound solution. The application of FLAC considering both constant and matrix alpha reduces the error boundary for the position and velocity states. However, the constant alpha leads to an inaccurate altitude, bias correction, and error covariance matrix. The matrix alpha parameter shows a final solution that improves the navigation accuracy for all states, preserving the stability of the error covariance matrix.

Keywords**:** INS/GNSS Fuzzy Adaptive weighted Kalman Filter

1. Introduction

Inertial navigation systems (INS) are used to indicate the real position, velocity, and attitude of an aircraft or spacecraft performing three-dimensional navigation. The INS operate continuously, showing low short-term noise and being invulnerable to jamming and interference [1]. However, this system will suffer degradation in long-term navigation as the errors presented in the sensor are integrated through the navigation equations.

On the other hand, Global Navigation Satellite System (GNSS) provides good accuracy for long-term navigation. However, in the short-term, errors are high, the signals are subject to obstruction and interference.

The advantages and disadvantages of INS and GNSS are complementary. Therefore, the integration of these technologies can benefit both, providing a navigation solution with high accuracy in long- and short-term [1]. In this paper, the fusion algorithm is based on Kalman Filter.

The KF uses the stochastic model to estimate, correct, and compensate for errors in the INS model, using the navigation solution and the errors from the INS and GNSS sensors.

Two main assumptions for Kalman filter implementation are that: i) the system is a linear dynamic system and ii) that all noise sources are white. However, in practice, this rarely occurs [2].

For the first assumption, Extended Kalman Filter (EKF), Error-State Kalman Filter (ESKF), and later Unscented Kalman Filter (UKF), have been extensively and successfully used for addressing non-linear systems.

For the second assumption, in some inertial measurement units (IMU) the noise may change during the time, showing a colored characteristic, which lead the KF to diverge or to converge to a high boundary [3]. This process is usually characterized by a 1/f power spectral density (PSD) - 1/f flicker noise [4]–[6]. To overcome this situation, widened noise or additional states must be considered. However, this can result in accuracy loss and in need to add additional states in the KF, which can be a demanding task [1].

An alternative could be applying one of Adaptive Kalman Filter (AKF) approaches in which the assumed process and measurement noise covariance may vary according to the measurement innovations.

With the advent of artificial intelligence (AI), Neural Networks (NN) [9], [10], Fuzzy Logic (FL) [3], [11], and Genetic Algorithms (GA)[12][13] techniques have been used to adapt the covariance matrices to the real noise situation in a more precise and efficient way. The Fuzzy Logic technique is used in adaptive controls to deal with nonlinear systems with uncertainties, where the errors can be compensated using a heuristic knowledge of the system. Therefore, no accurate mathematical model of the noise is needed.

Sasiadek, J.Z. et al.[3], [14] propose a Fuzzy Adaptive Extended Kalman Filter (FAEKF) for adapting the process and measurement noise covariance matrices, using an exponential data weighting performed by the Fuzzy Logic procedure to adapt the EKF. The exponential data weighting is a method that prevents the Kalman gain to diverge or reaching zero. If the Kalman gain reach zero, the KF ignores new measurements. Therefore, if the process noise change during time, the filter would not be able to compensate for it.

In this work, the Fuzzy Logic Weighted Kalman Filter is used in form of Error Feedback Error-State Kalman Filter (ESKF) and applied to integrate and fuse INS/GNSS signals. The objective is to improve the accuracy of the states when colored noise in the form of 1/f flicker noise is present in the IMU. For this purpose, a matrix notation for the weighting parameter alpha is introduced and compared against the single alpha value.

2. Methodology

A navigation profile of a vehiclet performing 3D navigation for 1000 seconds was generated using MATLAB [15]. A range of 5,000 m, a climb rate of 0.5 m/s, and an initial yaw and pitch of 90 and 10 degrees respectively were assumed. The simulation was done by defining an initial position, velocity and attitude, a constant angular velocity, and a constant acceleration, describing a spiraling circular trajectory. Gravity, Earth rotation rate and transport rate were also considered.

The raw gyroscope and accelerometer measurements were obtained using inverse kinematics and considering a measurement rate of 100Hz. The raw GNSS data was taken directly from the position and velocity, considering a measuring rate of 1Hz. For the GNSS error model, it was assumed that the estimated pseudo-range and pseudo-range rate error standard deviations will cause a variance of $\sigma_p = 5m$ in the position and a variance of $\sigma_v = 0.1m/s$ in the velocity.

For the IMU, the sensor measurement was modeled as follow, in order to consider noise in the measurements [1]:

$$\begin{aligned} \overline{\omega}^b &= (1 + M_g)\omega^b + b_g + Ga^b + w_g \\ \bar{a}^b &= (1 + M_a)a^b + b_a + w_a \end{aligned} \tag{1}$$

Where $\overline{\omega}^b$ and $\bar{a}^b$ are respectively the noisy gyro and accelerometer measurements, ω^b and a^b are the noise free measurements, M_g and M_a are the scale factors and cross coupling error, b_g and b_a are the bias, w_g and w_a are the noise, and G is the gyro g-dependent biases.

The error characteristics considered for the IMU is presented in Table 1 [1], [16].

Table 1 – IMU Error

Gyroscope error	Accelerometer	Gyroscope
Bias	$1x10^{-2}\ m/s^2$	$5x10^{-5}\ rad/s$
Random-noise root PSD	$1x10^{-3}\ m/s^{1.5}$	$5x10^{-6}\ rad/\sqrt{s}$
Scale Factor	$5x10^{-4}$	$4x10^{-4}$
Cross Coupling	$3x10^{-4}$	$3x10^{-4}$
G-dependent biases	N.A.	$0.5x10^{-6}\ rad - sec/m$

To simulate a colored noise, a $1/f$ filter was applied to the white noise w_g and w_a [17].

Although, sensor like gyroscope usually presents a dominant flicker noise in a frequency range of 0.1–0.001 Hz [18], in this work it is assumed that the flicker noise is dominant over the full frequency range.

To fuse the GNSS with the INS signals, an Error Feedback Error-State Kalman Filter (ESKF) was applied. The estimated position error, velocity error, attitude error, accelerometer bias, and gyroscope bias, are all feed back into the system to correct the INS states. The feedback correction occurs in each KF iteration. The coupled system implies that the KF will use the GNSS position and velocity solution as measurement inputs to perform the integration.

Often, a Total State Kalman Filter (TSKF) with absolute position, velocity, and attitude are used as an alternative to the ESKF. In this case, if the system is non-linear, the EKF must be used. However, the equations for a Total State

EKF are the same as for a closed-loop ESKF implementation. Therefore, the same performance is expected in both implementation[1].

2.1. ESKF Model

The full ESKF equation in Earth-Center Earth-Frame (ECEF) in discrete-time is presented as following [1]:

First phase - System Propagation:

i) Determine the first order transition matrix in discrete time:

$$\Phi = \begin{bmatrix} I_{3x3} - \Omega_{ie}^{e} dt & 0_{3x3} & 0_{3x3} & 0_{3x3} & C_b^e dt \\ 0_{3x3} & I_{3x3} & I_{3x3} dt & 0_{3x3} & 0_{3x3} \\ F_{21}^{e} dt & F_{23}^{e} dt & I_{3x3} - 2\Omega_{ie}^{e} dt & C_b^e dt & 0_{3x3} \\ 0_{3x3} & 0_{3x3} & 0_{3x3} & I_{3x3} & 0_{3x3} \\ 0_{3x3} & 0_{3x3} & 0_{3x3} & 0_{3x3} & I_{3x3} \end{bmatrix} \tag{2}$$

where Ω_{ie}^{e} is the skew symmetric matrix of Earth's rotation rate, $C_b^e$is the body to ECEF coordinate transformation matrix, dt is the sampling time and F_{21}^{e} and F_{23}^{e} are given by, respectively:

$F_{21}^{e} = \left[-\left(C_b^e a^b\right)\Lambda\right]$, $F_{21}^{e} = -\frac{2g\, r_m^{e\,T}}{r_{eS}^{e}(L)\, r_m^{e}}$, and $r_{eS}^{e}(L)$ is the geocentric radius at the surface of the Earth.

ii) Determine an approximate system noise covariance matrix:

$$\mathrm{Q}_{INS} = \begin{bmatrix} S_{gr} I_{3x3} & 0_{3x3} & 0_{3x3} & 0_{3x3} & 0_{3x3} \\ 0_{3x3} & 0_{3x3} & 0_{3x3} & 0_{3x3} & 0_{3x3} \\ 0_{3x3} & 0_{3x3} & S_{ar} I_{3x3} & 0_{3x3} & 0_{3x3} \\ 0_{3x3} & 0_{3x3} & 0_{3x3} & S_{gbd} I_{3x3} & 0_{3x3} \\ 0_{3x3} & 0_{3x3} & 0_{3x3} & 0_{3x3} & S_{gbd} I_{3x3} \end{bmatrix} \tag{3}$$

where S_{gr} and S_{ar} are the gyro and accelerometer random-noise PSD respectively; and the S_{gbd} and S_{abd} are the gyro and accelerometer bias random walk PSD respectively.

iii) Propagate the estimated states, by:

$$\hat{x}_k^- = \begin{bmatrix} \delta \hat{A}^e \\ \delta \hat{r}^e \\ \delta \hat{v}^e \\ \delta \widehat{ba}^e \\ \delta \widehat{bg}^e \end{bmatrix}_k^- = \Phi_{k-1} \begin{bmatrix} \delta \hat{A}^e \\ \delta \hat{r}^e \\ \delta \hat{v}^e \\ \delta \widehat{ba}^e \\ \delta \widehat{bg}^e \end{bmatrix}_{k-1}^+ \tag{4}$$

It can be assumed that all previous states are zero due to closed-loop correction, therefore, this step can be omitted.

iv) Propagate the state estimation error covariance matrix, using:

$$P_k^- \approx \Phi_{k-1}\left(P_{k-1}^+ + \frac{1}{2} Q_{k-1}\right)\Phi_{k-1}^T + \frac{1}{2} Q_{k-1} \tag{5}$$

Second phase – Measurements Update:

v) Set-up the measurement matrix by:

$$\mathrm{H}_k = \begin{bmatrix} 0_{3x3} & I_{3x3} & 0_{3x3} & 0_{3x3} & 0_{3x3} \\ 0_{3x3} & 0_{3x3} & I_{3x3} & 0_{3x3} & 0_{3x3} \end{bmatrix} \tag{6}$$

vi) Determine the measurement noise covariance matrix using:

$$R_k = E(w_m w_m^T) \tag{7}$$

where w_m is the GNSS measurement noise SD.

vii) Calculate Kalman gain matrix by:

$$\mathrm{K}_k = P_k^- H_k^T \left(H_k P_k^- H_k^T + R_k\right)^{-1} \tag{8}$$

viii) Formulate measurement innovations:

$$\delta z_k^{e-} = \begin{bmatrix} r_{INS}^e - r_{GNSS}^e \\ v_{INS}^e - v_{GNSS}^e \end{bmatrix} \tag{9}$$

ix) Update state estimates using:

$$\hat{x}_k^+ = \hat{x}_k^- + K_k \delta z_k^{e-} \tag{10}$$

x) Update state estimation error covariance matrix using:

$$P_k^+ = (I - K_k H_k) P_k^- \tag{11}$$

Third phase – Closed-loop correction:

xi) Correct attitude, velocity, and position of the INS solution using, respectively:

$$\hat{C}_b^{e+} = \delta \hat{C}_b^{e\,T} \hat{C}_b^{e-}\,,\ \hat{v}^{e+} = \hat{v}^{e-} - \delta \hat{v}^e, \text{ and } \hat{r}^{e+} = \hat{r}^{e-} - \delta \hat{r}^e \tag{12}$$

2.2. Weighted KF

Lewis [19] proposes the equations for a weighted error covariance matrix and weighted process, changing the KF model as following:

$$\begin{gathered} P_k^- \approx (\alpha)^2 \Phi_{k-1} \left(P_{k-1}^+ + \frac{1}{2} Q_{k-1}\right) \Phi_{k-1}^T + \frac{1}{2} Q_{k-1} \\ \mathrm{K}_k = P_k^{\alpha-} H_k^T \left(H_k P_k^{\alpha-} H_k^T + \frac{R_k}{(\alpha)^2}\right)^{-1} \end{gathered} \tag{13}$$

where α, the weighted parameter, is a constant value ≥ 1. For α >l, as time k increases, the R and Q decreases, given higher weighting for the recent measurement. When α=l, no weighing is applicable.

In this paper the weighted parameter α is considered as a matrix rather than a single number, in order to allow weighting the position (α_p) and velocity (α_v) individually while not affecting the other states (attitude and feedback-errors).

Consequently, the weighted parameter is given by:

$$\alpha_k = \begin{bmatrix} \alpha_{p_{3x3}} & 0_{3x3} \\ 0_{3x3} & \alpha_{v_{3x3}} \end{bmatrix} \tag{14}$$

Therefore, the weighted Kalman gain and the weighted error covariance matrix can be written as follows:

$$\begin{gathered} P_k^- \approx \left(I + H_k^T \alpha H_k\right)^2 \Phi_{k-1} \left(P_{k-1}^+ + \frac{1}{2} Q_{k-1}\right) \Phi_{k-1}^T + \frac{1}{2} Q_{k-1} \\ \mathrm{K}_k = P_k^{\alpha-} H_k^T \left(H_k P_k^{\alpha-} H_k^T + \frac{R_k}{(I+\alpha)^2}\right)^{-1} \end{gathered} \tag{15}$$

where I is the identity matrix. In this case α ≥ 0. Consequently when α=0, no weighing is applied in the system.

2.3. Fuzzy Logic Algorithm

The fuzzy logic used algorithm used in this paper is based on the work done by Sasiadek and Wang [14], where the FL is applied to define the value of the weighting parameter. This means that a fuzzy logic adaptive system (FLAS) is used to adjust the weight value α in the KF model.

For the Fuzzy logic procedure, two inputs vectors were used: the measurement innovations (δz) and the covariance of residuals matrix (P). Appling nine rules to the inputs, as presented in Table 2, the weighting value (α) was obtained. Therefore, for the constant alpha methodology, one state will be chosen to be observed by the FLAS, defining a constant weight that will be applied to the ESKF. For the matrix alpha methodology, each dimension of the position and velocity states will be used by the FLAS in order to define a matrix that will weight the KF.

Table 2 - Rule Table for FLAS - Alpha Value

		δz		
		Z	S	L
P	Z	S	Z	Z
	S	Z	L	M
	L	L	M	Z

Legend:
Z -Zero, S-Small, M-Medium, L-Large

The methodology for defining the Z, S, M, and L values of δz, P, and alpha, in order to build the membership functions are presented in the Table 3.

Table 3 - Values for FLAS

Parameter	Value	Rule
P	Z	Zero
	S	2 x Initial states uncertainties
	L	3 x Initial states uncertainties
δz	Z	Zero
	S	Initial states uncertainties
	L	3 x Initial states uncertainties
A	Z	Make no adjust in the system
	S	Make 2% of adjust in the system
	M	Make 10% of adjust in the system
	L	Make 20% of adjust in the system

3. Results and Discussions

Figure 1 presents the obtained results showed for when the dominant noise in the IMU is white noise and when it is a 1/f flicker noise. When the flicker noise is present, three different situations were simulated: the system with no correction, the system with a single number alpha correction, and the system with a matrix alpha correction. Table 4 shows the maximum error found for each state after the system reached stability.

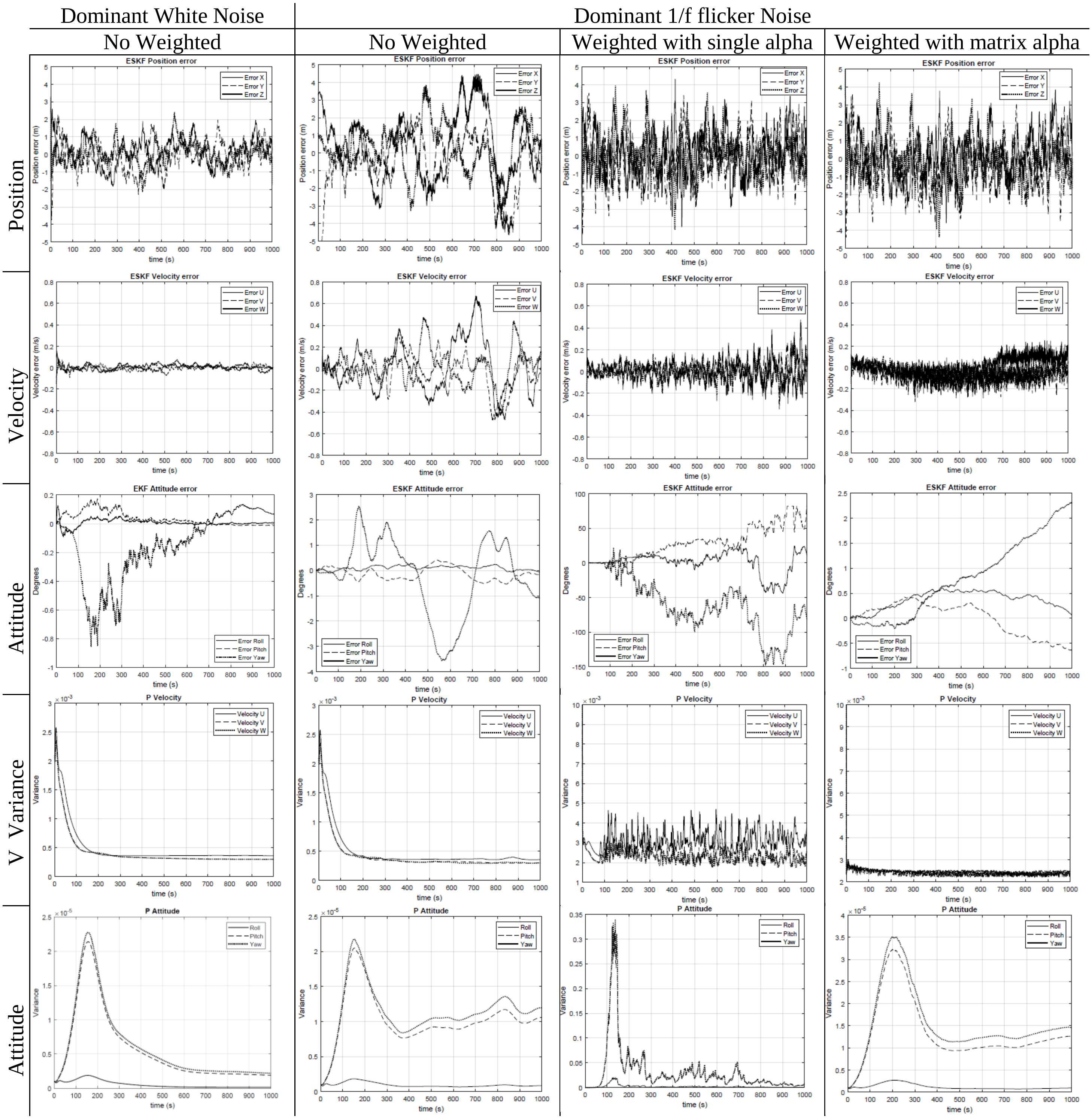

Figure 1 - Experimental Results

When white noise is the only type of noise present in the system, as shown in the first column of Figure 1, it is possible to confirm the improvement in the position and velocity determination using ESKF rather than any of the INS or GNSS sensor alone. For the position, the accumulated errors in the INS-alone position are in the range of +/-104 m after 1000 seconds of navigation, the GNSS-alone solution shows a deviation between +/- 15 m, and in the ESKF solution the final position error remains between +/-2.5 m. For the velocity, the accumulated errors in the INS-alone solution are in the range of +/- 50 m/s after 1000 seconds of navigation, the GNSS-alone solution shows a deviation between +/- 0.2 m/s, and in the ESKF solution the final velocity error is between +/- 0.08 m/s. For the attitude, although the ESKF solution shows a high error at the beginning, the tendency is that in the long term the error will be much lower.

The second column of Figure 1 shows the effect in the system when 1/f flicker noise is dominant in the IMU. Although, the change in the INS-alone solution is not significant, the colored noise makes the ESKF system reach a solution that converges to a large value. In this case, the position presents a maximum error of +/- 5 m, the velocity has a maximum error of +/- 0.7 m/s, and the attitude has a maximum error of +/- 4°. Therefore, the only state that benefited from the ESKF solution was the position. The ESKF solution for the velocity is worse than the GNSS-alone measurements and the attitude is worse than the INS-alone measurements. Also, it is possible to observe that the system cannot predict the correct bias of the gyroscope and accelerometer.

The third and fourth columns of Figure 1 show the effect of applying a constant alpha and a matrix alpha, respectively. For both cases, it is possible to observe an improvement when compared to the non-weighted solution previously described. The position presents a maximum error of +/- 4 m for both alphas and the velocity a maximum error of +/- 0.2 m/s for the matrix alpha and +/- 0.45 m/s for the constant alpha. However, it is possible to detect a considerable difference in the attitude determination between both methods. For the constant alpha, a high disturbance in the predicted altitude and gyro bias correction are registered. This occurs because the alpha is determined by the position error and covariance. Therefore, using this weighting factor to adjust other states can lead to a wrong correction. Furthermore, the use of a constant alpha generated a high instability in the error covariance matrix, which is not desirable.

Table 4 - Maximum error found for each state after stability

State	INS Alone	GNSS Alone	White Noise	Dominant 1/f flicker Noise		
			No Weighted	No Weighted	Single alpha	Matrix alpha
Position (m)	10^4	15	2.5	5	4	4
Velocity (m/s)	50	0.2	0.08	0.7	0.45	0.2
Attitude (°)	0.3	N.A.	0.2	4	150	2.5

The proposed alpha in matrix form, was confirmed as the best solution to deal with the colored noise using FLAC weighted ESKF. This led to a final solution that was able to improve the navigation accuracy for all the states, preserving the stability of the error covariance matrix and consequentially the stability of the system.

4. Conclusion

In this paper, a Fuzzy Logic Weighted Kalman Filter was used in the Error Feedback ESKF to fuse different INS/GNSS signals. The proposed algorithm allows to prevent the Kalman Filter from reaching a higher error boundary when the IMU present a dominant 1/f flicker noise.

First, the influence of adding colored noise in the system was shown, with no additional states modeled. This situation caused the ESKF to reach a large boundary value.

Secondly, the FLAC was applied in the model in order to adapt the ESKF to the flicker noise without considering additional states. For both methodologies considered, named constant alpha and matrix alpha, it was possible to register a significant improvement for position and velocity states. However, the constant alpha shows a high inaccuracies in the predicted altitude, bias correction, and error covariance matrix.

This occurs since the alpha is determined using only one state, the position error and covariance. Therefore, using this weighting factor to adjust other states can lead to a wrong correction. Furthermore, the use of a constant alpha generated a high instability in the error covariance matrix, which is not desirable.

An alpha parameter in matrix form, in which each dimension of the position and velocity states are corrected according to its own state error and covariance, results in the best solution to deal with the colored noise. This led to a final solution that improves the navigation accuracy for all the states, preserving the stability of the error covariance matrix.

Therefore, the use of FLAC to correct an ESKF system in which a 1/f flicker noise is dominant, was proven to be a viable solution, improving the accuracy of all states while preserving the stability of the system, without the need to consider additional states in the noise model.

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