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Dynamic Modelling of the Standard Neonatal Patient Transport System using a Newton-Euler Based Formulation in the Roll Plane

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***Abstract* -** Transport of neonatal patients between critical care units can expose patients to whole-body vibrations which may pose a risk to the vulnerable patients' health. The concern for patient safety has motivated a study on characterizing and mitigating vibrations transmitted by the Neonatal Patient Transport System (NPTS) that is used in ground and air ambulances in Ontario. To supplement in-vehicle testing, a simulation is being developed to replicate the motion of the NPTS. The first stage of developing this model involves simulating a planar representation of the NPTS in order to identify unknown system parameters. This paper outlines the derivation of equations of motion of the NPTS in the roll plane by applying the Newton-Euler method. The acceleration power spectral density (PSD) of the simulated motion is compared against recorded road test data to aid in tuning the system parameters. Simulated results show similar frequency responses for the vertical motion of the system. However, the roll direction deviates from the measured response. Further optimization is required to calibrate and validate this model to ensure it represents the angular motion of the system and reproduces behaviour exhibited in various transport conditions.

***Keywords*:** neonatal transport; frequency response; vibration analysis; dynamic model; power spectral density

1. Introduction

In recent years, a standardized Neonatal Patient Transport System (NPTS) has been introduced in Ontario to support the transport of neonates between medical facilities [1]. This system is used in both ground and air ambulances and ensures interoperable equipment is used by all operators within the province. Despite being vital to the transport of patients to receive critical care, substantial vibration of the system has been observed by transport teams. Motion in the vertical direction, as well as rotation about the roll axis, have been the greatest concern. The International Organization for Standardization identifies high-intensity whole-body vibration as a health risk [2]. In the specific care of neonatal patients, transport results in an elevated risk of intracranial haemorrhage [3]. To quantify, and ultimately reduce, the vibrations that neonatal patients are exposed to, a joint study is underway between Carleton University, the Children's Hospital of Eastern Ontario (CHEO), and other industry and government partners.

To characterize vibration within ground vehicles, on-road tests have been performed to record road data and motion at different locations across the NPTS. The road data were used to drive in-laboratory shaker tests, including tests of just the NPTS driven by the floor excitation, and a ground ambulance test on a four-post shaker. Preliminary mitigation tests have been performed as well, and shaker tests of the NPTS isolette are underway.

To assist in this study, a dynamic model is being developed. The objective of the model is to have a three-dimensional, seven-degree-of-freedom simulation that reproduces the dynamics of the system. The model is to use input floor motion to predict frequency responses across the stretcher and at the patient level. Before this complex model is completed, two planar models are being considered, one in the pitch plane and one in the roll plane. These models are used to determine unknown system parameters. This paper presents the derivation of the roll model using the Newton-Euler formulation, as the pitch model has been independently derived using Lagrange's equation [4]. The acceleration power spectral density (PSD) at multiple locations is compared against recorded data from on-road testing to evaluate the performance of the model and establish unknown stiffness and damping parameters. The paper details the development of the planar roll model and tests whether this model is representative of the frequency response exhibited by the physical system.

2. Neonatal Patient Transport System

The NPTS is a collection of medical equipment necessary for providing care to neonatal patients. The system includes an incubator, in which the patient is harnessed atop a mattress. The NPTS is secured to a deck stacked on top of a stretcher; the stretcher used in these studies is the Stryker Power-PRO™ XT (Stryker®, model #6506, MI, USA). In the ground vehicles used by Ottawa Paramedic Service (OPS), the stretcher is loaded and secured to the floor using the Stryker Power-PRO® loading system (Stryker®, model #6390, MI, USA).

The test data used to run the simulations was recorded during preliminary ground ambulance testing. This test involved driving on a variety of road types throughout Ottawa, Canada, and accelerometer and inertial measurement unit (IMU) data were recorded at locations on the floor of the vehicle and across the stretcher, NPTS, and baby manikin [5]. The test replicated by the current roll model corresponds to a 2.5 kg manikin transported over a high-speed arterial road, with the frequency response based off data recorded over a duration of 278 seconds.

3. Dynamic Model

The purpose of a simulation model is to replicate the dynamics of the physical system. Ultimately, a seven-degree-of-freedom multi-body dynamic model will be constructed to simulate motion of the NPTS in three dimensions. The model will be subjected to motion representing various transport conditions, and the resulting motion is to be recorded at various locations across the system, including the patient level within the incubator. To model this behaviour, the compliance of the stretcher system must be identified. Two planar models have been considered to reduce the number of generalized coordinates involved during parameter optimization. The frequency response of these models is to be driven to match data recorded during on-road tests to establish system parameters.

The dynamic model assumes rigid fixtures between the NPTS components, including the incubator, the interface deck, and stretcher. Throughout this derivation, this entire configuration of medical equipment, as well as the stretcher, will be considered a rigid body and will be referred to as the NPTS for simplicity. To model the interface between the NPTS and the floor of the vehicle, two spring-dampers are used, and a third is used to represent the viscoelastic properties of the mattress. The model is based off the orientation of the NPTS as used in ground ambulances by the OPS, in which the NPTS is longitudinally aligned with the vehicle body, slightly offset to the left of the centreline of the vehicle, with the foot of the stretcher pointing towards the rear and the incubator facing the front.

3.1. Roll Model

The model assumes the standard SAE vehicle coordinate system, where roll occurs about the vehicle's longitudinal X-axis, the Y-axis is directed laterally out the passenger side of the vehicle, and the Z-axis points downward [6]. The physical NPTS and diagram are presented in Fig. 1. The roll model of the NPTS is viewed from the rear, and it is assumed the centre of mass is along the longitudinal centreline of the NPTS. The height of the centre of mass, as well as the mass moment of inertia, are estimated based on the masses and relative positions of medical equipment on the NPTS [7, 8]. The mass and geometric properties of the roll model are presented in Table 1. The mass of the patient within the incubator, m, is treated as a point mass. Due to the near symmetry of the loading mechanism and stretcher about the longitudinal centreline, it is assumed that the equivalent spring-damper coefficients are equal at both attachments with the floor. The floor motion consists of three components: heave (z), sway (y), and roll (θ).

Table 1: Roll model properties.

Parameter	Symbol	Value
Initial height of patient from datum	h	0.6 m
Initial height of NPTS centre of mass from datum	r	0.43 m
Distance from centre of mass to side of stretcher	d	0.23 m
Mass of NPTS and stretcher	M	227.50 kg
Mass of patient	m	2.55 kg

Mass moment of inertia about the NPTS centre of mass	I_x	9.40 kg·m^2

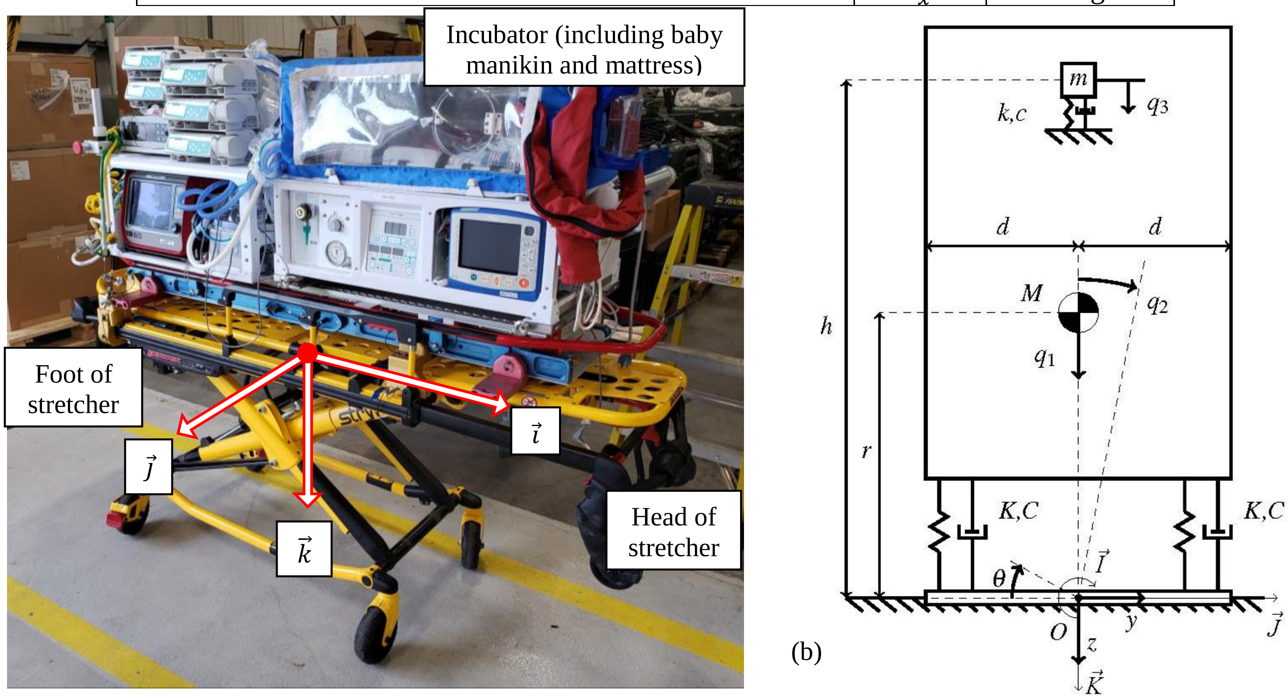


Fig. 1: (a) NPTS with body-fixed coordinate system; (b) Diagram of NPTS in the roll plane with global coordinate system.

4. Equations of Motion

This planar model is a three-degree-of-freedom problem with three generalized coordinates: q_1 represents the vertical displacement of the NPTS along its vertical axis; q_2 is the angular position of the NPTS; and q_3 is the displacement of the patient mass within the incubator, relative to its initial position in the NPTS. Coordinate q_2 is measured relative to the vehicle floor. Roll of the NPTS will be about a pivot point located on the floor, considering the constraint of the mechanical interface securing the stretcher to the floor.

The initial position of the vehicle floor is aligned with a global coordinate system fixed at Point *O*. The absolute positions, velocities, and accelerations of masses *M* and *m* are found relative to this point.

4.1. Kinematics

First, the positions of both masses are expressed in the global reference frame,

$$\vec{p}_M = -(q_1 - r)\sin q_2 \vec{J} + (q_1 - r)\cos q_2 \vec{K}, \tag{1}$$

$$\vec{p}_m = -(q_1 + q_3 - h)\sin q_2 \vec{J} + (q_1 + q_3 - h)\cos q_2 \vec{K}. \tag{2}$$

The position vectors are differentiated to obtain velocities,

$$\vec{v}_M = (-\dot{q}_1 \sin q_2 - (q_1 - r)\dot{q}_2 \cos q_2)\vec{J} + (\dot{q}_1 \cos q_2 - (q_1 - r)\dot{q}_2 \sin q_2)\vec{K}, \tag{3}$$

$$\vec{v}_m = (-(\dot{q}_1 + \dot{q}_3)\sin q_2 - (q_1 + q_3 - h)\dot{q}_2 \cos q_2)\vec{J} + \big((\dot{q}_1 + \dot{q}_3)\cos q_2 - (q_1 + q_3 - h)\dot{q}_2 \sin q_2\big)\vec{K}. \tag{4}$$

The velocity vectors are again differentiated to obtain accelerations

$$\vec{a}_M = \Big(-\ddot{q}_1 \sin q_2 - 2\dot{q}_1\dot{q}_2 \cos q_2 + (r - q_1)\big(\ddot{q}_2 \cos q_2 - \dot{q}_2{}^2 \sin q_2\big)\Big)\vec{J} \\ + \Big(\ddot{q}_1 \cos q_2 - 2\dot{q}_1\dot{q}_2 \sin q_2 + (r - q_1)\big(\ddot{q}_2 \sin q_2 + \dot{q}_2{}^2 \cos q_2\big)\Big)\vec{K}, \tag{5}$$

$$\vec{a}_m = \Big(-(\ddot{q}_1 + \ddot{q}_3) \sin q_2 - 2(\dot{q}_1 + \dot{q}_3)\dot{q}_2 \cos q_2 - (q_1 + q_3 - h)\big(\ddot{q}_2 \cos q_2 - \dot{q}_2{}^2 \sin q_2\big)\Big)\vec{J} \\ + \Big((\ddot{q}_1 + \ddot{q}_3) \cos q_2 - 2(\dot{q}_1 + \dot{q}_3)\dot{q}_2 \sin q_2 - (q_1 + q_3 - h)\big(\ddot{q}_2 \sin q_2 + \dot{q}_2{}^2 \cos q_2\big)\Big)\vec{K}. \tag{6}$$

To simplify these vectors, the accelerations are rotated about $\vec{I}$ to align them with a body-fixed reference frame, where $\vec{j}$ is along the base of the NPTS and $\vec{k}$ is along the vertical centreline of the NPTS, using

$$\vec{a}_{local} = \mathbf{R}\vec{a}_{global}, \\ \mathbf{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos q_2 & \sin q_2 \\ 0 & -\sin q_2 & \cos q_2 \end{bmatrix}. \tag{7}$$

The body-fixed accelerations are found to be

$$\vec{a}_M = (-2\dot{q}_1\dot{q}_2 - (q_1 - r)\ddot{q}_2)\vec{j} + \big(\ddot{q}_1 - (q_1 - r)\dot{q}_2{}^2\big)\vec{k}, \tag{8}$$

$$\vec{a}_m = (-2(\dot{q}_1 + \dot{q}_3)\dot{q}_2 - (q_1 + q_3 - h)\ddot{q}_2)\vec{j} + \big(\ddot{q}_1 + \ddot{q}_3 - (q_1 + q_3 - h)\dot{q}_2{}^2\big)\vec{k}. \tag{9}$$

4.2. Newton-Euler Formulation

The body-fixed reference frame considers forces that are acting along the axes of the NPTS body. To represent the interface between the NPTS and vehicle floor, a sliding pin joint is introduced to allow for rotation about the pivot point on the floor. The free-body diagrams of the system is shown in Fig. 2.

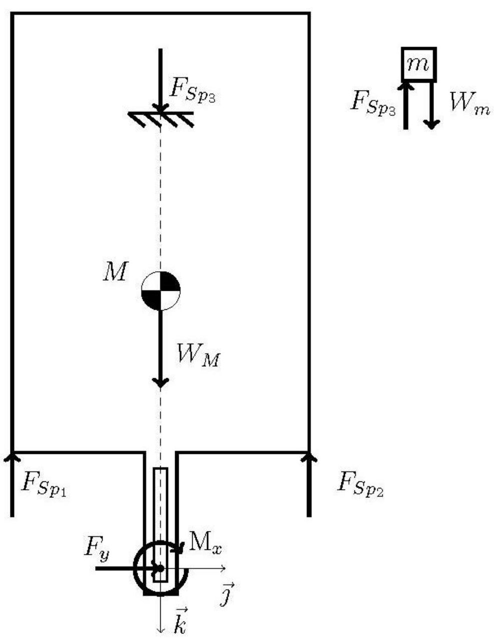


Fig. 2: Free-body diagram dynamic model of NPTS in roll plane.

Force balances in the horizontal and vertical directions, as well as a moment balance about the pivot point at the floor are expressed as,

$$\Sigma F_{M_y} = M a_{M_y} \tag{10}$$
$$\Sigma F_{M_z} = M a_{M_z} \tag{11}$$
$$\Sigma M_x = I \ddot{q}_2 \tag{12}$$

Beginning with Eq. (10),

$$\Sigma F_{M_y} = W_M \sin q_2 + F_y = M a_{M_y}, \tag{13}$$

where W_M is the weight of the NPTS. The resulting force due to horizontal floor motion, F_y, is found by considering a moment about the centre of mass of *M*,

$$\vec{r}_{floor/CM} \times \vec{F}_y = I_x \alpha_{floor/CM}. \tag{14}$$

Given the vertical distance between the floor and centre of mass is $r_{floor/CM} = r + z - q_1$, and the angular acceleration of the floor with respect to the centre of mass can be expressed as $\alpha_{floor/CM} = \frac{\ddot{y}}{(r+z-q_1)}$, Eq. (14) becomes

$$F_y = \frac{I_x \ddot{y}}{(r + z - q_1)^2}. \tag{15}$$

Substituting Eq. (15) into Eq. (13) gives

$$Mg \sin q_2 + \frac{I_x \ddot{y}}{(r + z - q_1)^2} = M(-2\dot{q}_1 \dot{q}_2 - (q_1 - r)\ddot{q}_2). \tag{16}$$

Considering Eq. (11),

$$\Sigma F_{M_z} = -F_{Sp_1} - F_{Sp_2} + F_{Sp_3} + W_M = M a_{M_z} \tag{17}$$

where F_{Sp_1}, F_{Sp_2}, and F_{Sp_3}, are the forces of the spring-dampers at the foot of the stretcher, the head of the stretcher, and the patient's mattress. Force F_{Sp_3} can be expressed using a vertical force balance on patient mass, *m*,

$$\begin{gathered} \Sigma F_{m_z} = W_m - F_{Sp_3} = m a_{m_z}, \\ mg - c\dot{q}_3 - kq_3 = m\left(\ddot{q}_1 + \ddot{q}_3 - (q_1 + q_3 - h)\dot{q}_2{}^2\right). \end{gathered} \tag{18}$$

where c and k are the damping and stiffness coefficients of the mattress. Eq. (18) is substituted into Eq. (17), and is simplified to become

$$-2C(\dot{q}_1 - \dot{z}) - 2K(q_1 - z) - m\left(\ddot{q}_1 + \ddot{q}_3 - (q_1 + q_3 - h)\dot{q}_2{}^2\right) + mg + Mg = M\left(\ddot{q}_1 - (q_1 - r)\dot{q}_2{}^2\right). \tag{19}$$

where C and K are the damping and stiffness coefficients of the stretcher.

Considering the moment balance occurs about Point *O*, and the mass moment of inertia is only known at the centre of mass of the NPTS, the parallel axis theorem must be applied. Given the vertical distance to the centre of mass is $q_1 - r$, then $I = I_x + M(q_1 - r)^2$, and the moment balance defined in Eq. (12) becomes

$$-F_y z + F_{Sp_1} d - F_{Sp_2} d - mg \sin q_2 (h - q_1 - q_3) + Mg \sin q_2 (r - q_1) = I_x \ddot{q}_2 + M(q_1 - r)^2 \ddot{q}_2. \tag{20}$$

A force balance of the patient mass *m* in the *y* direction gives

$$\Sigma F_{m_y} = m a_{m_y} \tag{21}$$
$$W_m \sin q_2 = m a_{m_y} \tag{22}$$
$$mg \sin q_2 = m(-2(\dot{q}_1 + \dot{q}_3)\dot{q}_2 - (q_1 + q_3 - h)\ddot{q}_2). \tag{23}$$

Considering the NPTS is in pure rotation about the pivot point, the weight of the NPTS and the horizontal force due to the floor become a couple, and the right-hand side of Eq. (16) becomes 0. Given $(q_1 - r)\ddot{q}_2$ is the vertical displacement of M multiplied by its angular acceleration, the tangential acceleration of M, the component in the y direction, may be substituted in Eq. (20), along with Eqs. (15), (16), (23), and the spring-damper forces. The final equation becomes

$$-\frac{I_x \ddot{y}}{(r+z-q_1)} - 2Cd^2\dot{q}_2 + 2Cd^2\dot{\theta}\cos\theta - 2Kd^2 + 2Kd^2\theta\sin\theta - 2m\dot{q}_2(\dot{q}_1+\dot{q}_3)(q_1+q_3-h) \\ - m\ddot{q}_2(q_1+q_3-h)^2 = I_x\ddot{q}_2 + M(q_1-r)(2\dot{q}_1\dot{q}_2 + (q_1-r)\ddot{q}_2). \tag{24}$$

Eqs. (19), (24), and (18), provide the three equations of motion. In matrix form these are expressed as:

$$\begin{bmatrix} M+m & 0 & m \\ 0 & Mr^2+mh^2+I_x & o \\ m & o & m \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{Bmatrix} + \begin{bmatrix} 2C & 0 & 0 \\ 0 & 2Cd^2 & 0 \\ 0 & 0 & c \end{bmatrix} \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{Bmatrix} + \begin{bmatrix} 2K & 0 & 0 \\ 0 & 2Kd^2 & 0 \\ 0 & 0 & k \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix} \\ = \begin{Bmatrix} M\dot{q}_2^2(q_1-r) + m\dot{q}_2^2(q_1+q_3-h) + 2C\dot{z} + 2Kz + Mg + mg \\ -2M\dot{q}_1\dot{q}_2(q_1-r) - M\ddot{q}_2(q_1^2 - 2q_1 r) - 2m\dot{q}_2(\dot{q}_1+\dot{q}_3)(q_1+q_3-h) \\ -m\ddot{q}_2(q_1^2 + 2q_1q_3 - 2q_1h + 2q_3h) + 2Cd^2\dot{\theta}\cos\theta + 2Kd^2\sin\theta - \dfrac{I_x\ddot{y}}{(r+z-q_1)} \\ m\dot{q}_2^2(q_1+q_3-h) + mg \end{Bmatrix} \tag{25}$$

These equations have been validated by an independent derivation using the Lagrange formulation. This previous derivation was conducted in the pitch plane of the NPTS, where the centre of mass was offset from the centreline of the NPTS. The patient mass was offset from the centre of mass, and the floor spring-dampers were assumed to be not equal. Once solved, the terms for these offsets were cancelled out and the spring-damper parameters equated, and the equations of motion reduced to those presented in Eq. (25).

5. Results

Using IMU data recorded during on-road testing, random vibration representative of floor motion during a high-speed segment of road was used for the input z, y, and θ directions in a MATLAB simulation based on Eq. (25). The components of the input motion are shown in Fig. 3. The equations of motion were used to predict the vertical response at the NPTS centre of mass and patient, and the rotational response of the NPTS body. The positions and accelerations of each generalized coordinate are presented in Fig. 4.

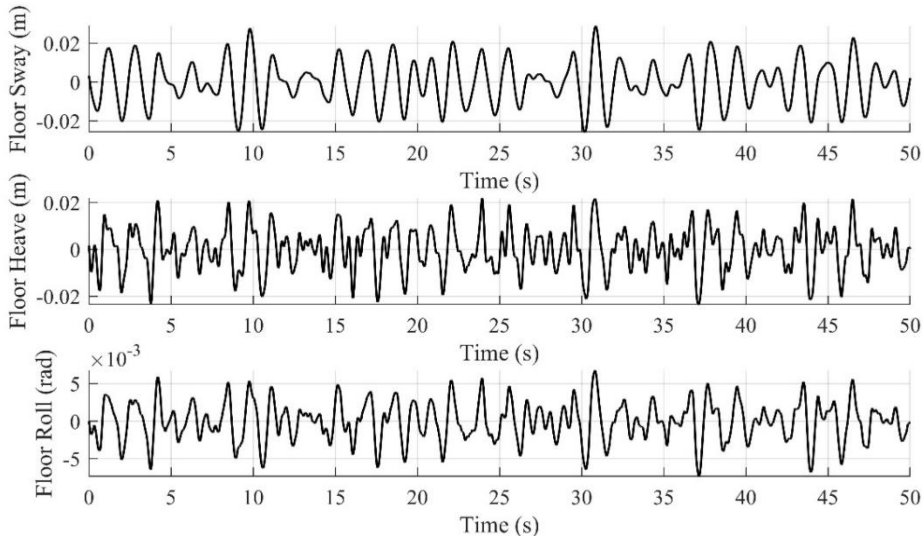
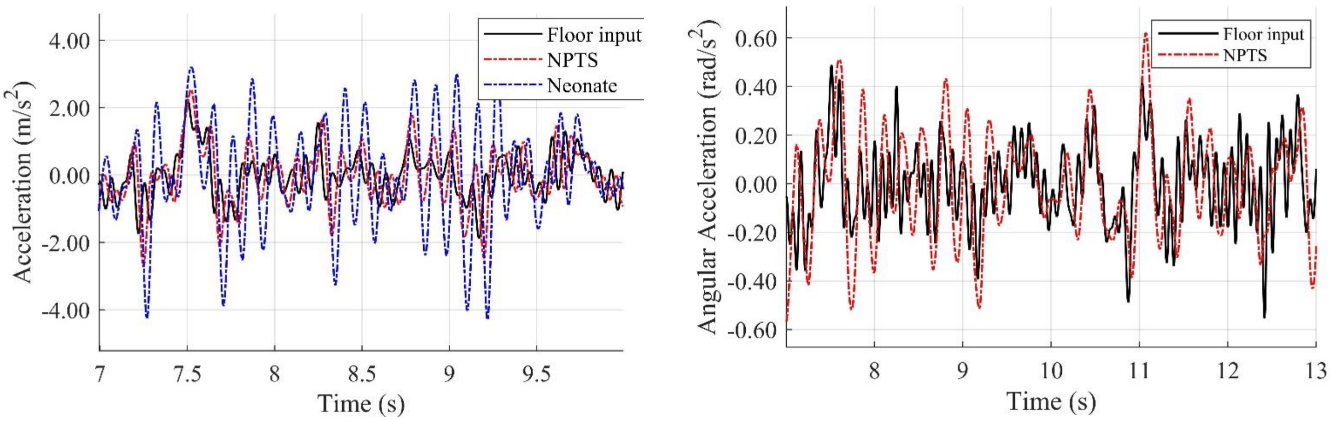


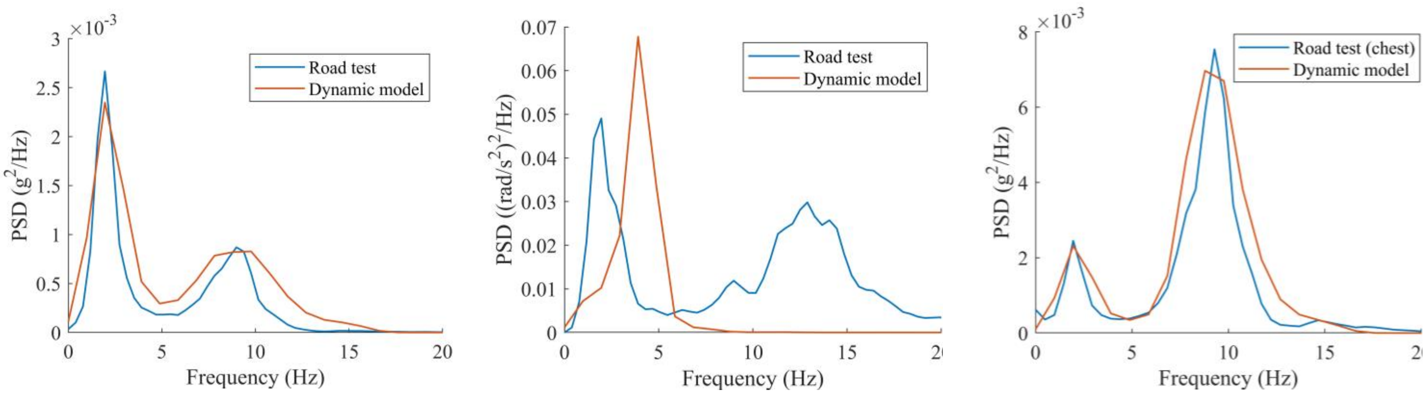
Fig. 3: Floor motion in the y, z, and θ directions.



(a) (b)

Fig. 4: Simulated acceleration of NPTS in the (a) vertical direction; and (b) roll direction.

To compare against road data, the PSDs of the simulated accelerations in the vertical direction were driven towards the measured PSDs by tuning the stiffness and damping coefficients. The PSDs were solved using Welch's power spectral density estimate in MATLAB, using data segmentation windows of 1000 points with 50% overlap. The spring-damper parameters used for these distributions are K = 330000 N/m, k = 8000 N/m, C = 1700 Ns/m, and c = 60 Ns/m. Figure 5 presents the acceleration PSDs exhibited by the NPTS in the vertical and roll directions, and the patient mass in the vertical direction. Since the model represents the neonate with respect to the NPTS, plots of neonate motion include a summation



(a)

of q_1 and q_3 for the absolute position and its derivatives. Solving for the eigenvalues of the equations of motion, the natural frequencies of the system were found to be 4.11 Hz, 8.33 Hz, and 9.26 Hz. The corresponding damped natural frequencies were 4.10 Hz, 8.24 Hz, and 9.07 Hz.

Fig. 5: Power spectral densities: (a) NPTS, vertical; (b) NPTS, roll; (c) patient, vertical.

6. Discussion

The vertical acceleration PSDs of the NPTS and neonate closely resemble those for the recorded data, as seen in Fig. 5 (a) and (c). Both of these curves exhibit the greatest magnitudes around the same excitation frequencies. The amplification experienced by the neonate is around 9 Hz, and it was found that this is one of the largest vibrations of the model. This suggests the model is providing the amplification of this mode that was observed in the physical system. The angular acceleration PSD of the NPTS is presented in Fig. 5(b). The simulated results show a high magnitude peak at a lower frequency; however it does not align with the high vibration experienced during the on-road test. The magnitude is greater than that of the recorded data, and it appears that frequencies higher than 9 Hz are not contributing to the simulated PSD.

The parameters used for the spring-dampers in this roll model are the same as those used in the previously-developed pitch model. Further tuning is required to ensure they are suitable for various transport conditions; however the vertical acceleration PSDs in the pitch plane model were also very close to representing the real world system. Both models exhibit lower amplitudes in the angular frequency response, and shifts in frequency, suggesting further refinement is required to ensure it is suitable in this direction. Other factors that may affect the angular motion of the NPTS, including the floor attachment mechanism, will be considered.

7. Conclusion

A dynamic model has been developed to represent the motion of the NPTS and stretcher system in the roll plane. Equations of motion were derived using the Newton-Euler formulation and were compared against a previous derivation using Lagrange's equation. The frequency response of the roll model has been compared against recorded data, and it was found that the PSDs of the NPTS and patient in the vertical direction closely resemble those of the road data. Differences in magnitude and frequency are present in the roll PSD, suggesting further adjustments to the model may be required. The system parameters are to be tuned to ensure the model satisfies different transport conditions experienced in ground and air ambulances.

Future plans for further developing the dynamic model include testing both the pitch and roll models against various transport conditions to ensure system parameters are suitable over a range of operating conditions. A three-dimensional multi-body dynamic model will be constructed based on the properties of the two planar models, and further simulation, and refinement of model parameters, will be performed. This dynamic model is intended to replicate motion in seven-degrees-of-freedom and can be used to support the design of vibration mitigation efforts to reduce motion at the incubator level. Ultimately, this model can be a tool to further understand the dynamic properties of the NPTS in versatile transport environments, with the goal of increasing the safety of neonates during transport.

Acknowledgements

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