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Study of Energy Efficiency for Satellite Communication Subsystems by Differential Game

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Abstract **-** This paper studies a satellite transponder's communication channel, in which there exist multiple-user terminals, who compete for limited radio resources to meet their own data rate needs. Because inter-user interference limits on the satellite transponder's performance, the transponder's power-control system needs to coordinate all its users to reduce interference and maximizes overall performance. A non-cooperative Differential Game (DG) is set up to model the users' competition in a transponder's communication channel. Each user's utility function is a trade-off between transmission data rate and power consumption. Nash Equilibrium (NE) is defined to be the solution of the DG model. The optimality condition of NE is derived to be a set of Differential Algebraic Equations (DAE). An algorithm based on minimizing Hamiltonians is developed to solve the DAE system. The numerical solution of the DG model provides the transponder's power control system with each user's power-control strategy at the equilibrium.

Keywords**:** Spectrum and Power Allocation, Energy-Efficiency, Satellite Communication, Differential Game

1. Introduction

Satellites most commonly use the C band (6/4 GHz). Each C band typically has 24 channels. Each satellite transponder represents an individual communication channel. Within a 36-MHz bandwidth channel, each transponder can handle an enormous amount of information by using different multiple-access schemes, so each channel contains many pairs of senders and receivers [1], [2]. This study assumes each pair is selfish to maximize its own performance by a specific power-allocation scheme. The interference from other pairs also affects the channel performance [3]. Furthermore, the C band's heavier use leads to more interference. Shifting satellite communication to higher frequencies is one effective way to minimize interference, but crowding and interference problems will still exist, which motivates this study to develop a technique that increases bandwidth efficiency and signal-caring capacity, and decreases interference of satellite communication subsystems.

This paper models a transponder's communication channel as an interference channel with aim to optimize the trade-off between transmission data rate and power consumption. Section II reviews a transponder's communication channel and static energy-efficient power control games. Section III models the power-allocation optimization problem for all users in a transponder as a Differential Gaussian Interference Channel Game (DGICG) based on the special properties of satellite wireless communications. Section IV and Section V derive and analyse the DGICG model's optimality condition, and develop numerical methods to solve the optimality condition of NE and then solve the model. The numerical solution from the model provides all users in a transponder's channel with the optimal power-allocation scheme at the equilibrium.

2. Preliminaries

2.1. Satellite Wireless Communications Subsystem

A transponder is a repeater that implements a wideband communication channel that can carry many simultaneous one-to-one communication transmissions [1], so it can be modelled as a multiuser interference channel as Fig. 1 [4], [5]. This interference channel is an M-to-M network where a one-to-one correspondence exists between senders and receivers such that each sender communicates information only to its corresponding receiver [4]. This study models each pair of sender-receiver in a transponder channel as a user (a player). The interference limits the system's performance. Interference cancellation is an option when the interference signal is sufficiently strong, but its implementation is complex, requiring prior knowledge of users' transmission schemes is accessible by other users [5], [6]. This study assumes that each user applies power to affect the cross-coupling gain and then reduce interference without any interference cancellations.

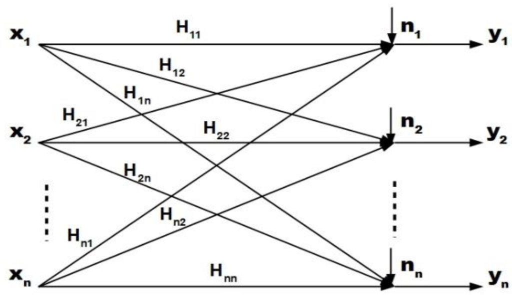

Fig. 1: Multiuser Interference Channel

2.2. Static Power Control Game

Goodman and Mandayam [7] study a static energy-efficient power control game on a distributed multiple-access channel with a finite number of users, denoted by K. Each user chooses its own power control policy p_i to maximize its energy-efficiency $u_i = \frac{R_i f(SINR_i)}{p_i}$, where R_i is the information transmission rate in bit/s for user i, and f is an efficiency function representing the block success rate, which is assumed to be sigmoidal and identical for all the users [7], [8]. The channel model is given by

$$y(n) = \sum_{i=1}^{K} H_i(n) x_i(n) + \sigma(n) \quad (1)$$

where $x_i(n)$ is the symbol transmited by sender i at time n, $\sigma(n)$ is a Gaussian random variable with zero-mean and variance σ^2. It is called a static game because (a) it assumes that the users transmit data over quasi-static or block-fading channels at the same time and in the same frequency band, assuming each channel gain $H_i(n)$ to be constant over each block. (b) Each user in the game applies a fixed power policy, once per block, to maximize its utility. However, for long-distance wireless communication such as satellite communication, channel gain varies with time, so its modulus is usually assumed to be in a compact set $|H_i|^2 \in [\eta_i^{min}, \eta_i^{max}]$. A variable power policy is expected to be designed to control channel gain. Furthermore, with assumption of complete information and rationality, the existence of Nash Equilibrium is guaranteed by Debreu-Fan-Glicksberg existence theorem [9]. The Nash Equilibria are found by solving equations $\frac{\partial u_i}{\partial p_i}(\bar{p}) = 0, i = 1, \cdots, K$. And the static power game has unique pure Nash Equilibrium, which is discussed by Yates [10], and Saraydar [11].

Besides the energy-efficient game for communication channel, there are other types of noncooperative games [12], [13] constructed for different utility, which are generally called Gaussian Interference Games (GIGs). The water-filling algorithm also solves for Nash Equilibrium of GIG without the need for centralized control [13]. Amir Leshem applied cooperative game theory for analysing interference channels [14]. Wei Wan [15] created a cooperative static game for a transponder's centralized power control to maximize overall channel data transmission rate.

3. Differential Game for a Transponder

All users $(x_i, y_i), i \in \kappa$ in a transponder's channel simultaneously choose their power-control policy before establishing communication. This implies an open-loop power control policy, which is a function of time. Each user's communication is through N sub-frequency channels simultaneously, and each user applies independent power control policy in each sub-frequency channel. Furthermore, each user has two types of power consumption policy: the first improves its own channel gain, and the second decreases interference. The major variables are defined as follows:

$H_{ii}^f(t)$: the direct channel gain from the transmitter to the receiver of user i over frequency f at time t.

$H_{ji}^f(t)$: the cross-coupling gain from the transmitter j to the receiver of user i over frequency f at time t.
$p_i^f(t)$: the transmit power spectrum density used by user i to increase direct channel gain over frequency f at time t.
$v_i^f(t)$: the transmit power spectrum density used by user i to decrease cross-coupling channel gain over frequency f at time t.
$\sigma_i^f(t)$: the noise power spectrum density at user i over frequency f at time t.

Construction of objective function: Since the first and most interesting objective for each transponder user is to optimize the trade-off between the achievable data rate and energy consumption. With an assumption of no channel interference cancellation, the interference from other users is consequently noise. Then, the achievable rate for user i at time t over frequency (f_1, f_2) is as follows [5], [3]:

$$R_i(t) = \int_{f_1}^{f_2} log_2\left(1 + \frac{p_i^f(t)|H_{ii}^f(t)|^2}{\sigma_i^f(t) + \sum_{j\neq k} p_j^f(t)|H_{ji}^f(t)|^2}\right) df \cong log_2\left(1 + \frac{p_i^f(t)|H_{ii}^f|^2}{\sigma_i^f(t) + \sum_{j\neq k} p_j^f(t)|H_{ji}^f|^2}\right)\Delta f \tag{2}$$

, where approximation assumes the variables to be constant over small bands. The energy efficiency for user $i, i \in \kappa$ over time $[0, T]$ is

$$\int_0^T \sum_{f=1}^N \left[R_i(t) - c_i^f(p_i^f(t))^2 - d_i^f(v_i^f(t))^2\right] dt \tag{3}$$

, where c_i^f, d_i^f are weights between power spectrum density and data rate in evaluation of energy efficiency, which is the log transformation of ratio of information bits that are transmitted without error per unit time to the transmit power. It is to be maximized. The second goal of transponder power control is to control the direct channel gain to reach a certain channel-capacity level and also to reduce the cross-coupling gain to certain level. This second objective is to minimize the following expression:

$$\sum_{f=1}^N w_1^{(f,i)}(|H_{ii}^f(T)|^2 - r_{ii}^f\eta_{ii}^f)^2 + w_2^{(f,i)}(|H_{ji}^f(T)|^2 - r_{ji}^f\eta_{ji}^f)^2 \tag{4}$$

, where $w_1^{(f,i)}, w_2^{(f,i)}$ are weights between different objectives; η_{ii}^f, η_{ji}^f are constants, and upper bound of $|H_{ii}^f(T)|^2, |H_{ji}^f(T)|^2$; and r_{ii}^f, r_{ji}^f are targeted channel-gain levels.

Construction of dynamics: Generally, $|H_{ii}^f(t)|^2, |H_{ij}^f(t)|^2$ belong to a compact set $[\eta_i^{min}, \eta_i^{max}]$, and can be approximated by Kronecker's delta function [16]. In long-distance wireless communication, satellite transponders can apply energy $p_i^f(t)$ to impact channel gain. This analysis assumes that the growth rate is proportional to power consumption. Thus, logistic growth with carrying capacity is adopted to approximate the dynamics of $|H_{ii}^f(t)|^2$:

$$\frac{d|H_{ii}^f|^2}{dt} = \alpha_i^f p_i^f(t)\left(\eta_{ii}^f - |H_{ii}^f(t)|^2\right) \tag{5}$$

, where α_i^f represents the growth rate. Furthermore, when user i applies $p_i^f(t)$ to improve the channel gain $|H_{ii}^f(t)|$, it also increases the cross-coupling gain $|H_{ij}^f(t)|$. However, user j is able to cost power $v_i^f(t)$ to decrease interference brought by $p_i^f(t)$. At last, because of threshold effects existing in channel gain, cross-coupling gain has a lower bound. Thus, the dynamics of $|H_{ij}^f(t)|^2$ is approximated by:

$$\frac{d|H_{ij}^f(t)|^2}{dt} = \beta_{ij}^f\left(p_i^f(t) - v_j^f(t)\right)\left(\eta_{ij}^f - \left|H_{ij}^f(t)\right|^2\right)\left(\left|H_{ij}^f(t)\right|^2 - \xi_i^f\right) \tag{6}$$

, where $i \neq j$, and β_i^f represents the growth rate. Thus, the DGICG model $(\kappa, \{p_i^f, v_i^f\}_{i\in\kappa}, \{J_i\}_{i\in\kappa})$ is set up as follows:

$$\begin{cases} J_1 = \min_{p_1^f(t), v_1^f(t)} \int_0^T \left\{ \sum_{f=1}^{N} c_1^f (p_1^f(t))^2 + d_1^f (v_1^f(t))^2 - log_2 \left(1 + \frac{p_1^f(t)|H_{11}^f|^2}{\sigma_1^f(t) + \sum_{j \neq 1} p_j^f(t)|H_{j1}^f|^2} \right) dt \right\} \\ \qquad + \sum_{f=1}^{N} \left[w_1^{(f,1)} (|H_{11}^f(T)|^2 - r_{11}^f \eta_{11}^f)^2 + \sum_{j \neq 1} w_2^{(f,1)} (|H_{j1}^f(T)|^2 - r_{j1}^f \eta_{j1}^f)^2 \right] \\ \qquad \vdots \\ J_K = \min_{p_K^f(t), v_K^f(t)} \int_0^T \left\{ \sum_{f=1}^{N} c_K^f (p_K^f(t))^2 + d_K^f (v_K^f(t))^2 - log_2 \left(1 + \frac{p_K^f(t)|H_{KK}^f|^2}{\sigma_K^f(t) + \sum_{j \neq k} p_j^f(t)|H_{jK}^f|^2} \right) dt \right\} \\ \qquad + \sum_{f=1}^{N} \left[w_1^{(f,K)} (|H_{KK}^f(T)|^2 - r_{KK}^f \eta_{KK}^f)^2 + \sum_{j \neq K} w_2^{(f,K)} (|H_{jK}^f(T)|^2 - r_{jK}^f \eta_{jK}^f)^2 \right] \end{cases} \tag{7}$$

$$s.t. \quad \begin{cases} \frac{d|H_{ii}^f|^2}{dt} = \alpha_i^f p_i^f(t) (\eta_{ii}^f - |H_{ii}^f(t)|^2) \\ \frac{d|H_{ij}^f(t)|^2}{dt} = \beta_{ij}^f (p_i^f(t) - v_j^f(t)) \left(\eta_{ij}^f - \left| H_{ij}^f(t) \right|^2 \right) \left(\left| H_{ij}^f(t) \right|^2 - \xi_i^f \right), \quad j \neq i \\ \sum_{f=1}^{N} P_i^f(t) \leq \boldsymbol{P}_i^{\boldsymbol{max}} \\ |H_{ij}^f(0)|^2 \, given, \quad i,j = 1, \dots, K, \quad f = 1, \dots, N. \end{cases}$$

4. Analysis of Optimality of Equilibrium

Nash Equilibrium (NE) is defined as the solution of above non-Cooperative DGICG among all users in one transponder's channel. The necessary condition for controls $\{p_i^f(t), v_i^f(t)\}$ to be NE of non-Cooperative differential game models is derived from Pontryagin's Minimum Principle [17], [18], which is composed of following system: (In order to save notations, we set $x_{ij}^f(t) = |H_{ij}^f(t)|^2$ in the following expressions.)

- Player i's Hamiltonian H_i :

$$H_i = \sum_{f=1}^{N} \left[c_i^f \left(p_i^f(t) \right)^2 + d_i^f \left(v_i^f(t) \right)^2 - log_2 \left(1 + \frac{p_i^f(t) x_{11}^f(t)}{\sigma_i^f(t) + \sum_{j \neq i} p_j^f(t) x_{ji}^f} \right) \right] + \\ \sum_{f=1}^{N} \sum_{i=1}^{K} \lambda_{ii}^f \left[\alpha_i^f p_i^f(t) (\eta_i^f - x_{ii}^f(t)) \right] + \sum_{f=1}^{N} \sum_{i \neq j} \lambda_{ij}^f \left[\beta_{ij}^f (p_i^f(t) - v_j^f(t)) (\eta_{ij}^f - x_{ij}^f(t)) (x_{ij}^f(t) - \xi_{ij}^f) \right] \tag{8}$$

- A System of State equations:

$$\begin{cases} \frac{dx_{ii}^f}{dt} = \alpha_i^f p_i^f(t) (\eta_{ii}^f - x_{ii}^f(t)) \\ \frac{dx_{ij}^f}{dt} = \beta_{ij}^f (p_i^f(t) - v_j^f(t)) (\eta_{ij}^f - x_{ij}^f(t)) (x_{ij}^f(t) - \xi_{ij}^f), j \neq i \\ \sum_{f=1}^{N} P_i^f(t) \leq \boldsymbol{P}_i^{\boldsymbol{max}} \\ x_{ij}^f(0) \; given. \quad i,j = 1, \dots, K. \quad f = 1, \dots, N. \end{cases} \tag{9}$$

- A System of Co-state equations:

$$\begin{cases} \frac{d\lambda_{ii}^f}{dt} = -\frac{dH_i}{dx_{ii}^f} = \frac{1}{ln2}\frac{p_i^f(t)}{\sigma_i^f + \sum_{j\neq i} p_j^f(t)x_{ji}^f + p_i^f(t)x_{ii}^f} + \lambda_{ii}^f \alpha_i^f p_i^f(t) \\ \frac{d\lambda_{jj}^f}{dt} = -\frac{dH_i}{dx_{jj}^f} = \lambda_{jj}^f(t)\alpha_j^f p_j^f(t), j \neq i \\ \frac{d\lambda_{ij}^f}{dt} = -\frac{dH_i}{dx_{ij}^f} = \lambda_{ij}^f(t)\beta_{ij}^f\left(p_i^f(t) - v_j^f(t)\right)\left(2x_{ij}^f(t) - \eta_{ij}^f - \xi_{ij}^f\right), j \neq i \\ \frac{d\lambda_{ij}^f}{dt} = -\frac{dH_i}{dx_{ji}^f} = \frac{-1}{ln2}\frac{p_i^f(t)x_{ji}^f(t)p_j^f(t)}{\left(\sigma_i^f + \sum_{j\neq i} p_j^f(t)x_{ji}^f(t)\right)^2 + p_i^f(t)x_{ii}^f(t)(\sigma_i^f + \sum_{j\neq i} p_j^f(t)x_{ji}^f(t))} \\ \qquad\qquad\qquad + \lambda_{ji}^f(t)\beta_{ji}^f\left(p_j^f(t) - v_i^f(t)\right)\left(2x_{ji}^f(t) - \eta_{ji}^f - \xi_{ji}^f\right) \end{cases} \tag{10}$$

where $f = 1, \ldots, N$, and has the following boundary condition:

$$\begin{cases} \lambda_{ii}^f(T) = \frac{dh_i}{dx_{ii}^f} = 2w_1^{(f,i)}(x_{ii}^f(T) - r_{ii}^f \eta_{ii}^f) \\ \lambda_{jj}^f(T) = \frac{dh_i}{dx_{jj}^f} = 0 \\ \lambda_{ij}^f(T) = \frac{dh_i}{dx_{ij}^f} = 0 \\ \lambda_{ji}^f(T) = \frac{dh_i}{dx_{ji}^f} = 2w_2^{(f,i)}(x_{ji}^f(T) - r_{ji}^f \eta_{ji}^f) \end{cases} \tag{11}$$

- The candidates for player i's control policy at NE are those $\{p_i^f(t), v_i^f(t)\}$, which make $\{\frac{dH_i}{dp_i^f}, \frac{dH_i}{dv_i^f}\}$ vanish by assuming other players are using their NE control policies:

$$\begin{cases} \frac{dH_i}{dp_i^f} = 2c_i^f p_i^f(t) - \frac{1}{ln2}\frac{x_{ii}^f(t)}{\sigma_i^f(t) + \sum_{j\neq i} p_j^f(t)x_{ji}^f + p_i^f(t)x_{ii}^f} \\ \quad + \lambda_{ii}^f(t)\alpha_i^f\left(\eta_{ii}^f - x_{ii}^f(t)\right) + \lambda_{ij}^f(t)\beta_{ij}^f\left(\eta_{ij}^f - x_{ij}^f(t)\right)\left(x_{ij}^f(t) - \xi_{ij}^f\right) = 0 \\ \frac{dH_i}{dv_i^f} = 2d_i^f v_i^f(t) - \sum_{j\neq i}\lambda_{ji}^f(t)\beta_{ji}^f\left(\eta_{ji}^f - x_{ji}^f(t)\right)\left(x_{ji}^f(t) - \xi_{ji}^f\right) = 0 \end{cases} \tag{12}$$

5. Numerical Solution of DGICG Model

In nonlinear system (12), the optimal control policy cannot be solved explicitly. Thus, solving the n players' open-loop DGICG is equivalent to solving $n^2 + 2n$ Differential-Algebraic Equation (DAE) system (9)-(12). The design of following algorithm is based on the fact that each player optimizing its own Hamiltonian with its own control by assuming all other players have adopted optimal control policy is necessary for its control to reach NE. This implies searching NE could be achieved by each player searching to optimize its own Hamiltonian simultaneously. The procedure of following iterative algorithm starts with a randomly generated discretized control for each player, and then solved the DAE system by these control policies. Next, each player updates its control in its own Hamiltonian's steepest descent direction, that is each player optimizes its own Hamiltonian simultaneously in its own space of controls. Finally, the algorithm terminates when all players' $\{\frac{dH_i}{dp_i^f}, \frac{dH_i}{dv_i^f}\}$ vanish, or there is little improvement of objective utilities.

Algorithm 1:

Step 1: Generate randomly a discrete approximation to control $\{p_i^f(t), v_i^f(t)\}$ over $t \in [0, T]$.

$$\begin{cases} p_i^{f,n}(k) = p_i^f(t), t \in [t_k, t_{k+1}), k = 1, \cdots, M \\ v_i^{f,n}(k) = v_i^f(t), t \in [t_k, t_{k+1}), k = 1, \cdots, M \end{cases} \tag{17}$$

, where $i \in 1, \cdots, K.$ $f = 1, \ldots, N$. And n stands for the number of iterations, and $n = 1$.

Step 2: Use discretized control (17) to integrate the state equation (9) over $[0, T]$ forward by Runge-Kutta (RK4) method with given initial condition $x_{ij}^f(0)$.

Step 3: Evaluate the terminal value of costate variables in (11) by terminal value of state variables $x_{ij}^f(T)$, and then integrate costate equations (10) backward by RK4.

Step 4: Evaluate each player's objective function $J_i(k)$ by the discrete values of state, costate and controls from Step 2-3.

Step 5: If $|J_i(n+1)\text{-}J_i(n)| < \epsilon$ for all i or $\left\|\frac{dH_i}{dp_i^f}\right\| < \epsilon$ and $\left\|\frac{dH_i}{dv_i^f}\right\| < \epsilon$, then the iterative procedure terminates and output the optimal controls and state equations. If the stopping criterion is not satisfied, then new piecewise constant controls are generated by

$$\begin{cases} p_i^{f,n+1}(k) = p_i^{f,n}(k) - \tau_i^f \dfrac{dH_i}{dp_i^f}(k) \\ v_i^{f,n+1}(k) = v_i^{f,n}(k) - \delta_i^f \dfrac{dH_i}{dv_i^f}(k) \end{cases} \tag{18}$$

, where step length τ_i^f and δ_i^f are chosen independently for each player by line search procedure to guarantee all objective values decrease at each iteration, and where $i \in 1, \cdots, K.$ $f = 1, \ldots, N$.

6. Numerical Experiments

Above algorithm was tested on a two-player DGICG over one sub-frequency channel. These two players are symmetric. Convergence of the algorithm is shown by the vanishing of $\{\frac{dH_i}{dp_i^f}, \frac{dH_i}{dv_i^f}\}$ and convergence of objective functions (Fig.2). The numerical solution of optimal controls and the state trajectories are shown in Fig. 3 and Fig. 4.

- Convergence of objective functions:

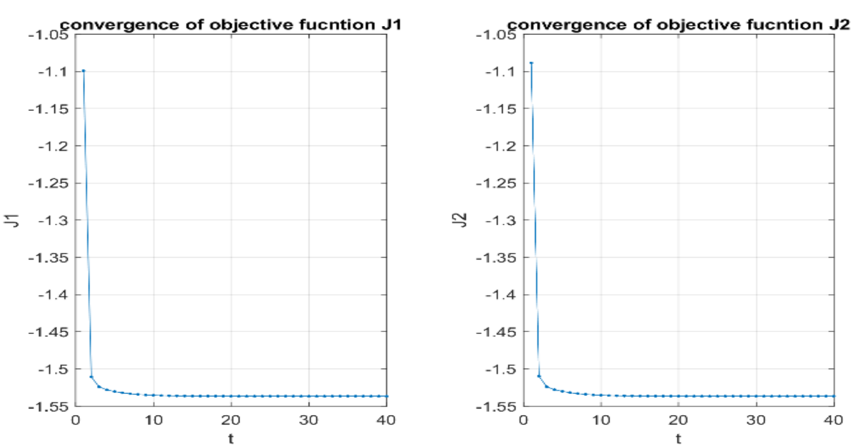

Fig. 2: Convergent Trajectories of Player 1 and 2's Objective Function Values

- Optimal controls of two players:

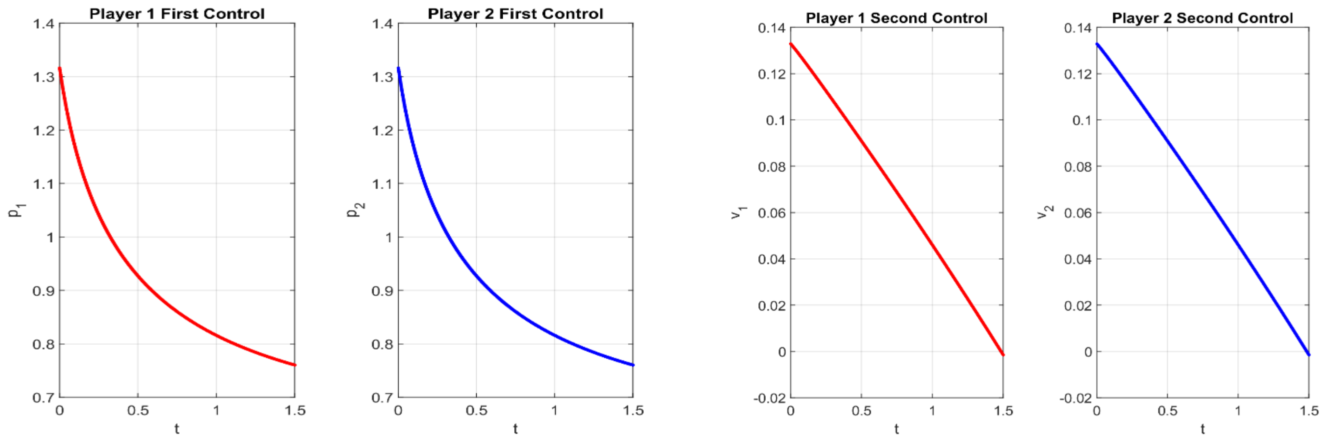

Fig. 3: Trajectories of Player 1 and 2's Optimal Control $p_i(t)$ and $v_i(t)$

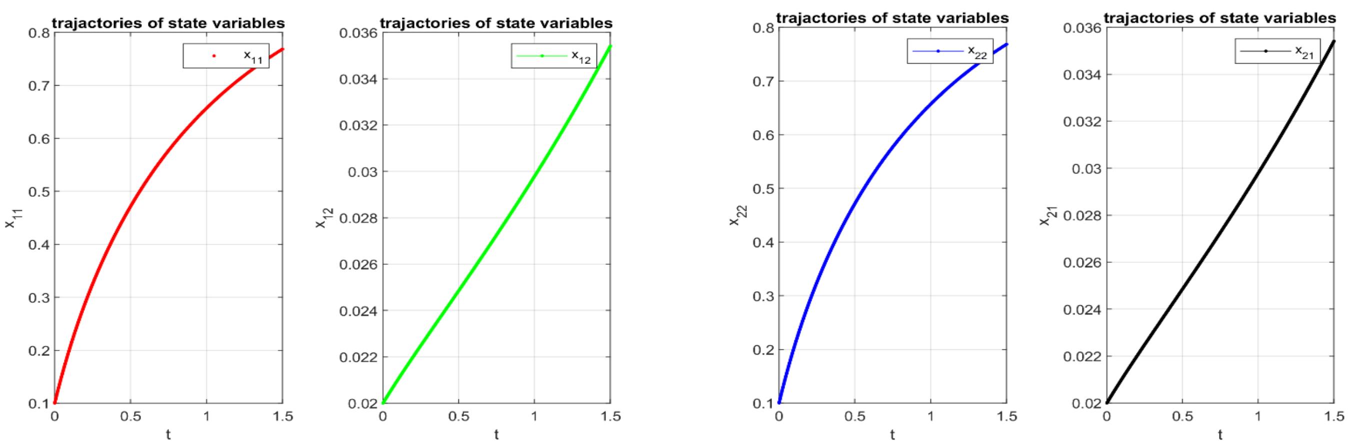

Fig. 4: Trajectories of $|H_{11}^f|^2 and\ |H_{12}^f|^2$, $|H_{22}^f|^2 and\ |H_{21}^f|^2$ at Nash Equilibrium

5. Summary and Conclusion

This paper models a satellite transponder's communication channel as a multiuser interference channel and focuses on its power allocation to improve energy efficiency. In a transponder's channel, the performance of each pair of transmitters and receivers depends not only on its own power allocation, but also on the other pairs' and their interference. Each user in the channel would be competing for limited radio resources to meet their selfish data rates with less energy consumption. Another feature of satellite communication is its long-distance, so the channel gain is not constant. Thus, each user can use power to improve its own channel gain and reduce interference. This paper introduced a noncooperative DGICG model for all users in a transponder's channel. In this game model, each user's energy efficiency is redefined, and logistic growth is adopted to approximate the changing of channel gain under specific energy consumption. The objective function of each user is a weighted sum of energy efficiency and targeted channel gain. Then, the optimality condition for NE of the model is derived to be a DAE system. At last, an algorithm is developed solve for NE. The design of algorithm is based on a steep-descent method and optimizes all players' objective functions simultaneously. This algorithm is especially efficient to solve differential game model even if the optimal controls are not able to be solved explicitly. The numerical solution of the game model can be used to support designing power allocation scheme of transponders.

In the end, a game model could have multiple NEs, and a NE may not be Pareto optimal, so mathematical analysis of NE will be the next step of research. Furthermore, the differential game model in this paper assumes that all users in the channel are symmetrical, but in practice there exists some users who have priority to communicate, so leader-follower game analysis is also expected in the near future research work.

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