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Attitude Task Allocation and Control in a Swarm of Magnetically Controlled CubeSats

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Abstract **–** The paper delineates the magnetic controller gains tuning procedure for an active magnetic attitude control system of a CubeSat in the ISS orbit. The optimization is conducted for arbitrary required attitude given in the orbital reference frame of the spacecraft. The study has been conducted as a part of the Skoltech University project to deploy a swarm of 3U CubeSats for collective gamma-ray bursts detection, which requires the satellites' coordinated attitude control. We consider four identical CubeSats exhibiting swarm behaviour by optimal attitude task allocation on receipt of a command from the mission control center. The first part of this study shows how to obtain the controller gains via linearization of the spacecraft rotational dynamics in the vicinity of the required attitude regime and subsequent numerical optimization (carried out in terms of Floquet theory). The second part shows the collective attitude control scenario. The task of the swarm is to ensure maximum sky coverage around a principal direction uplinked to the spacecraft. The task extends to ensuring the maximum collective stability of the swarm through implementing the optimization algorithm.

Keywords**:** Magnetic Attitude Control, Swarm of CubeSats, Floquet Theory, Coverage Optimization

1. Introduction

This study was carried out as a part of the Skoltech University project to deploy a swarm of four 3U CubeSats in LEO [1]. The mission's principal objective is collective gamma-ray bursts or terrestrial gamma-ray flashes detection, which requires the satellites' coordinated attitude control. As the pointing accuracy requirements are not very stringent magnetically actuated attitude control system was chosen during the early design stage.

Magnetic control systems are widely used nowadays for attitude stabilization. Being low-cost, reliable, lightweight, and small, they present an attractive solution for small satellites, often linked to educational research platforms. These benefits often outdo their under-actuation and low accuracy problem. However, complementing magnetorquers with various additional compact, lightweight, and fuel-independent actuators or mechanical concepts, such as spin stabilization, bias momentum wheel, gravitational stabilization, and reaction wheels, often solves the problem entirely. However, this does add some unavoidable complications involving restricted attitude patterns [2].

Another approach is to have a three-axis magnetic attitude control system with solely magnetic actuation. This setup is usually considered a complicated task in terms of both control synthesis and its implementation. Limited controllability of magnetic systems has opened a wide field of possible research on the topic [3]. However, one major problem is that magnetic control torque is restricted in certain directions because of its inability to function along the geomagnetic field vector. The quaternion feedback control, although simple, is fully susceptible to this control restriction problem. An approach to solving a problem of solely three axes magnetic control is proposed in [4].

Different tools are available to further improve the magnetic control system's performance, such as; optimization methods, model predictive control, and adaptive methods. Optimizations are good at achieving the best results in terms of cost functions and their ability to incorporate constraints. One of the most dependable methods is a linear quadratic regulator, which is studied as a periodic LQR [5] and as an algorithm for the periodic Riccati equation solving with the application to the magnetic control [6]. Model predictive control provides the essential control direction by utilizing the geomagnetic field vector rotation in the sliding control theory. This dynamic model is used to predict the satellite's motion, which is further utilized in the convergence of necessary attitude using error and cost function. Satellite control using general model predictive control is studied in [7]. Lastly, adaptive methods also can incorporate restriction in the control direction. For example, the adaptive method can be based on neural network-driven control, which can partly utilize a feedback approach as mentioned in [8] and gives better performance in simulation while also effectively canceling uncertainties in the dynamical model.

Three-axis magnetic control requires extensive in-flight validation and further research. For instance, there is also a problem of residual magnetization, which affects the satellite operation and is studied for Skoltech swarm mission in [9]. The effects of the duty cycle on stability and the steady-state error in attitude maneuvers are studied in [10]. A detailed review covering the main algorithms for the active magnetic attitude control system is given in [3].

Lastly, attitude maneuvers that involve magnetic actuators rely heavily on control gains. Finding suitable values for these gains is not only time taking computationally expensive procedure but also requires human assistance for fine-tuning. An approach that uses periodicity of the equations of motion coefficients for the simplified magnetic field model and hence employs the Floquet theory to find the quaternion feedforward magnetic controller gains was proposed in [11]. Our previous study [12] took this point further and showed how deep neural networks can aid the procedure as a means of gains approximation. This study reformulates the approach of [11] in terms of quaternions (thus making it valid for arbitrary orientations) and goes on to focus on using the obtained maps of magnetic controller gains to be used by a swarm of CubeSats coordinating their attitude for maximum sky coverage.

2. Problem Statement

Each one of the four spacecraft is considered to be in a circular orbit of 400 km attitude and 52 degrees inclination. The spacecraft are assumed to have a mass of 3 kg with dimensions 100x100x340 mm in *X*, *Y* and *Z* directions respectively. The inertia tensor of the satellite is diag[0.031,0.031,0.005]kg·m^2.

The following reference frames are used in the paper:

- the Earth-Centered Inertial reference frame (ECI) F^I whose origin is at Earth's center and axes coincide with those of the J2000 frame.
- the Earth-Centered Earth-Fixed reference frame (ECEF) F^F, whose origin is at the Earth's center, *z*-axis directed along the mean rotational axis of the Earth, *x*-axis pointing to the Greenwich meridian, and *y*-axis completes the right-handed system.
- the Orbital reference frame F^O with the origin at the center of mass of the satellite, *z*-axis pointing away from the center of the Earth, *y*-axis along the cross product of the satellite's center of mass position and velocity vectors, and *x*-axis completing the frame according to the right hand rule.
- the Body-fixed reference frame F^B with the origin at the satellite's center of mass, its three axes coincide with the three principal axes of inertia of the satellite.

All vector transformations between reference frames are described by unit quaternions.

Letting $\boldsymbol{\Omega}^b$ be the satellite's absolute angular velocity (between the F^B frame and the F^I frame) and $\boldsymbol{\omega}^b$ be its relative angular velocity (between the F^B frame and the F^O frame), the two angular velocities can be related as follows:

$$\boldsymbol{\Omega}^b = \boldsymbol{\omega}^b + \boldsymbol{\omega}_0^b = \boldsymbol{\omega}^b + \tilde{\boldsymbol{q}}^{ob} \circ \boldsymbol{\omega}_0^o \circ \boldsymbol{q}^{ob}, \tag{1}$$

where $\boldsymbol{\omega}_0^b$ is the relative angular velocity between the F^O frame and the F^I frame represented in the F^B frame, $\boldsymbol{\omega}_0^o = (0 \quad \omega_0 \quad 0)^T$ is the relative angular velocity between the F^O frame and the F^I frame being represented in the F^O frame, ω_0 is the mean motion of the satellite in the orbit, and $\boldsymbol{q}^{ob} = (q_0^{ob} \quad \boldsymbol{q}^{ob})$ is the unit quaternion that transforms from the F^B frame to the F^O frame.

The kinematics of the spacecraft are described in terms of unit quaternions as follows:

$$\dot{\boldsymbol{q}}^{ob} = \frac{1}{2}\boldsymbol{q}^{ob} \circ \boldsymbol{\omega}^b. \tag{2}$$

Treating the satellite as a rigid body, and ignoring all the environmental disturbances except for the gravity gradient, the satellite's dynamical equations of motion (relative to the non-inertial orbital frame) can be written as:

$$\begin{gathered}\mathbb{I}^b\dot{\boldsymbol{\omega}}^b = \boldsymbol{\omega}^b \times \mathbb{I}^b\boldsymbol{\omega}^b + \mathbf{M}_{\mathrm{rel}}^b + \mathbf{M}_{\mathrm{ctrl}}^b + \mathbf{M}_{\mathrm{gg}}^b,\\ \mathbf{M}_{\mathrm{rel}}^b = \mathbb{I}^b\left(\boldsymbol{\omega}^b \times \boldsymbol{\omega}_0^b\right) - \boldsymbol{\omega}^b \times \mathbb{I}^b\boldsymbol{\omega}_0^b - \boldsymbol{\omega}_0^b \times \mathbb{I}^b\left(\boldsymbol{\omega}^b + \boldsymbol{\omega}_0^b\right),\end{gathered} \tag{3}$$

where $\mathbb{I}^b$ denotes the inertia tensor of the spacecraft, $\mathbf{M}_{\mathrm{ctrl}}^b$ is the control torque provided by the actuators, and $\mathbf{M}_{\mathrm{gg}}^b$ is the gravity-gradient torque. The gravity-gradient torque is modeled as follows:

$$\mathbf{M}_{\mathrm{gg}}^b = 3\omega_0^2\mathbf{e}_3^b \times \mathbb{I}^b\mathbf{e}_3^b. \tag{4}$$

The control torque is given by the following equation:

$$\mathbf{M}_{\mathrm{ctrl}}^b = \mathbf{m}^b \times \mathbf{B}^b. \tag{5}$$

Here $\mathbf{B}^b$ is the geomagnetic induction vector, $\mathbf{m}^b$ is the control dipole moment generated by the magnetorquers. The control dipole moment is derived from Lyapunov based PD-controller [11]:

$$\mathbf{m}^b = -4q_{e,0}K_p\mathbf{B}^b \times \mathbf{q}_e - K_d\mathbf{B}^b \times \boldsymbol{\omega}^b, \tag{6}$$

where K_p and K_d are the controller gains and $\boldsymbol{q}_{e,0} = (q_{e,0} \quad \mathbf{q}_e)^T$ is the error quaternion defined in the following equation:

$$\mathbf{q}_e = \tilde{\boldsymbol{q}}_d \circ \boldsymbol{q}^{ob}, \tag{7}$$

where $\boldsymbol{q}_d$ is the desired quaternion (desired $\boldsymbol{q}^{ob}$ at steady state). The aim of the controller is to drive the scalar component of the error quaternion to either 1 or -1.

Introducing the $\mathbb{W}_{\boldsymbol{x}}$ skew-symmetric operator for any vector $\mathbf{x} = (x_1 \quad x_2 \quad x_3)^T$ as:

$$\mathbb{W}_{\boldsymbol{x}} = \begin{bmatrix} 0 & -x_3 & x_2\\ x_3 & 0 & -x_1\\ -x_2 & x_1 & 0\end{bmatrix}. \tag{8}$$

Eqs. (2)-(3) can be linearized in the vicinity of $\boldsymbol{\omega}^b = \hat{\boldsymbol{\omega}}^b$ ($\hat{\boldsymbol{\omega}}^b = (0 \quad 0 \quad 0)^T$ for slew maneuvers), and $\boldsymbol{q}^{ob} = \boldsymbol{q}_d$ [13] as follows:

$$\begin{gathered}\dot{\boldsymbol{\delta}\mathbf{q}}^{ob} = -\mathbb{W}_{\hat{\boldsymbol{\omega}}^b}\boldsymbol{\delta}\mathbf{q}^{ob} + \frac{1}{2}\boldsymbol{\delta}\boldsymbol{\omega}^b,\\ \dot{\boldsymbol{\delta}\boldsymbol{\omega}}^b = \left(\mathbb{I}^b\right)^{-1}\left(\mathbb{F}_{rel}^q + \mathbb{F}_{ctrl}^q + \mathbb{F}_{gyr}^q\right)\boldsymbol{\delta}\mathbf{q}^{ob} + \left(\mathbb{I}^b\right)^{-1}\left(\mathbb{F}_{rel}^\omega + \mathbb{F}_{ctrl}^\omega + \mathbb{F}_{gyr}^\omega\right)\boldsymbol{\delta}\boldsymbol{\omega}^b,\end{gathered} \tag{9}$$

where

$$\mathbb{F}_{ctrl}^q = 4K_p\mathbb{W}_{\hat{\mathbf{B}}^b}\mathbb{W}_{\hat{\mathbf{B}}^b}, \quad \mathbb{F}_{gg}^q = 6\omega_0^2\left(\mathbb{W}_{\hat{\mathbf{e}}_3^b}\mathbb{I}^b\mathbb{W}_{\hat{\mathbf{e}}_3^b} - \mathbb{W}_{\mathbb{I}^b\hat{\mathbf{e}}_3^b}\mathbb{W}_{\hat{\mathbf{e}}_3^b}\right),$$

$$\mathbb{F}_{rel}^{\omega} = -\mathbb{I}^b \mathbb{W}_{\widehat{\boldsymbol{\omega}}_0^b} + \mathbb{W}_{\mathbb{I}^b \widehat{\boldsymbol{\omega}}_0^b} - \mathbb{W}_{\widehat{\boldsymbol{\omega}}_0^b} \mathbb{I}^b, \qquad \mathbb{F}_{ctrl}^{\omega} = K_d \mathbb{W}_{\widehat{\mathbf{B}}^b} \mathbb{W}_{\widehat{\mathbf{B}}^b}, \qquad \mathbb{F}_{gyr}^{\omega} = -\mathbb{I}^b \mathbb{W}_{\widehat{\boldsymbol{\omega}}^b} + \mathbb{W}_{\mathbb{I}^b \widehat{\boldsymbol{\omega}}^b}.$$

Again, in Eq. (9), $\widehat{\boldsymbol{\omega}}^b$ is the relative angular velocity around which the linearization is carried out, and any vector be expressed as $\hat{\mathbf{x}}^b = \widetilde{\boldsymbol{q}}_d \circ \hat{\mathbf{x}}^o \circ \boldsymbol{q}_d$, where $\boldsymbol{q}_d$ is the desired quaternion around which the linearization is carried out.

For geomagnetic field modeling, direct dipole model is used, which in the orbital frame, is given by the following equation [11]:

$$\mathbf{B}^o = B_0 \begin{pmatrix} \cos(u)\sin(i) \\ \cos(i) \\ -2\sin(u)\sin(i) \end{pmatrix}, \tag{10}$$

where $B_0 = \mu_e / r^3$, $\mu_e = 7.812 \cdot 10^6$ $\mathrm{km^3 kg\, s^{-2} A^{-1}}$, r is the radius of the circular orbit of the spacecraft. The direct dipole model is presented here because it leads to a periodic right-hand side of Eq. (9), which forms a crucial aspect for the implementation of Floquet theory.

3. Floquet Analysis

It has been established in [4, 11] that the performance of the attitude control system is sensitive to the choice of the controller gains in Eq. (6), K_p and K_d. The choice of the gains depends on the orbit of the satellite, its inertia tensor and, lastly, the attitude around which its stabilization is required (q_d). It is due to the physical limitation of the magnetic control system that the satellite might not be able to stabilize around certain orientations. At some orbits it is impossible to generate torques in certain directions as the magnetic field vector is aligned with the direction of the desired torque.

Numerical experiments have led us to conclude that for a fixed orbit, there is no set of gains that can stabilize the spacecraft around some desired attitudes (the unstable attitudes pool). However, there is a stable attitudes pool for which the gains need to be optimized in order to attain optimal performance. Our simulations results corroborate that within the stable attitudes pool and after finding the optimal gains, the satellite's performance around some desired orientations is better than that around some others.

In this section, a method to optimize the gains, $K_p$and K_d, is presented. We follow the approach outlined in [11], but reformulate it in quaternion representation for convenience of obtaining the gains for arbitrary required attitude. The method is based on Floquet theory, which gives the shape of solution to ordinary differential equations with periodic Jacobian matrices. For the following n-dimensional linear system of ordinary differential equations:

$$\frac{d}{dt}\mathbf{x}(t) = \mathbb{A}(t)\mathbf{x}(t), \tag{11}$$

where $\mathbb{A}(\mathrm{t})$ is periodic with a period T, and letting $\mathbb{X}(\mathrm{t})$ be the principal fundamental matrix solution (i.e. $\dot{\mathbb{X}}(\mathrm{t}) = \mathbb{A}(\mathrm{t})\mathbb{X}(\mathrm{t}), \mathbb{X}(0) = \mathbb{I}$), the fundamental solution is expressed as follows:

$$\mathbb{X}(t) = \mathbb{P}(t)\exp(\mathbb{B}t), \tag{12}$$

where $\mathbb{P}(t)$ is a periodic matrix with period T, and $\mathbb{B}$ is a constant complex matrix.

Eq. (12) suggests that if the matrix $\mathbb{B}$ is negative definite, then we can expect the system in Eq. (11) to settle at zero in the steady state. One important note here is that if $\mathbb{X}(0) = \mathbb{I}$, the monodromy matrix of the system $\mathbb{M}$ can be expressed as $\mathbb{M} = \mathbb{X}(T) = \exp(\mathbb{B}t)$.

In order to apply the Floquet analysis to the gain choice problem, the equations of motion have to be linearized in the vicinity of the desired attitude and angular velocity, which has been introduced by Eq. (9). The second step in the procedure is to minimise the maximum eigenvalue of the logarithm of the monodromy matrix $\mathbb{X}(T)$ taken at one orbital period. In this step, we are searching for a negative maximum eigenvalue of $\log[\mathbb{X}(T)]$. If a negative maximum eigenvalue is not found, the required attitude is said to belong to the unstable attitudes pool. Letting f be the cost function that needs to be minimized, it can be written as

$$f = \max\big(\Re(\log[\lambda_i])\big). \tag{13}$$

where $\lambda_i = \lambda_i(K_p, K_d)$ are eigenvalues of $\mathbb{X}(T)$.

The optimization problem can now be formulated as finding the minimum of the cost function f given that both controller gains are positive:

$$\min_{K_p,K_d>0} f. \tag{14}$$

Adapting Eq. (11) for the linearized attitude dynamics given by Eq. (9), the state vector can be defined as follows

$$\mathbf{x}(t) = \begin{pmatrix} \boldsymbol{\delta}\mathbf{q}^{ob} \\ \boldsymbol{\delta}\boldsymbol{\omega}^{b} \end{pmatrix}, \tag{15}$$

while the Jacobian matrix following (9) is given by

$$\mathbb{A}(t) = \begin{bmatrix} -\mathbb{W}_{\widehat{\boldsymbol{\omega}}^b} & \frac{1}{2}\mathbb{I}_{3\times 3} \\ (\mathbb{I}^b)^{-1}\big(\mathbb{F}^q_{rel} + \mathbb{F}^q_{ctrl} + \mathbb{F}^q_{gyr}\big) & (\mathbb{I}^b)^{-1}\big(\mathbb{F}^\omega_{rel} + \mathbb{F}^\omega_{ctrl} + \mathbb{F}^\omega_{gyr}\big) \end{bmatrix}. \tag{15}$$

As an example, the contour plot of the objective function defined in Eq. (13) for a desired attitude $\boldsymbol{q}_d = (1\ \ 0\ \ 0\ \ 0)^T$ (gravity-gradient orientation for this case) is shown in Fig. 1. It is found after the optimization that this attitude belongs to the stable attitudes pool. The red dot indicates the position of optimized eigenvalues.

In order to make K_p and K_d comparable, the latter is substituted with $K_d' = K_d/\omega_0$.The performance of the magnetic control system using the optimized gains for $\boldsymbol{q}_d = (1\ \ 0\ \ 0\ \ 0)^T$ is presented in Fig. 2.

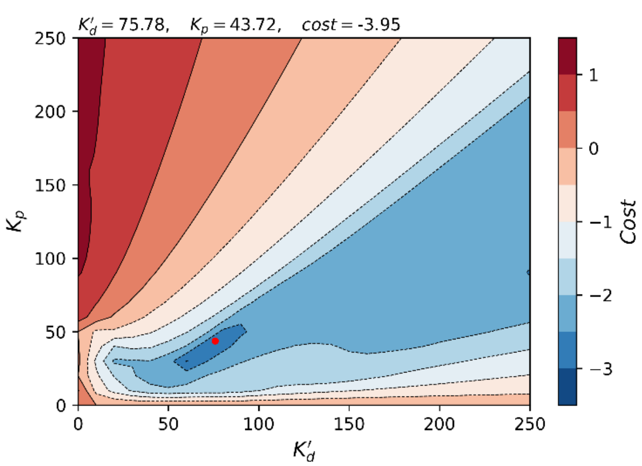

Fig. 1: Contour plot of the cost function for $\boldsymbol{q}_d = (1\ \ 0\ \ 0\ \ 0)^T$

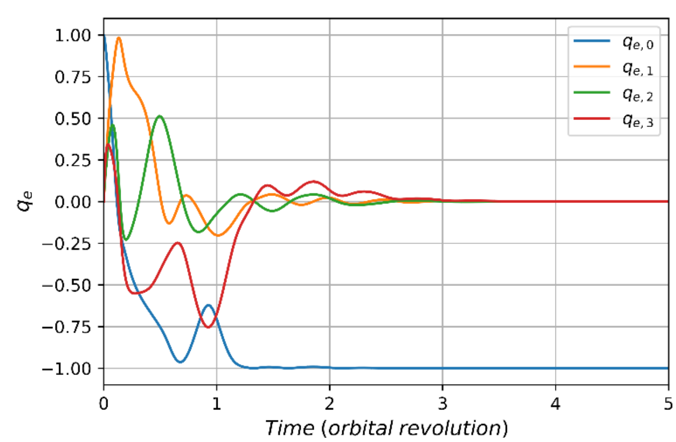

Fig. 2: Simulation example using optimized control gains

4. Generation of the Gains Data Set

This section presents the results of optimization problem stated in the Eq. (14) for a sufficiently large number of desired spacecraft attitudes. This data is used in the next section to provide an optimized attitude set points for a swarm CubeSats seeking maximum sky coverage. Although the attitudes in this study are parameterized by unit quaternions, it been concluded that it would be more intuitive to express the desired attitude of the spacecraft as a set of Euler angles sequence of intrinsic rotations of 1-2-3 sequence, $\boldsymbol{\alpha}_{d,123} = (\alpha_x\ \alpha_y\ \alpha_z)$. The desired Euler angles can be transformed to $\boldsymbol{q}_d$ in order to proceed with solving the minimization problem.

It is important to note that the inertia tensor defined in Section 2 suggests that the considered spacecraft is dynamically symmetrical around the Z-axis of the F^B frame, which allows us to use only two Euler angles to fully parameterize any desired attitude, as the satellite is invariant to the rotation with respect to the Z-axis. This suggests $\boldsymbol{\alpha}_{d,123} = (\alpha_x\ \alpha_y\ \alpha_z) = (\alpha_x\ \alpha_y\ 0)$. All possible orientations can now be thought of as points on a unit sphere with α_x being the elevation and α_y being the azimuth.

A differential evolution global optimization algorithm has been used to run the optimization problem stated in the Eq. (14) for 1700 points on the sphere, and the results are shown in Fig. 3 and 4. Fig. 3 shows a scatter plot of only those orientations that belong to the stable attitudes pool (whose optimized values of cost function are negative) on a sphere, each point of which corresponds to particular values of α_x and α_y. Fig. 4, presents a projection of all the same points onto a plane. The color bar in both figures represents the cost function of the optimization problem as given by Eq. (14).

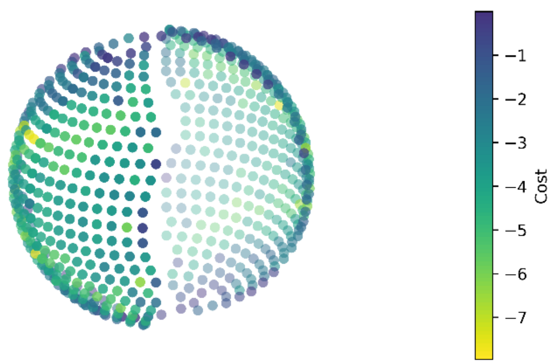

Fig. 3: Scatter plot showing area of successful convergence on a sphere

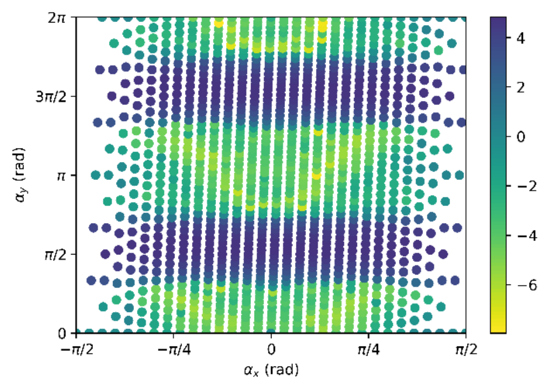

Fig. 4: A grid plot showing area of successful convergence in spherical coordinates

5. Test Case for algorithm Application

The described procedure determines the magnetically controllable and uncontrollable areas of attitude for the considered spacecraft at a given orbit. In missions that are not very stringent on a specific desired attitude of the spacecraft, the presented routine helps in the decision of the best nominal orientation to carry out the mission. One consideration is that the spacecraft has to stabilize its orientation around the required one, and another consideration is to provide a suitable attitude to achieve the mission objectives. To illustrate this usage, Skoltech's swarm prospective mission comprising four CubeSats is considered. The satellites have the same parameters as described in Section 2. The aim of the mission is to observe certain astronomical events through mounted gamma-ray detectors with maximum possible sky coverage. The reference orientation of each spacecraft is provided by the ground station based on the direction vector of the event of interest, and is given in the form of a unit vector $\boldsymbol{e}_O$ represented in the orbital frame of any of the spacecraft. The CubeSats are assumed to be operating at small distances from one another (the distances are 500-1000 m so as the swarm orbital configuration is compatible with other planned experiments described in [1]) and the events of interests (e.g. gamma-bursts produced by neutron stars mergers) are sufficiently distant to neglect the difference between the positions of the swarm spacecraft and their orbital frames. Let us denote by θ the field of view of the employed sensors (assuming conical field of view and that the sensors are mounted to have the field of view cone axis $\boldsymbol{e}_{f,i}$coinciding with the z-axis of the i_{th} spacecraft body frame).

Let us define the coverage maximization metric in terms of the closest direction to $\boldsymbol{e}_O$ which does not fall within field field of view of any of the sensors. If this direction is represented by a unit vector $\boldsymbol{e}_c$ our goal is to maximize the dot product $\boldsymbol{e}_c \cdot \boldsymbol{e}_O$. From symmetry the arrangements of the spacecraft providing maximum coverage is such that the lines defined by $\boldsymbol{e}_{f,i}$ intersect any plane orthogonal to $\boldsymbol{e}_O$ in points that belong to a circle (se Fig. 5). The points are distributed on this circle symmetrically, however the maximum coverage problem solution is invariant to rotation of this circle (which changes the required orientation of the four spacecraft). This invariance lets us add another objective to the optimization problem, which is expressed in minimization of the maximum cost function (as given by Eq. (14)) value associated with the four required attitudes. Thus, we can find the position of the four points on the circle which ensure the best stability of the required configuration within the maximum coverage solution. The case when the optimization yields a solution that requires one or more of the spacecraft to take an attitude that belongs to the unstable pool is not considered here.

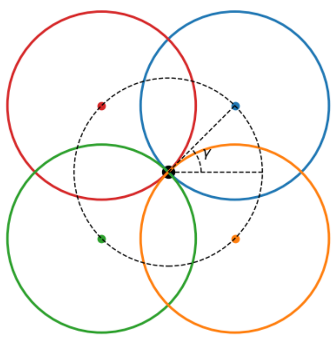

Fig. 5: Distribution of sensors FOV cones

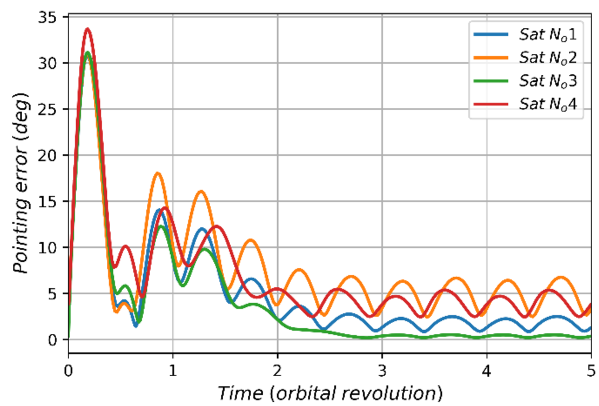

Fig. 6: Sky coverage simulation example

An example simulation for the four CubeSats' attitudes converging to an optimized coverage solution is presented in Fig. 6. The simulation is conducted for $\boldsymbol{e}_O = (0.3691 \quad 0.0134 \quad 0.929)$, the field of view angle is assumed to be $\theta = 20$ deg. The plots in Fig. 6 shows the four lines representing the angular errors (angles between the required and actual $\boldsymbol{e}_{f,i}$).

It should be noted that running the searching algorithm will not always result stabilizable reference orientations for the four spacecraft. In fact, as the observation vector changes, the level of stability of the of the resulted orientations varies from unstable to a stability level with a sub-degree accuracy. Such performance can hardly be expected in the presence of environment, actuator, and estimation disturbances, but the analysis of disturbances effect lies beyond the scope of the paper. If the searching problem is run for a certain observation vector and one satellite is found to be not stabilizable, the observation vector is said to be un-achievable with the given resources.

6. Conclusion

The paper presented a technique to mitigate the problem of magnetic control systems being sensitive to the choice of control gains through choosing an optimized set of gains for any desired orientation. The optimization problem is defined based on Floquet analysis as the equations of motion were found to have a periodic Jacobian matrix after linearization. Gains were optimized for 1700 possible reference attitudes to serve as a data set that can be interpolated for any reference attitude. Although the optimized gains work perfectly fine to stabilize the system around the required attitude in most cases, it is noted that some attitudes are generally poorly stable even with a concise choice of gains while others are impossible to stabilize the spacecraft around using pure magnetic control.

The presented algorithm is indeed useful in determining the initial gain set from which the tuning process should start for a magnetically controlled satellite, however, it is particularly important for missions comprising more than one satellite. It has been shown that the gain selection technique can be used in a swarm of satellites that are required to point to a certain event with maximum coverage. The attitude set point of each satellite uses the generated data to determine these orientations with two considerations in mind, maximum coverage of the event and stability of the reference orientation.

Although the study case of Skoltech University swarm mission comprises four satellites, it is believed that the gain optimization algorithm is scalable to larger swarms.

A prospective step is to use the obtained data set as an input a genetic algorithm that further optimizes the gains in a high-fidelity nonlinear dynamical and environmental model. Another future step is to extend the test case scenario so that swarm spacecraft would act as four agents to solve a multi-objective optimization problem maximizing the coverage and negotiating to allocate their individual attitudes so as to maximize the resulting levels of stability given the attitude feasibility constraint. This would be achieved through a communication and negotiation algorithm among the swarm agents.

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