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A Model-Free Control System Based on the Sliding Mode Control with Automatic Tuning Using as On-Line Parameter Estimation Approach

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Abstract - A model-free sliding mode control technique is solely based on observable measurements and therefore does not require a theoretical model of the dynamic system in developing the controller form. In this work, a unique model-free sliding mode control technique has been developed solely based on previous control inputs and system state measurements. The traditional switching gain drives the system states into a sliding surface in the presence of disturbance and uncertainties. Lyapunov's stability theorem is used in the controller formulation process to ensure closed-loop asymptotic stability. High frequency chattering of the control effort is reduced by using a smoothing boundary layer into the control law. If the controller tuning parameters are chosen arbitrarily in model-free sliding mode control scheme, then the tracking performance of the controller becomes unacceptable due to the noise effect. An approach is proposed for estimating the increment to the switching gain in real-time to ensure the sliding condition (which guarantees closed-loop tracking stability) is satisfied using a control law form that assumes a strictly unitary input influence gain. In formulating the on-line estimation law, an exponential forgetting factor is incorporated into the least-squares estimator to ensure the usage of the updated data and exclusion of the past data generated by the past parameters. An automatic bounded forgetting tuning technique is developed to maintain the benefits of data forgetting while avoiding the possibility of gain unboundedness in absence of persistent excitation. The tuning estimator is assured that the resulting gain matrix is upper bounded regardless of the persistent excitation by suspending the data forgetting if the gain matrix reaches the specified upper bound. The controller is tested for a nonlinear second order system.

***Keywords*:** Sliding Mode Control, Real-Time, Parameter Estimation, Switching Gain, Automatic Bounded Forgetting.

1. Introduction

Recently advanced control techniques have been developed for increased system performance. Sliding mode control (SMC) is a powerful method due to the control law's robustness and ability to control higher-order nonlinear dynamic systems operating under uncertainty conditions. The method splits the control problem down into two phases: the *reaching phase* drives the system states towards the sliding surface, where the *sliding phase* reacts and slides the states towards the equilibrium. Lyapunov's Direct Method is used to ensure asymptotic tracking stability of state trajectories.

Pai [1] proposed a discrete-time integral sliding mode control scheme to track dynamic inputs of a non-delay reference model. Kai et al. [2] mentioned how sliding surface can affect the overall performance of the sliding mode control when it relates to the uncertain physical quantities. Examples of parameters estimation for designing controller can be found in work done by Vahidi et al. [7] proposed a recursive least squares (RLS) with multiple forgetting, and Toshio Yoshimura [8] considered an adaptive discrete sliding mode control method and estimated the uncertain states by using weighted least squares estimator (WLSE).

In spite of claimed robustness, high frequency oscillations of the state trajectories are the major obstacle for the implementation of SMC in a wide range of applications. In addition, the SMC methodology requires derivation of unique mathematical models to each system. Therefore, several researchers have investigated various types of model-free SMC in order to simplify the process. Crassidis and Mizov [3] developed a model-free sliding model controller which relies on previous control inputs and state measurements to drive systems. Crassidis and Reis [4] derived a similar controller that was introduced in [3] but employed a different approach and applied it to SISO linear and nonlinear systems, in the presence of measurement noise. Crassidis and Fares [5] used also similar approach but implemented on fully-actuated and underactuated MIMO systems. Crassidis, Sreeraj, and Kaputa [6] were applied model-free approach to an unmanned aircraft system with unknown control input gain parameters and untuned control gains.

This work is an extension of previous work done by [4, 5] where a model-free approach is developed with an assumption that the bounds of the input influence gain matrix are known although the dynamics of the system is assumed to be unknown. Research performed previously showed that the approach is feasible but is not truly model-free since the upper and lower bounds of the input influence gains were assumed. The method proposed here extends previous work by assuming a unitary input influence gain type measurement model and update the control law in real-time to satisfy the sliding condition ensuring tracking stability with minimal chattering. The outline of this paper is as follows; following the introduction in section 1, section 2 will describe the system, section 3 will develop the control law, section 4 will create real-time estimator law, and finally, section 5 will simulate a second order nonlinear system with the developed controller.

2. System Description

Consider the following n^{th}-order single-input dynamic system:

$$\mathrm{x}^{\mathrm{n}} = \mathrm{f}(\mathrm{x}) + \mathrm{b}(\mathrm{x})\mathrm{u} \tag{1}$$

where the scalar x is the output to be controlled, the scalar u is the control input, n is the system's order and x is the state vector. The functions $f(\mathrm{x})$ and $b(\mathrm{x})$ are not exactly known but are bounded by some known prior values.

The above system is redefined to the following form:

$$\mathrm{x}^{\mathrm{n}} = \mathrm{x}^{\mathrm{n}} + \mathrm{bu} - \mathrm{bu}_{\mathrm{k-1}} - \mathrm{bu} + \mathrm{bu}_{\mathrm{k-1}} \tag{2}$$

where u_{k-1} is the previous control input. To compute the control law, and to avoid an algebraic loop within the controller algorithm, an estimation of error between the current and the previous control input are defined as:

$$\hat{\varepsilon}(\mathrm{u}) = \mathrm{u}_{\mathrm{k-1}} - \mathrm{u}_{\mathrm{k-2}} \tag{3}$$

where u_{k-2} is the previous control input of the previous input. Although the control input error is not known exactly, the error is assumed to be bounded as follows:

$$(1 - \sigma_{\mathrm{l}})\hat{\varepsilon}(\mathrm{u}) \le \hat{\varepsilon}(\mathrm{u}) \le (1 + \sigma_{\mathrm{u}})\hat{\varepsilon}(\mathrm{u}) \tag{4}$$

where σ_u is the upper bound and σ_l is the lower bound of the estimation error. At high sampling times, the values of the error bounds will be near zero since the estimation error will be approximately equal to the actual error.

2.1. Sliding Surface

A sliding surface, as defined by Slotine and Li [9] for a n^{th}-order single-input-single output system, is as follows:

$$\mathrm{s} = (\mathrm{d}/\mathrm{dt} + \lambda)^{\mathrm{n-1}}\tilde{\mathrm{x}}(\mathrm{t}) \tag{5}$$

where λ is the slope of the sliding surface, which is assumed to be a positive constant and $\tilde{x}(t)$ is the difference between the desired state and the actual state, i.e., $\tilde{x}(t) = x(t) - x_d(t)$. To ensure the system's trajectories will be asymptotically stable during the reaching phase, Lyapunov's direct method is used. Lyapunov's direct method states, if the total energy of a system is continuously dissipated, then the system, whether linear or nonlinear, must eventually settle down to an equilibrium point. The energy of the system will be defined as:

$$V(x) = (1/2)s^2 \tag{6}$$

which is positive definite, implies that the system has positive energy. The rate of energy is obtained as:

$$\dot{V}(x) = \dot{s}s \le 0 \tag{7}$$

Eq. (7) is then redefined as:

$$\dot{V}(x) = -\eta|s| \le 0 \tag{8}$$

where η is a small positive constant and Eq. (8) guarantees that the closed loop system will be asymptotically stable.

3. Controller

A set of assumptions must be satisfied to derive the model-free control law. All system states must be both observable and controllable. Additionally, if the control input is not generated digitally by a computer or microprocessor, assumed that it is measurable. Furthermore, the discontinuous term used to handle the system's uncertainties ensures closed loop asymptotic stability.

3.1. Control Law

The control law is obtained by differentiating Eq. (5) with respect to time and setting the equation equal to zero, to ensure that once the system's states are on the sliding surface, they will remain there. For a 2^{nd}-order system, results in:

$$\dot{s} = \ddot{\tilde{x}} + \lambda\dot{\tilde{x}} = (\ddot{x} - \ddot{x}_d) + \lambda(\dot{x} - \dot{x}_d) \tag{9}$$

Substituting Eq. (2) into Eq. (9) and added a discontinuous term to achieve controller robustness against system's uncertainties. The following control law is obtained:

$$u = b^{-1}[-\lambda(\dot{x} - \dot{x}_d) - (\ddot{x} - \ddot{x}_d) - \eta \mathrm{sgn}(s)] + u_{k-1} - \varepsilon(u) \tag{10}$$

where η is a small positive constant and $sgn(s)$ is the signum function of the sliding surface.

3.3. Switching Gain

The control law, described at Eq. (10), is now redefined as:

$$\hat{u} = \hat{b}^{-1}[-\lambda(\dot{x} - \dot{x}_d) - (\ddot{x} - \ddot{x}_d) - K \mathrm{sgn}(s)] + u_{k-1} - \hat{\varepsilon}(u) \tag{11}$$

where K is the switching gain and $\hat{b}$ is the input gain estimation, obtained by on-line estimation approach. To ensure closed-loop stability sliding condition must be satisfied. Substituting Eq. (7) into Eq. (8) sliding condition yields:

$$\dot{s}s \le -\eta|s| \tag{12}$$

Using the definition of the sliding surface, system model, and the control law Eq. (11) can be written as follows:

$$K = |\beta - 1||\ddot{x} - \ddot{x}_d| + |\beta - 1|\lambda|\dot{x} - \dot{x}_d| + \left|\hat{b}\sigma_u(u_{k-2} - u_{k-1})\right| + \beta\eta \tag{13}$$

where β is an auxiliary variable obtain from estimation.

3.4. Boundary layer

To minimize the chattering effect, introduced by the discontinuous term, a smoothing layer is included in the control law. Boundary layer works as a low pass filter to the dynamics of the sliding surface and eliminates the chattering. To maintain attractiveness of the boundary layer, the sliding condition, Eq. (12) is updated to as follows:

$$|s| \geq \phi \rightarrow 1/2(ds^2/dt) \leq (\dot{\phi} - \eta)|s| \tag{14}$$

where ϕ is the boundary layer thickness. Eq. (14) guarantees that the distance to the boundary layer is always decreasing. The switching gain is then updated to include the boundary layer and the control law becomes:

$$\hat{u} = \hat{b}^{-1}\left[-\lambda(\dot{x} - \dot{x}_d) - (\ddot{x} - \ddot{x}_d) - (K - \dot{\phi})sat(s/\phi)\right] + 2u_{k-1} - u_{k-2} \tag{15}$$

where if $|s/\phi| \leq 1$ then $sat(s/\phi) = s/\phi$, otherwise, $sat(s/\phi) = sgn(s/\phi)$.

4. On-Line Parameter Estimation

Since most physical systems involve slow time-varying parameters, an on-line estimation method is proposed in this paper to estimate the increment to the switching gain and to automatically update the control law in real time.

4.1. Standard Least-squares Method

Slotine and Li [9] defined the output of a linear parametrization model as follows:

$$y(t) = bu(t) \tag{16}$$

where b is the input gain and requires to estimate, $u(t)$ is the signal matrix, and $y(t)$ is the system output. Standard least-squares method generates an estimator by minimizing the following cost function:

$$J = \int_0^t \left\|\hat{b}^T(t)u(\tau) - y(\tau)\right\|^2 d\tau \tag{17}$$

To achieve computational efficiency and evaluate inverting the integral, estimation gain matrix P is introduced and computed recursively. The above equation can be simplified as follows:

$$\frac{d}{dt}[P^{-1}(t)] = \frac{d}{dt}\left[\int_0^t u(\tau)u^T(\tau)d\tau\right] = u(t)u^T(t) \tag{18}$$

By using identity and Eq. (18), the estimation parameter obtain as follows:

$$\dot{\hat{b}} = -Pu(u^T\hat{b} - y) \tag{19}$$

and the estimation gain matrix is obtained:

$$\dot{P} = -Puu^TP \tag{20}$$

The standard least-squares estimator and estimation gain matrix are described by Eq. (19) and Eq. (20) require an initial parameter value $\hat{b}(0)$ and an initial estimator gain value $P(0)$ respectively.

4.2. Least-squares Estimator with Exponential Forgetting

The standard least-squares estimator has good robustness with respect to noise and disturbances, but poor ability in tracking time-varying parameters. The reason is that standard least-squares estimator attempts to fil all the data up to the current time, while in reality, the old data is generated by old parameters. To deduct the old data, an exponential forgetting factor is introduced in the cost function. The new cost function is chosen as follows:

$$J = \int_0^t \exp\left[-\int_\tau^t \lambda(r)dr\right] \left\|\hat{b}^T(t)u(\tau) - y(\tau)\right\|^2 d\tau \tag{21}$$

where $\lambda(t) \geq 0$ is the time-varying forgetting factor. The exponential term in the integral represents the weighting for the data. Then the estimation parameter law is obtained in the same form as Eq. (19).

For implementation in a more efficient way the estimation gain matrix is updated to the following form:

$$d/dt[P] = \lambda(t)P - Pu^T(t)u(t)P \tag{22}$$

The exponential forgetting guarantees the asymptotic convergence of the estimated parameters. However, a constant forgetting factor may diminish magnitude in certain direction of the estimated gain in the absence of persistent excitation (PE) due to the exponential decaying terms which leads to oscillations in the parameter estimation law.

4.3. Bounded gain Forgetting Factor Tuning

To avoiding the possibility of gain unboundedness, it is desirable to tune the forgetting factor with variation, so the data forgetting is activated when $u(t)$ is non-contstant, and suspended when $u(t)$ is not.

A specific technique for achieving this purpose is to choose:

$$\lambda(t) = \lambda_0(1 - \|P(t)\|/k_0) \tag{23}$$

where λ_0 is the maximum forgetting rate, and k_0 is the pre-specified bound for gain matrix. Essentially, if the norm of $P(t)$ implies a strong PE, the forgetting factor in Eq. (23) discards the data at maximum rate λ_0. As $\|P(t)\|$ becomes larger the forgetting speed is reduced and when the norm reaches the pre-specified upper bound k_0, the forgetting is suspended. Initially the gain matrix must be chosen such that $\|P(0)\| \leq k_0$.

5. Simulation

To validate the sliding mode controller proposed in this paper, a second order nonlinear nonunitary input gain system using a unitary input gain assumption for estimation of increment to switching gain within a smoothing boundary layer was used as an illustrative example.

Consider the following second-order nonlinear model to be controlled:

$$\ddot{x} + 3x\dot{x} + 5x^2 = 3u \tag{24}$$

Simulations were performed using Simulink, Matlab, and the ode5 (Dormand-Price) solver for 30 seconds, with a fixed sample time 0.001 seconds. The controller parameters were $\lambda = 20$, $\phi = 5$, $\sigma_u = 0.2$, $\lambda_b(0) = 2$, $p_0 = 5$, and $k_0 = 500$. The reference signal was defined as $x_d(t) = \sin(\pi t/2)$. The following results were obtained.

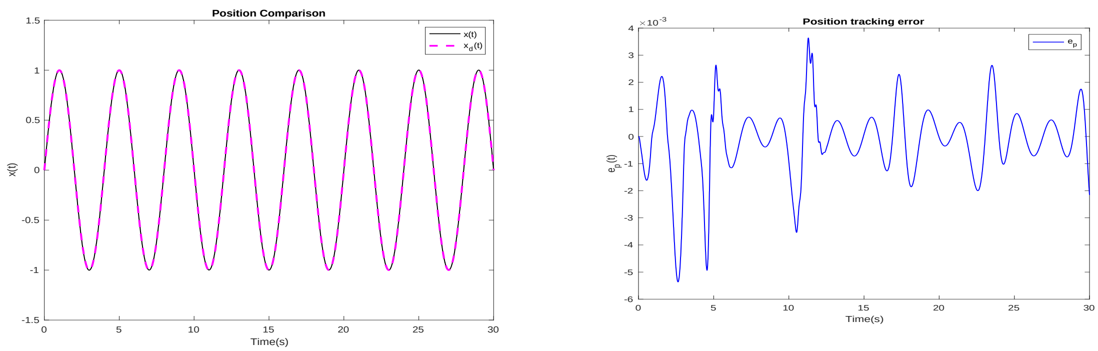

Figure 1: Position comparison and position tracking error

Figure 1 displays the position comparison and position tracking error of the closed-loop system. The position state follows the desired position almost perfectly. The position error is less than 4e-3, which proves precise tracking of the sliding mode controller.

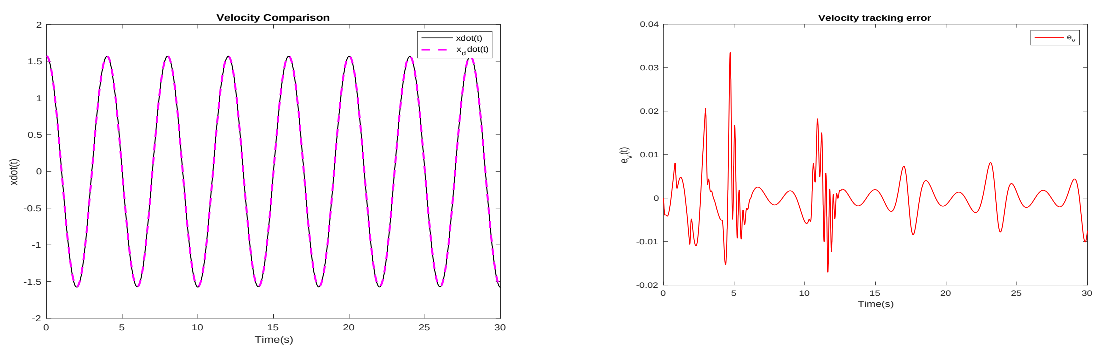

Figure 2: Velocity comparison and velocity tracking error.

The velocity comparison and velocity tracking error are shown in Figure 2. The difference between the velocity state and desired one is negligible. The velocity tracking error is greater compared to the position error, but it is still minimal, with values less than 0.04.

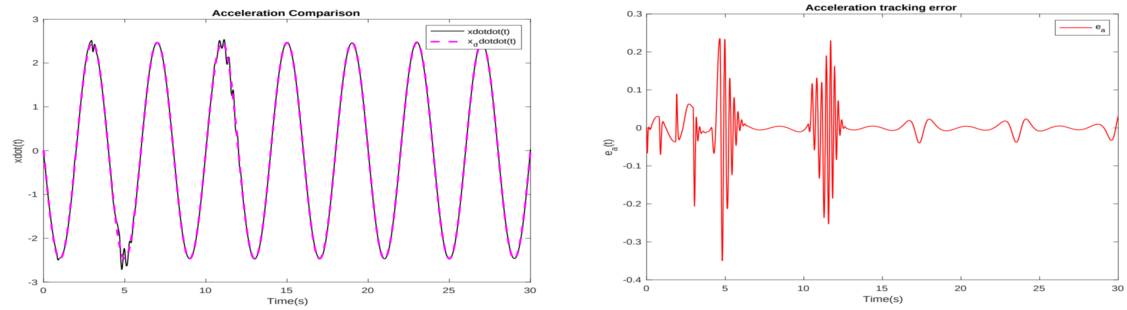

Figure 3: Acceleration comparison and acceleration tracking error.

Figure 3 shows the acceleration comparison and acceleration tracking error. The acceleration state agrees with the reference signal with slight deviation on peaks initially. The maximum acceleration tracking error is less than 0.30 and chattering exist. Perfect tracking for the acceleration response is not achieved, but can be considered as acceptable.

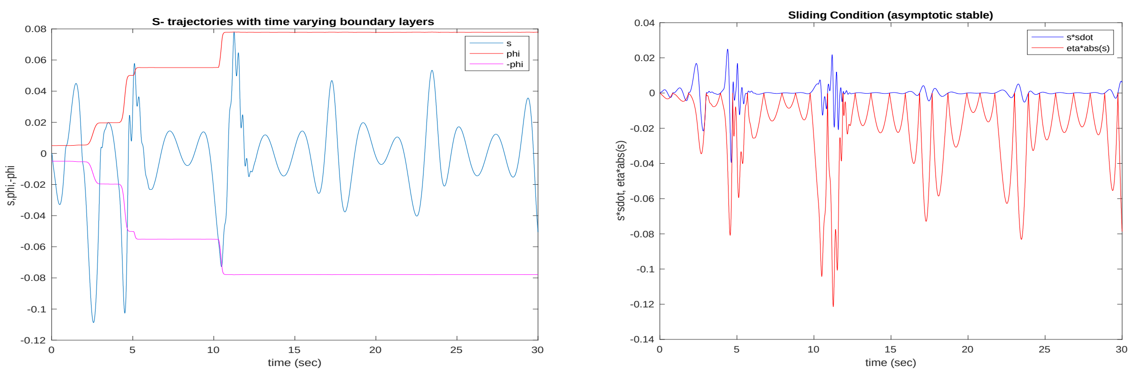

Figure 4: Sliding surface trajectories and sliding condition.

Figure 4 displays *s*-trajectories along with the boundary layer and sliding condition. The sliding surface initially was outside the boundary layer. However, as the increment to the switching gain, η is estimated and updated boundary layers automatically to satisfy the sliding condition, the sliding surface converges within the boundary layer and remains there. The updated sliding condition was not initially satisfied but becomes satisfied as the estimation process proceeds and remains satisfied.

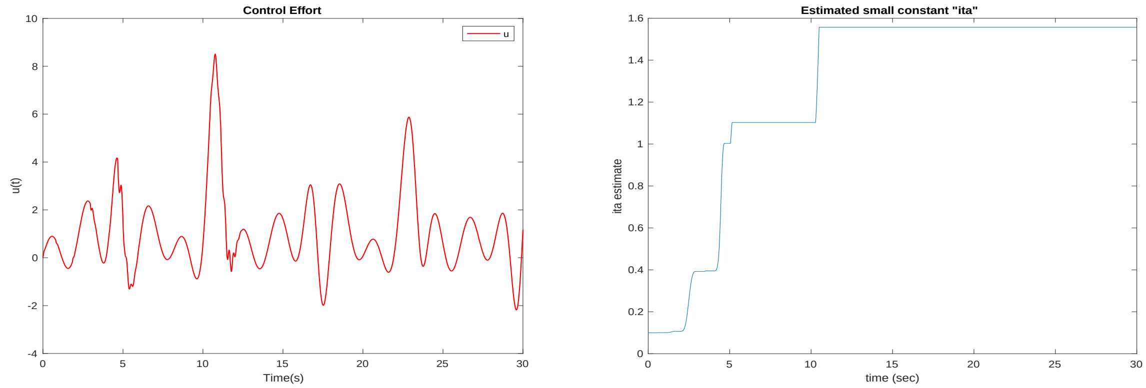

Figure 5: Control effort and estimated η.

Figure 5 shows the control effort and estimation of increment to the switching gain. The control effort is reasonable and smooth without any effect of chattering. The estimated increment to the switching gain, η resulting from the least-squares estimation process maintained the sliding condition. The increment to the switching gain converges as the sliding condition is satisfied verified by the results shown in Figure 4. The input influence gain varies in time and the least-squares estimation algorithm achieved outstanding tracking performance.

4. Conclusion

The model-free sliding mode controller which was implemented for the nonlinear system with varying boundary layer achieved outstanding results. The focus of the paper was on estimating the increment to the switching gain in real time and updating the model-free control law automatically to maintain sliding condition and ensure asymptotic stability. The estimated increment to the switching gain, η perfectly captured the initial deficiency, the sliding condition was not satisfied since the control law assumed a unitary input influence gain which was not the case for the actual system. The estimated increment to the switching gain was varied by the least-squares algorithm so that the sliding condition satisfied ensuring asymptotic tracking stability. In the entire simulation time, switching gain remained positive and control effort was reasonable without chattering and the response was smooth. Overall, the concept of estimating the increment to the switching gain, to compensate for assuming a unitary gain control law, appeared feasible. Thus, a true model-free approach for controlling both nonlinear and liner systems possible.

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