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Scaled Consensus Of Hybrid Multi-Agent Systems

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Abstract **-** This paper studies the scaled consensus problem of a class of multi-agent systems called hybrid multi-agent systems, which consist of both continuous-time and discrete-time dynamic agents. Based on the interactions among agents, two scaled consensus protocols are proposed to solve the scaled consensus problems in the hybrid multi-agent systems employing directed communication graphs that contain a spanning tree. Some numerical examples are provided to illustrate the effectiveness of the theoretical results.

Keywords**:** Scaled consensus; Hybrid multi-agent system; Spanning tree

1. Introduction

Over the past decades, multi-agent systems have been widely studied in many disciplines such as engineering, computer science, and mathematics. They are typically referred to software agents in computer science, while in robotic society, multi-agent systems are referred to multi-robot systems (as the agents can be robots), which have been extensively studied since the early 1990s. For example, Weiss (1999,[1]) replaced the single agents by the multiple agents as the computing paradigm in artificial intelligence, Sugihara and Suzuki (1990,[2]) studied the distributed motion coordination of multiple robots. In the recent years, many research topics in multi-agent systems have been actively studied, such as consensus or agreement problems (2007,[3]), flocking(2006,[4]), formation control (2009,[5]), coverage control (2002,[6]), containment control (2011,[7]), distributed estimation (2008,[8]), and others.

A multi-agent system is a dynamical system consisting of a group of agents which can interact with each other or their environment. Consensus is one of the fundamental problems in multi-agent coordination, which implies that all agents reach an agreement on some common features, which can be velocities, positions, attitudes, and many other quantities. As a result, a consensus algorithm, an appropriate control input based on the local information that enables all agents to reach consensus, has been actively studied for a long time. For example, the original work of consensus problems was proposed by Degroot (1974,[9]), and similar idea were found in distributed computing in the works of Tsitsiklis (1984,[10]) and extensive references.

In recent years, many consensus algorithms were proposed based on the dynamic model of agents. In 1995, for example, Vicsek et al.,[11] presented the discrete time model of agents moving with the same speed and proved by simulation that all agents can move to one direction. In 2003, Jadbabaie et al.,[12] used the nearest neighbor rules for proving the model of Vicsek in [11]. Moreover, Wang et al.,(2007,[13]) proposed the new method for solving consensus problems of discrete time multi-agent systems with time-delays, more results about consensus seeking in discrete time multi-agent systems can be seen in ([14–16]) and references therein.

For continuous-time dynamic agents, many consensus algorithms have been proposed, such as in 2004, Olfati-Saber and Murray[17] showed the consensus results of continuous time dynamic agents with switching topology and time-delays. In addition, Ren and Beard(2005,[18]) also studied the consensus in continuous time multi-agent systems and used some concepts from graph theory and matrix theory to extend the results in [17], which gave more relaxation conditions than the previous works. More results about consensus seeking in continuous time multi-agent systems can be seen in ([19],[20]) and extensive references.

Inspired by the result of Halloy et, al,.([21], 2007) who studied a group decision-making between animals and autonomous robots, specifically, a group of cockroaches and autonomous robots, which share the shelter together under some conditions. Hence, it is reasonable to study consensus problems in the dynamical systems involving the interaction of continuous and discrete dynamics, which is typically called the hybrid systems (Antsaklis(2000),[22]). In the recent years, the study of consensus problems under switching topologies, one of the classic hybrid systems, has been actively attracted by many researchers. For example, in 2008, Sun et al.,[23] showed the average consensus results of dynamic agents with switching topologies and time-varying delays.

There are, however, some practical applications, the states of all agents are not necessary to achieve consensus on a common quantity, but of their own scales due to the constraints of physical environments, such as, compartmental mass-action systems [24], water distribution systems and multiscale coordination control between spacecrafts and their simulating vehicles on the ground [25]. In order to deal with these problems, Roy[26] proposed the out standing idea of scaled consensus, allowing prescribed ratios among the final convergent values of all agents. It can be seen that the scaled consensus is more general than the standard consensus i.e., it can achieve standard consensus when all ratios are one. Furthermore, by adopting appropriate scales, the scaled consensus provides a less conservative framework and solves many consensus problems, for example, bipartite (or sign) consensus[27] and cluster consensus [28], where all agents in the same subnetwork share a common value while there is no agreement between different subnetworks. In 2018, Zheng et al.[29] studied consensus problems of hybrid multi-agent systems consisting of continuous time dynamic agents and discrete-time dynamic agents. By assuming that all agents interact with their neighbours at the sampling time tk, two classes of consensus protocols were proposed for solving consensus in the hybrid multi-agent system. However, to our knowledge, there has been no systematic study on scaled consensus problems for hybrid multi-agent systems, so this work aims to study scaled consensus to the hybrid multi-agent system under directed

communication networks.

The rest of this paper is organized as follows. Some preliminaries and the problem formulation are provided in Section II. In Section III, two scaled consensus protocols have been introduced to solve the scaled consensus problems of hybrid multi-agent systems via directed topology. In Section IV, numerical examples are provided to illustrate the effectiveness of our main results. Finally, some conclusions are drawn in Section V.

2. Preliminaries And Problem Formulation

In this section, we introduce some basic concepts from algebraic graph theory. We also give some definitions and lemmas for later use. For more details, refer to [30, 31].

Throughout this paper, an interaction among n agents is described as a weighted directed graph $G = (V, E, A)$ that consists of a set of nodes $V = \{v_1, v_2, \dots, v_n\}$ and the set of edges $E = \{e_{ij} = (v_i, v_j)\} \subseteq V \times V$ a nonnegative matrix $A = [a_{ij}]_{n\times n}$. The set of all neighbours of an agent i is denoted by $N_i = \{k: a_{ik} > 0\}$. The out-degree and in-degree of node v_i are denoted by $\deg_{out}(v_i)$ and $\deg_{in}(v_i)$ which is the number of edges $e_{ij} = (v_i, v_j)$ and $e_{ki} = (v_k, v_i)$, respectively. A graph G is balanced if the out-degree and in-degree of each node are equal. A directed path of G is a sequence of edges (v_{i1}, v_{i2}), (v_{i2}, v_{i3}), (v_{i3}, v_{i4}),… in a digraph G. A digraph G is called strongly connected if there is a directed path connecting any two arbitrary nodes in G. A directed tree is a digraph such that there is only one root (that is, no edge points to this node) in it, and every node except the root has exactly one parent. A spanning tree of G is a directed tree that connects all the nodes of G .

Moreover, we denote by R, the real number set, N the positive integer set, R^n the n dimensional real vector space.

For a given vector or matrix X, X^T denotes its transpose, $||X||$ denotes the Euclidean norm of a vector X. A vector is nonnegative if all its elements are nonnegative and the column vector with all entries equal to one or zeroes are denoted by 1_n and 0_n, respectively. I_n is an $n-$dimensional identity matrix and the diagonal matrix with diagonal elements being $a_1, a_2, a_3, \dots, a_n$ is denoted by $\mathrm{diag}(a_1, a_2, a_3, \dots, a_n)$. Moereover, $[a_{ij}]_{n\times n}$ is an n by n matrix with a_{ij} representing its (i, j)th entry.

A matrix $B = [b_{ij}]_{n\times n}$ is said to be nonnegative, denoted by $B \geq 0$, if all its entries are nonnegative. For the set of nonnegative matrices, we define an order as follows: if A and B are nonnegative matrices, then $A \geq B$ implies $A - B$ is a nonnegative matrix. A is a stochastic matrix if A is nonnegative and all its row sums are 1. A stochastic matrix P is called indecomposable and aperiodic (SIA) if there exists a column vector y such that $\lim_{n\to\infty} P^n = 1_n y^T$, where $1_n = = (1, 1, \dots, 1)^T$ is an $n \times 1$ vector. Some useful definitions, lemmas, and properties are provided as follows:

Lemma 1. [24] A stochastic matrix has algebraic multiplicity equal to one for eigenvalue $\lambda = 1$ if and only if the graph associated with matrix has a spanning tree. Furthermore, a stochastic matrix with positive diagonal elements has the property that $|\lambda| < 1$ for every eigenvalue not equal to one.

Lemma 2. [24] Let $A = [a_{ij}]_{n\times n}$ be a stochastic matrix. If A has an eigenvalue $\lambda = 1$ with algebraic multiplicity equal to one, and all the other eigenvalues satisfy $\lambda < 1$, then A is SIA, that is, $\lim_{n\to\infty} A^n = 1_n y^T$, where y is nonnegative and satisfies $A^T y = y, 1^T y = 1$.

Lemma 3. Given the scalar scale $\beta = (\beta_1, \beta_2, \ldots, \beta_n), \beta_i \neq 0$. Define $\beta_{max} = \max_{1\le i\le n} |\beta_i|, H = \mathrm{diag}\{h_1, h_2, \ldots, h_n\}$ such that $0 < h_i \le \frac{1}{d_{max}\beta_{max}}$, $i \in I_n$, and $|\beta| = \mathrm{diag}(|\beta_1|, |\beta_2|, \ldots, |\beta_n|)$. Then, $I_n - H|\beta|L$ is SIA, i.e., $\lim_{k\to\infty}[I_n - H|\beta|L]^k = 1_n y^T$ if and only if a graph G has a spanning tree. Furthermore, $[I_n - H|\beta|L]^T y = y$, $1_n y^T = 1$ where each element of y is nonnegative.

Proof. (Sufficiency): Since $h \in (0, \frac{1}{d_{max}\beta_{max}})$, one obtains $I_n - H|\beta|L = (I_n - H|\beta|D) + H|\beta|A$ is a stochastic matrix with positive diagonal entries, where $D = \mathrm{diag}(d_1, d_2, \ldots, d_n)$ and A is the degree matrix and adjacency matrix of G , respectively. Obviously, for all $i, j \in I_n,; i \neq j$, the (i, j)th entry of $I_n - H|B|L$ is positive if and only if $a_{ij} > 0$. Then, G is the graph associated with $I_n - H|B|L$. Combining Lemma 1 and Lemma 2, gives $\lim_{k\to\infty}[I_n - H|\beta|L]^k = 1_n y^T$,when G has a spanning tree, where y is nonnegative vector. Moreover, $\lim_{k\to\infty}[I_n - H|\beta|L]^k = 1_n y^T$

(Necessary): From Lemma 1, if G does not have a spanning tree, the algebraic multiplicity of eigenvalue $\lambda = 1$ of $I_n - H|\beta|L$ is $m > 1$. Then, it can be seen that the rank of $\lim_{k\to\infty}[I_n - H|\beta|L]^k$ is greater than 1, which implies $\lim_{k\to\infty}[In - H|\beta|L]^k \neq 1_n y^T$.

In this work, we assume that the hybrid multi-agent system consists of n agents which are continuous-time and discrete-time dynamic agents, labelled 1 through n, where the number of continuous-time dynamic agents is $c, c < n$. Without loss of generality, we assume that agent 1 through c are continuous-time dynamic agents.

Moreover, $I_c = \{1, 2, \ldots, c\}, I_n / I_c = \{c + 1, c + 2, \ldots, n\}$. Then, each agent has the dynamics as follows:

$$\begin{cases} \dot{x}_i(t) = u_i, & i \in I_c \\ x_i(t_{k+1}) = x_i(t_k) + u_i(t_k), t_k = kh, & i \in I_n / I_c, \end{cases} \tag{2.1}$$

where h is the sampling period, $x_i \in R$ and $u_i \in R$ are the state and control input of agent i, respectively. The initial conditions are $x_{i0} = x_i(0)$, and $x(0) = [x_1(0), x_2(0), \ldots, x_n(0)]^T$.

Moreover, the hybrid multi-agent system (2.1) is modelled as a connected directed graph, where all agents are regraded as the nodes and the interaction between two agents has been represented by the edge in a graph. This implies that (v_i, v_j) corresponds to an available information link from agent i to agent j. Besides, each agent updates its current state based on the information received from its neighbours. Furthermore, we suppose that there exists communication behaviour as in hybrid multi-agent system (2.1), that is, there are agent i and agent j which make $a_{ij} > 0$.

Definition 1. Given any scalar scale $\beta_i \neq 0$ for the agent i, the hybrid multi-agent system (2.1) is said to reach scaled consensus to $(\beta_1, \beta_2, \ldots, \beta_n)$ if for any initial conditions, we have

$$\lim_{t_k\to\infty} ||\beta_i x_i(t_k) - \beta_j x_j(t_k)|| = 0 \tag{2.2}$$

and

$$\lim_{t\to\infty} ||\beta_i x_i(t) - \beta_j x_j(t)|| = 0 \tag{2.3}$$

Remark 1 If a scalar scale $\beta_i = 1$ for all i, It is easy to see that the scaled consensus can reduce to the standard consensus, that is, it is more general than the standard consensus problems.

3. Main Results

In this section, the scaled consensus problems of hybrid multi-agent system (2.1) are studied under two kinds of control inputs (consensus protocols), respectively.

Case I: We assume that all agents communicate with their neighbours and update their control inputs in a sampling time t_k. Then, the consensus protocol for hybrid multi-agent system (2.1) is defined as follows:

$$\begin{cases} u_i(t) = sgn(\beta_i)\sum_{j\in N_i} a_{ij}[\beta_j x_j(t_k) - \beta_i x_i(t_k)], & for\ t \in (t_k, t_{k+1}], \quad i \in I_c \\ u_i(t_k) = h\cdot sgn(\beta_i)\sum_{j\in N_i} a_{ij}[\beta_j x_j(t_k) - \beta_i x_i(t_k)], & i \in I_n/I_c, \end{cases} \tag{3.1}$$

where $A = [a_{ij}]_{n\times n}$ is the weighted adjacency matrices associated with the graph G, $h = h_i = t_{k+1} - t_k$ for all i is the sampling period. For any real number α, the $sgn(\alpha)$ is the signum function such that $sgn(0) = 0$; $sgn(\alpha) = 1\ if\ \alpha > 0\ sgn(\alpha) = -1\ if\ \alpha < 0$. Furthermore, $\alpha \cdot sgn(\alpha) = |\alpha|$ for any real number α.

Theorem 1. Let G be a directed connected communication network of the hybrid multi-agent system (2.1) and $\beta_i \neq 0$ be any scalar scale of agent i. Assume that $0 < h < \frac{1}{d_{max}\beta_{max}}$. Then, the hybrid multi-agent system (2.1) with the protocol (3.1) reaches scaled consensus to $(\beta_1, \beta_2, \dots, \beta_n)$ if and only if G contains a spanning tree.

Proof. (Sufficiency) Let $\beta_i \neq 0$, we have, for $t \in (t_k, t_{k+1}]$, be any scalar scale of agent i, we first show that equation (2.2) holds. From (3.1)

$$\begin{cases} \beta_i x_i(t) = \beta_i x_i(t_k) + (t - t_k)|\beta_i|\sum_{j\in N_i} a_{ij}[\beta_j x_j(t_k) - \beta_i x_i(t_k)], & i \in I_c \\ \beta_i x_i(t_{k+1}) = \beta_i x_i(t_k) + h|\beta_i|\sum_{j\in N_i} a_{ij}[\beta_j x_j(t_k) - \beta_i x_i(t_k)], & i \in I_n/I_c, \end{cases} \tag{3.2}$$

Therefore, it follows that

$$\beta_i x_i(t_{k+1}) = \beta_i x_i(t_k) - h|\beta_i|\textstyle\sum_{j\in N_i} a_{ij}[\beta_j x_j(t_k) - \beta_i x_i(t_k)], \qquad \text{for } i \in I_n \tag{3.3}$$

Let $x(t_k) = (x_1(t_k), x_2(t_k), \dots, x_n(t_k))^T \in R^n$, $\beta = diag(\beta_1, \beta_2, \dots, \beta_n) \in R^{n\times n}$, $H = diag\{h_1, h_2, \dots, h_n\}$, and $|\beta| = diag(|\beta_1|, |\beta_2|, \dots, |\beta_n|)$. Then, equation (3.3) can be written as

$$\beta x(t_{k+1}) = [I_n - H|\beta|L]\beta x(t_k) \qquad \text{for } i \in I_n. \tag{3.4}$$

Since G contains a directed spanning tree and $0 < h < \frac{1}{d_{max}\beta_{max}}$, by Lemma 3, we have $\lim_{k\to\infty}[I_n - H|\beta|L]^k = 1_n y^T$, where y is nonnegative and satisfies $[In - H|\beta|L]^T y = y$. Thus, $\lim_{k\to\infty} \beta x(t_k) = \lim_{k\to\infty}[I_n - H|\beta|L]^k \beta x(0) = 1_n y^T \beta x(0)$.

As a consequence, equation (2.2) holds. Furthermore,

$$\lim_{t_k\to\infty} \beta_i x_i(t_k) = y^T \beta x(0), \quad \text{for} \quad i \in I_n. \tag{3.5}$$

On the other hand, for each $i, j \in I_c$ and $\beta_i \neq 0$,

$$||\beta_i x_i(t) - \beta_j x_j(t)|| \leq ||\beta_i x_i(t) - \beta_i x_i(t_k)|| + ||\beta_i x_i(t_k) - \beta_j x_j(t_k)|| + ||\beta_j x_j(t_k) - \beta_j x_j(t)||. \tag{3.6}$$

From equation (3.2), one obtains, for $t \in (t_k, t_{k+1}]$,

$$||\beta_i x_i(t) - \beta_i x_i(t_k)|| \leq |\beta_i| \sum_{j\in N_i} a_{ij} [\beta_j x_j(t_k) - \beta_i x_i(t_k)]. \tag{3.7}$$

As $t \to \infty$, we have $t_k \to \infty$. Thus, $\lim_{t\to\infty} ||\beta_i x_i(t) - \beta_i x_i(t_k)|| = 0$, for $i \in I_c$. Taking the limit as $t \to \infty$ on both sides of equation (3.6), one obtains

$$\lim_{t\to\infty} ||\beta_i x_i(t) - \beta_j x_j(t)|| = 0, \text{ for each } i, j \in I_c.$$

This implies that equation (2.3) holds. Therefore, the hybrid multi-agent system (2.1) with protocol (3.1) reaches scaled consensus.

(Necessity) Suppose that G does not contain a spanning tree. Then, by Lemma 3, we have $\lim_{k\to\infty}[I_n - H|\beta|L]^k \neq 1_n y^T$, which implies that $\lim_{t_k\to\infty} ||\beta_i x_i(t_k) - \beta_j x_j(t_k)|| \neq 0$, for some $i, j \in I_n$. Hence, the hybrid multi-agent system (2.1) cannot achieve scaled consensus.

Case II: All agents communicate with their neighbours and update their control inputs in a sampling time t_k. However, different from **Case I**, we assume that each continuous-time dynamic agent can observe its own state in real time. Then, the consensus protocol for hybrid multi-agent system (2.1) is defined by:

$$\begin{cases} u_i(t) = sgn(\beta_i) \sum\limits_{j\in N_i} a_{ij} [\beta_j x_j(t_k) - \beta_i x_i(t)], & for\ t \in (t_k, t_{k+1}], \quad i \in I_c \\ u_i(t_k) = h \cdot sgn(\beta_i) \sum\limits_{j\in N_i} a_{ij} [\beta_j x_j(t_k) - \beta_i x_i(t_k)], & i \in I_n / I_c, \end{cases} \tag{3.8}$$

where $A = [a_{ij}]_{n\times n}$ is the weighted adjacency matrices associated with the graph G, $h = h_i = t_{k+1} - t_k$ for all i is the sampling period and $sgn(\cdot)$ is the signum function defined as above.

Theorem 2. Let G be a directed connected communication network of the hybrid multi-agent system (2.1) and $\beta_i \neq 0$ be any scalar scale of agent i. Assume that $0 < h < \frac{1}{d_{max}\beta_{max}}$. Then, the system (2.1) with protocol (3.8) reaches scaled consensus to $(\beta_1, \beta_2, \ldots, \beta_n)$ if and only if G contains a spanning tree.

Proof. (Sufficiency) Since $\beta_i \neq 0$ and by equation (3.8) , we have

$$\begin{cases} \beta_i x_i(t) = \beta_i x_i(t_k) + |\beta_i| \left(\frac{1 - e^{-\sum_{j=1}^{n} a_{ij} |\beta_i| (t - t_k)}}{\sum_{j=1}^{n} a_{ij} |\beta_i|} \right) \sum_{j\in N_i} a_{ij} [\beta_j x_j(t_k) - \beta_i x_i(t_k)], & i \in I_c \\ \beta_i x_i(t_{k+1}) = \beta_i x_i(t_k) + h |\beta_i| \sum_{j\in N_i} a_{ij} [\beta_j x_j(t_k) - \beta_i x_i(t_k)], & i \in I_n / I_c. \end{cases} \tag{3.9}$$

Accordingly, at time t_{k+1}, the states of agents are

$$\begin{cases} \beta_i x_i(t) = \beta_i x_i(t_k) + |\beta_i| \left(\dfrac{1 - e^{-\sum_{j=1}^{n} a_{ij}|\beta_i| h}}{\sum_{j=1}^{n} a_{ij}\, |\beta_i|} \right) \displaystyle\sum_{j \in N_i} a_{ij} [\beta_j x_j(t_k) - \beta_i x_i(t_k)], & i \in I_c \\ \beta_i x_i(t_{k+1}) = \beta_i x_i(t_k) + h|\beta_i| \displaystyle\sum_{j \in N_i} a_{ij} [\beta_j x_j(t_k) - \beta_i x_i(t_k)], & i \in I_n / I_c. \end{cases} \tag{3.10}$$

Let $x(t_k) = (x_1(t_k), x_2(t_k), \ldots, x_n(t_k))^T \in R^n$, $|\beta| = \mathrm{diag}(|\beta_1|, |\beta_2|, \ldots, |\beta_n|)$, and $\beta = \mathrm{diag}(\beta_1, \beta_2, \ldots, \beta_n) \in R^{n\times n}$. Then, equation (3.3) can be written as

$$\beta x(t_{k+1}) = [I_n - H|\beta|L]\beta x(t_k) \qquad \text{for } \; i \in I_n, \tag{3.11}$$

where $H = \mathrm{diag}\{\frac{1-e^{-\sum_{j=1}^{n} a_{1j}|\beta_1| h}}{\sum_{j=1}^{n} a_{1j}|\beta_1|}, \ldots, \frac{1-e^{-\sum_{j=1}^{n} a_{cj}|\beta_c| h}}{\sum_{j=1}^{n} a_{cj}|\beta_c|}, h, \ldots, h\}$.

Because $\frac{1-e^{-\sum_{j=1}^{n} a_{ij}|\beta_i| h}}{\sum_{j=1}^{n} a_{ij}|\beta_i|} < \frac{1}{|\beta_i| d_{ii}}$ for $i \in I_c$, and $h < \frac{1}{d_{max}\beta_{max}}$, one obtains $0 < h_i < \frac{1}{d_{max}\beta_{max}}$ for H.

Since G contains a directed spanning tree and $0 < h < \frac{1}{d_{max}\beta_{max}}$, by Lemma 3, we have $\lim_{k\to\infty}[I_n - H|\beta|L]^k = 1_n y^T$, where y is nonnegative and satisfies $[In - H|\beta|L]^T y = y$. Thus, $\lim_{k\to\infty} \beta x(t_k) = \lim_{k\to\infty}[I_n - H|\beta|L]^k \beta x(0) = 1_n y^T \beta x(0)$. As a consequence, equation (2.2) holds. Moreover,

$$\lim_{t_k\to\infty} \beta_i x_i(t_k) = y^T \beta x(0), \quad \text{for } \; i \in I_n. \tag{3.12}$$

From equation (3.10), one obtains, for $t \in (t_k, t_{k+1}]$,

$$||\beta_i x_i(t) - \beta_i x_i(t_k)|| \leq |\beta_i| \left(\frac{1-e^{-\sum_{j=1}^{n} a_{ij}|\beta_i| h}}{\sum_{j=1}^{n} a_{ij}|\beta_i|} \right) \sum_{j\in N_i} a_{ij} [\beta_j x_j(t_k) - \beta_i x_i(t_k)]. \tag{3.13}$$

As $t \to \infty$, we have $t_k \to \infty$. Thus, $\lim_{t\to\infty} ||\beta_i x_i(t) - \beta_i x_i(t_k)|| = 0$, for $i \in I_c$.

Using the same argument of the proof of Theorem 1, once obtains

$$\lim_{t\to\infty} ||\beta_i x_i(t) - \beta_j x_j(t)|| = 0, \text{ for each } i, j \in I_c$$

Therefore, the hybrid multi-agent system (2.1) with protocol (3.9) reaches scaled consensus.

(Necessity) The proof is similar to the proof of Necessity part of Theorem 1.

4. Simulations

In this section, an example have been provided to demonstrate the effectiveness of theoretical results in this work.

Example 1. Assume that there are 8 agents consisting of six continuous-time dynamic agents and two discrete-time dynamic agents, denoted by $1 - 6$ and $7 - 8$, respectively. Let $x(0) = [\ 6\ \ 4\ \ 2\ \ 1\ \ 1\ \ 2\ \ 4\ \ 6]^T$. The communication network with $0 - 1$ weights is shown in Fig. 4.1.1, where the dashed lines mean that each agent exchanges information at time $t = t_k$.

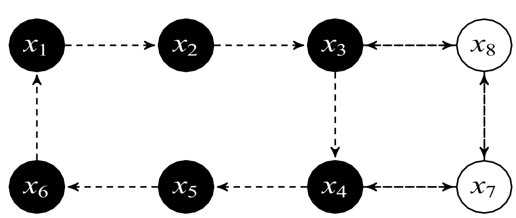

Fig. 4.1.1: A connected directed network G.

Let the scalar scales be (2,-2, 1,-1, 3, 1.5, -2, -1). It can be noted that G is balanced and contain a directed spanning tree with $d_{\max} = 2$ and $\beta_{\max} = 3$. Since the sampling period $h = 0.2 < 0.33 = (d_{\max}\beta_{\max})^{-1}$. By using the consensus protocol (3.1), the state trajectories of all agents are shown in Fig. 4.1.2, which is consistent with the sufficiency of Theorem 1.

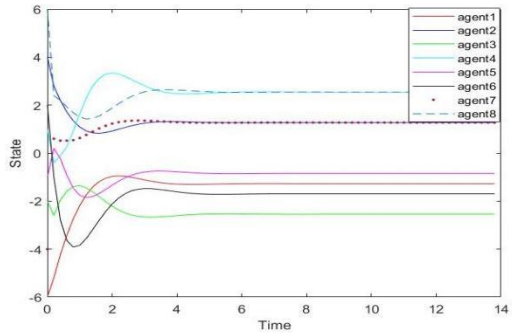

Fig. 4.1.2. The state trajectories of all agents with scalar scales $(2, -2, 1, -1, 3, 1.5, -2, -1)$ using the consensus protocol (3.1) and communication network G with $h = 0.2$.

However, if sampling period $h = 0.4 > 0.33 = (d_{\max}\beta_{\max})^{-1}$, the state trajectories of all agents can not reach scaled consensus under the protocol (3.1) (see Fig. 4.1.3).

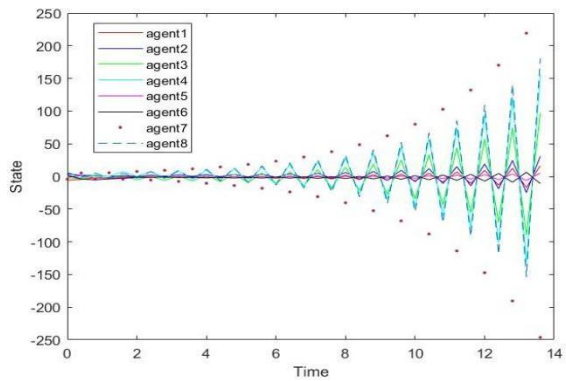

Fig. 4.1.3. The state trajectories of all agents with scalar scales (2,-2, 1,-1, 3, 1.5, -2, -1) using the consensus protocol (3.1) and communication network G withh = 0.4.

5. Discussion And Conclusion

In this work, scaled consensus problems for the hybrid multi-agent system (2.1) consisting of directed communication networks have been studied. Two consensus protocols are proposed based on the interactions among agents. Firstly, we assume that the directed communication networks contains a spanning tree with $h < (d_{\max}\beta_{\max})^{-1}$ and interactions among agents occur in the sampling time t_k. Hence, by Theorem1 and protocol (3.1), the hybridmulti-agent system (2.1) achieves scaled consensus to $(\beta_1, \beta_2, \ldots, \beta_n)$.

Secondly, assume that the directed communication networks contains a spanning tree with $h < (d_{\max}\beta_{\max})^{-1}$ and interactions among agents occur in the sampling time t_k but the continuous-time dynamic agents can observe their own states in real time. By Theorem 2 and protocol (3.8), we show that the hybrid multi-agent system (2.1) achieves scaled consensus to $(\beta_1, \beta_2, \ldots, \beta_n)$.

Moreover, under the consensus protocols (3.1) and (3.8), we see that if $\beta_i = 1$ for all i, the scaled consensus results can reduce to the general consensus, which shows generalization of our main results (compare with Zheng et al.,[29]). However, if $h > (d_{max}\beta_{max})^{-1}$, the hybrid multi-agent system (2.1) cannot achieves scaled consensus to $(\beta_1, \ldots, \beta_n)$ under protocols (3.1) and (3.8).

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