NDI/INDI Control Scheme for Tilt-rotorcraft Unmanned Aircraft Systems

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Abstract - In this paper, a new control architecture is proposed to enable advanced Unmanned Aerial Vehicles to maneuver inside confined spaces to perform operations in Urban Search and Rescue missions and hazardous industrial spaces where sending humans is highly risky. The proposed controller is tested on a revolutionary highly maneuverable VTOL system first conceptualized by 4Front Robotics Ltd. presenting highly coupled dynamics enabling it to perform unique maneuvers such as pitch hover. To deal with the associated complexities and coupling motion effects inherent to such aircraft, a Nonlinear Dynamic Inversion (NDI) controller developed for position control, coupled with an Incremental Nonlinear Dynamic Inversion (INDI) controller designed for attitude control is described. The results for different flight maneuvers demonstrate effective the ability of the controller to control the position and orientation independently while tracking complex trajectories.

Keywords: Unmanned Aerial Vehicle, Nonlinear Dynamic Inversion, Incremental Nonlinear Dynamic Inversion.

1. Introduction

Unmanned Aerial Vehicles (UAVs) have been recognized in multiple conferences and academic literature as a powerful tool to save lives and reduce recovery costs after disasters. To be truly effective in response to natural disasters, the next generation of UAV will require advance design configuration and advanced control systems to navigate in helicopter-impenetrable environments and cope with the aerodynamic disturbances that affect the aircraft. While there has been significant work on developing diverse control systems for UAVs in open spaces, few studies have focused on the unique challenges of controlling UAVs inside restricted GPS-denied spaces. Existing control methods have primarily been developed for rotorcraft and fixed-wing aircraft, focusing on stabilization problems associated with altitude control and trajectory tracking. This paper presents new control formulations for the specific control of agile UAV maneuvering inside restricted, GPS-denied environments such as collapsed buildings.

Due to the fact that aircraft systems (especially those flying inside cluttered spaces) are affected by diverse characteristics (e.g., propellers' performances, air pressure, wind, etc.) obtaining an exact mathematical model of aircraft is practically impossible. To simplify the development of control systems, linear models are typically generated from nonlinear system models, enabling the use of simpler (linear) control techniques [1-3]. Although linear control systems have been successful for typical flight missions they have been proven ineffective for controlling UAVs during untypical flight conditions (e.g., extreme wind gusts) or aggressive flight maneuvers. In this domain Marconi et al. proposed a robust control algorithm for landing VTOL aircraft on an oscillating deck ship using an internal model-based feedback dynamic regulator [4]. Mokhtari et al. [5] introduced a robust feedback linearization method with a linear generalized H ∞ controller to improve robustness against disturbances and uncertainties. Although many other linear-based controllers have been attempted for the operation of aircraft under untypical conditions, all such control developments have focused on traditional unmanned aircraft experiencing simple external disturbances. Thus, they are not applicable to new unmanned aircraft concepts such as supersonic or transitional UAVs that are currently being developed [6-8] where obtaining an accurate mathematical model of the aircraft is not possible.

Sensor-based control approaches, such as Incremental Dynamics (ID) techniques, have been developed to generate controllers less dependent on the system model. The Incremental Nonlinear Dynamic Inversion (INDI) method, in particular combined with PID/PD controllers, has been successfully employed to control the attitude and position of advanced tilt-rotor bi-copter drones, which cannot be fully controlled using traditional methods [9].

In [10], an INDI controller was developed for the 3D acceleration trajectory tracking of tail-sitter UAVs. The results presented in [11] show that INDI control is effective in controlling aircraft operating under diverse flight modes. According to work reported in [9], the Incremental Dynamics (INDI) control method is the most appropriate

mechanism for the control of complex nonlinear systems. INDI is less dependent on the dynamic model of the UAV under consideration, less sensitive to model uncertainties, and increases both system robustness and flexibility with respect to changes in the aircraft's operation mode. This paper proposes an extension of previous INDI formulations for the control of advanced highly maneuverable VTOL UAVs in geometrically complex GPS-denied confined spaces subject to aerodynamic ground and wall effects. The proposed control method decouples the control parameters, which reduces the time to compute a proper control signal and improves the aircraft's performance.

In this paper first, the nonlinear mathematical model of a novel highly maneuverable UAV, named Navig8, is developed. Then the proposed control architecture is described in Section 3 followed by the simulation results and conclusions presented in Section 5.

2. Dynamic Model

The drone under consideration is a 6 Degrees of Freedom (DOF) tilt rotorcraft having three propellers each having a Variable Pitch Propeller (VPP) mechanism. The two main (left & right) propellers (see Fig. 1a) have a constant dihedral angle, γ , to reduce aerodynamic ground/wall effects. The third propeller is a horizontal tail rotor. The thrust produced by each propeller can be controlled in diverse ways by either changing its rotational speed, its angle of attack, or a combination of them. Thus, the aircraft has eight control variables ω_1 , ω_2 , ω_3 , α_1 , α_3 , β_1 , β_2 , and β_3 which correspond to the left, tail, and right propellers' speed, the propellers' tilt angles around the y-axis of the UAV's body frame, and the propellers' angle of attack, Fig.1. Due to the control allocation challenges that are available for the control of such UAV, in this paper we only consider controlling the UAV using the following five variables: ω_1 , ω_2 , ω_3 , α_1 , and α_3 .



Fig. 1. Navig8 UAV: a) Frames of reference, b) Variable pitch propeller mechanism.

The Newton-Euler formulation is used to derive the system's dynamic model. In order to describe the rotation of the drone the Euler angles (roll (φ), pitch (θ), yaw (ψ)) are used. The motion for the Navig8 is described by Eq. (1), in which *p*, *q*, and *r* represent the body's angular velocity in the original reference frame.

The terms *X*, *Y*, *Z*, *L*, *M*, and *N* in Eq. (1) denote the total forces and torques applied to the UAV's center of mass in the *x*, *y*, and *z* directions, respectively. Such terms are obtained as per Eq. (2) where orthogonal distances between the center of mass of the UAV and the left, right and back propellers are described by vectors $p_1 = (a, b, c)$, $p_2 = (-d, 0, -e)$ and $p_3 = (a, -b, c)$, respectively.

$$\begin{cases} \ddot{x} = \frac{1}{m} \left(X_b \cos\theta \cos\psi + Z_b (\sin\phi \sin\psi + \cos\phi \sin\theta \cos\psi) \right) \\ \ddot{y} = \frac{1}{m} \left(X_b \cos\theta \sin\psi + Z_b (-\sin\phi \cos\psi + \cos\phi \sin\theta \sin\psi) \right) \\ \ddot{z} = \frac{1}{m} \left(-X_b \sin\theta + Z_b \cos\phi \cos\theta \right) - g \\ \dot{p} = \frac{I_{XZ}}{I_{XX}} \left(\dot{r} + pq \right) + \frac{I_{YY} - I_{ZZ}}{I_{XX}} qr + \frac{L}{I_{XX}} \\ \dot{q} = \frac{I_{XZ}}{I_{YY}} \left(r^2 - p^2 \right) + \frac{I_{ZZ} - I_{XX}}{I_{YY}} pr + \frac{M}{I_{YY}} \\ \dot{r} = \frac{I_{XZ}}{I_{ZZ}} \left(\dot{p} - qr \right) + \frac{I_{XX} - I_{YY}}{I_{ZZ}} pq + \frac{N}{I_{ZZ}} \end{cases}$$
(1)

$$\begin{bmatrix} L\\M\\N\\X\\Z \end{bmatrix} = \begin{bmatrix} k_{T1}b\cos\gamma & -k_{T01} & -k_{T3}b\cos\gamma & k_{T03} & 0\\ -k_{T1}a\cos\gamma & -k_{T1}c & -k_{T3}a\cos\gamma & -k_{T3}c & k_{T2}d\\ k_{T01}\cos\gamma & k_{T1}b & -k_{T03}\cos\gamma & -k_{T3}b & k_{T02}\\ 0 & -k_{T1} & 0 & -k_{T3} & 0\\ k_{T1}\cos\gamma & 0 & k_{T3}\cos\gamma & 0 & k_{T2} \end{bmatrix} \begin{bmatrix} \omega_1^2\cos\alpha_1\\ \omega_1^2\sin\alpha_1\\ \omega_2^2\sin\alpha_3\\ \omega_3^2\sin\alpha_3\\ \omega_2^2 \end{bmatrix} = KU$$
(2)

K is a fixed matrix defined by the physical characteristics of the UAV including k_{Ti} and k_{Toi} which represent the thrust and torque coefficients of the "*i*" th propeller, and *U* is a vector defined by control variable as:

$$U = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix} = \begin{bmatrix} \omega_1^2 \cos \alpha_1 \\ \omega_1^2 \sin \alpha_1 \\ \omega_3^2 \cos \alpha_3 \\ \omega_3^2 \sin \alpha_3 \\ \omega_2^2 \end{bmatrix} = K^{-1} \begin{bmatrix} L \\ M \\ N \\ X \\ Z \end{bmatrix}$$
(3)

Therefore, the five control inputs of interest are obtained as:

$$\begin{bmatrix} \omega_1^2 \\ \omega_3^2 \\ \omega_2^2 \\ \alpha_1 \\ \alpha_3 \end{bmatrix} = K^{-1} \begin{bmatrix} \sqrt{u_1^2 + u_2^2} \\ \sqrt{u_3^2 + u_4^2} \\ u_5 \\ \tan^{-1} \frac{u_2}{u_1} \\ \tan^{-1} \frac{u_4}{u_3} \end{bmatrix}$$
(4)

3. Control Technique

The dynamic model of the UAV described in Section 2 can be represented as a general form of a nonlinear state-space system:

$$\dot{\boldsymbol{x}} = F(\boldsymbol{x}, \boldsymbol{u}) \tag{5}$$

where x and u represent the robot's state ($x = [x, y, z, \phi, \theta, \psi, \dot{x}, \dot{y}, \dot{z}, \dot{\phi}, \dot{\theta}, \dot{\psi}]'$) and the input control vector ($u = [\omega_1, \omega_2, \omega_3, \alpha_1, \alpha_3]'$, respectively. By calculating the first order Taylor series of Eq. (5), such formulation can be written as:

$$\dot{\boldsymbol{x}} = \dot{\boldsymbol{x}}_0 + \frac{\partial F}{\partial \boldsymbol{x}}|_{\boldsymbol{x}_0, \boldsymbol{u}_0} (\boldsymbol{x} - \boldsymbol{x}_0) + \frac{\partial F}{\partial \boldsymbol{u}}|_{\boldsymbol{x}_0, \boldsymbol{u}_0} (\boldsymbol{u} - \boldsymbol{u}_0)$$
(6)

in which, x_0 and u_0 are the state and input vectors of the system time, t_0 which is used to denote the UAV's motion at the previous time step. By using a small sampling time to capture the UAV's motion and assuming that a high performance of the actuators used in the drone exist, the changes of the state of the UAV within the given time step can be considered negligible with respect to the large changes of the input parameters. That is, $(x - x_0)$ can be considered negligible (ignored) and therefore, Eq. (6) becomes:

$$\dot{\boldsymbol{x}} \approx \dot{\boldsymbol{x}}_0 + \frac{\partial F}{\partial \boldsymbol{u}}|_{\boldsymbol{x}_0, \boldsymbol{u}_0} (\boldsymbol{u} - \boldsymbol{u}_0)$$
⁽⁷⁾

From the previously described motion model, it is known that the motion of the UAV of interest is highly coupled, and any desired motion will produce additional (coupled) motions. To deal with such complexities, the approach taken in to effectively control the aircraft is to employ an inner loop INDI controller to control the orientation (attitude or angular rotation) of the Navig8, and a combination of NDI and INDI controllers as the outer loop to control the UAV's position. This approach provides an improved control solution which is less dependent to the dynamic model of the system (i.e., replacing sensors' data with the UAV's dynamic model response on the previous time step) enabling the aircraft to effectively manage its dynamic couplings. The developed control scheme is illustrated in Fig. 2.



Fig. 2. Proposed NDI/INDI controller architecture.

3.1. Attitude Control

From the dynamic model, it is noted that the lateral motion of the UAV is highly coupled with the roll angle of the body and pure lateral movement without rolling and vice versa is not possible. Therefore, for the propose of this paper the roll angle is not considered as a controllable state, but it is used as an input to reach a desired lateral translation, y. By defining the UAV's state and the control input per Eqn. (8) one can map the state of the UAV, \bar{x} , to Euler angles of the UAV, $\omega =$ $[\phi, \theta, \psi]$, via a transformation matrix *H* as illustrated in Eq. (9). As a result, it is possible to define ϕ, θ and ψ as the output variables which can then be used as the control variables, as shown in Eq. (8).

$$\bar{x} = \begin{bmatrix} \vartheta \\ \omega \end{bmatrix} = \begin{bmatrix} \phi \\ \theta \\ \psi \\ p \\ q \\ r \end{bmatrix} \quad \bar{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$
(8)

$$\vartheta = H\bar{x} = \begin{bmatrix} I_{3\times3} & 0_{3\times3} \end{bmatrix} \bar{x} \tag{9}$$

Because the first order time derivative of the control variable vector, ϑ , does not contain the control input vector, \bar{u} , the relationship between the input and output variables can be obtained by using the second order derivative of the control vector, ϑ , as shown in Eqs. (10) and (11):

$$\frac{d\vartheta}{dt} = \frac{dH\bar{x}}{dt} = H\bar{x} = H\begin{bmatrix}J(\vartheta)\omega\\\dot{\omega}\end{bmatrix} = J(\vartheta)\omega$$
(10)

$$\frac{d^{2}\vartheta}{dt^{2}} = \frac{d(J(\vartheta)\omega)}{dt} = \frac{d}{d\bar{x}} \left(\begin{bmatrix} 1 & tan\theta sin\phi & tan\theta cos\phi \\ 0 & cos\phi & -sin\phi \\ 0 & sin\phi/cos\theta & cos\phi/cos\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \right) \dot{x}$$
(11)

From Eq. (11) the term $J(\vartheta)$ is jacobian matrix transfering Euler speed to the UAV's rotational speed in the aircraft's body reference frame. From Eq. (11) and following Eq. (7) an INDI controller can be formulated by performing a Taylor series expansion in the current time step, *t*, as described in Equations (12) and (13).

$$\dot{\vartheta} = F_{\vartheta}(\bar{x}, \bar{u}) \tag{12}$$

$$\ddot{\vartheta} \approx F_{\vartheta}(\bar{x}_0, \bar{u}_0) + \frac{\partial F_{\vartheta}(\bar{x}, u)}{\partial \bar{x}}|_{\bar{x}_0, u_0}(\bar{x} - \bar{x}_0) + \frac{\partial F_{\vartheta}(\bar{x}, \bar{u})}{\partial \bar{u}}|_{\bar{x}_0, \bar{u}_0}(\bar{u} - \bar{u}_0)$$
(13)

Per Eq. (7), Eq. (13) can be used to obtain the control law.

$$\ddot{\vartheta} \simeq \ddot{\vartheta}_0 + \bar{K}^{-1}(\bar{x}_0, \bar{u}_0) \Delta \bar{u} \tag{14}$$

Solving for $\Delta \bar{u}$ from Eq. (14) results in $\Delta \bar{u} = \bar{K}(\bar{x}_0, \bar{u}_0)^{-1}(\ddot{\vartheta} - \ddot{\vartheta}_0)$ where matrix $\bar{K}(\bar{x}_0, \bar{u}_0)$ represents the derivative of the rotational equation of the motion with respect to the control inputs of the system, resulting in:

$$\overline{K}(\overline{x}_{0}, \overline{u}_{0}) = J(\overline{x}_{0}) \begin{bmatrix} \frac{k_{T1}b\cos\gamma}{I_{XX}} & 0 & 0\\ 0 & \frac{-k_{T1}c}{I_{YY}} & 0\\ 0 & 0 & \frac{-k_{T03}\cos\gamma}{I_{ZZ}} \end{bmatrix}$$
(15)

With the above formulations it is possible to effectively compute the control inputs based on the vehicle's angular acceleration measurements. Thus, Equations (14) and (15) represent the attitude (orientation) INDI controller which guides the UAV on how to attain the desired orientation over a desired path.

3.2. Position Control

The position controller for the x and z aircraft motions follows a similar approach as the one developed for attitude (Section 3.1). However, a different virtual control, \hat{u} , and corresponding state vector chosen for this controller (Eq. (16)).

$$\hat{u} = \begin{bmatrix} X \\ Z \end{bmatrix} \qquad \hat{x} = \begin{bmatrix} x \\ z \end{bmatrix} \tag{16}$$

Considering the dynamic model of the system in the x - z plane, Eq. (17), and defining states x and z as vector \hat{x} the second order time derivative of matrix \hat{x} for the INDI position controller is given per Eq. (18).

$$\begin{bmatrix} \ddot{x} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} 0 \\ -g \end{bmatrix} + \frac{1}{m} \begin{bmatrix} \cos\theta \, \cos\psi & \sin\phi \, \sin\psi + \cos\phi \, \sin\theta \, \cos\psi \\ -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} X_b \\ Z_b \end{bmatrix}$$
(17)

$$\ddot{\hat{x}} \simeq \ddot{\hat{x}}_0 + \frac{\partial}{\partial u} (f_P(\hat{x}, \hat{u}))|_{\hat{x}_0, \hat{u}_0} (\hat{u} - \hat{u}_0) = \ddot{\hat{x}}_0 + G(\hat{x}_0, \hat{u}_0) \Delta \hat{u}$$
(18)

in which $G(\hat{x}_0, \hat{u}_0)$ is defined by Eq. (19).

$$G(\hat{x}_0, \hat{u}_0) = \frac{\partial}{\partial u} (f_P(\hat{x}, \hat{u}))|_{\hat{x}_0, \hat{u}_0} = \frac{1}{m} \begin{bmatrix} \cos\theta \cos\psi & \sin\phi \sin\psi + \cos\phi \sin\theta \cos\psi \\ -\sin\theta & \cos\phi \cos\theta \end{bmatrix}$$
(19)

$$\hat{u} = \hat{u}_0 + G(\hat{x}_0, \hat{u}_0)^{-1} (\ddot{\hat{x}} - \ddot{\hat{x}}_0)$$
⁽²⁰⁾

As before, because of the high coupling between the roll angle, ϕ , and the motion of the aircraft in y direction it is not possible to independently control both parameters at the same time. By defining the variable, u_y , as per Eq. (21), an NDI controller can then be designed to control the aircraft's y position. The roll output of this controller block, ϕ , can be fed to the attitude control block which will then have full information to control the side motion of the drone.

$$u_y = \sin\phi \tag{21}$$

The second order time derivative of the side motion of the drone given by Eq. (1) is used to develop the NDI position controller that interfaces with a complementary INDI controller responsible for controlling x and z motion. term $sin\phi$ in Eq. (1) has been replaced with u_y as shown in Eq. (22).

$$\ddot{y} = \frac{1}{m} \Big(X_b \cos\theta \sin\psi + Z_b \Big(-u_y \cos\psi + \cos\phi \sin\theta \sin\psi \Big) \Big) = f(\vartheta) + G(\vartheta)u_y$$
(22)

A virtual control input v can then be chosen as $v = \ddot{y}$ provided det($G(\vartheta)$) $\neq 0$ from where Eq. (23) and (24) are generated.

$$u_{y} = G(\vartheta)^{-1}(\nu - f(\vartheta))$$
⁽²³⁾

$$G(x,u) = \frac{-Z_b}{\cos b} \tag{24}$$

With the above formulation one can then formulate a NDI controller with an INDI scheme (see Fig. 2) as an outer loop, to control the slow dynamics of the y motion and then generate the state command used in the inner INDI block as represented by Equation (25).

$$\phi_d = \sin^{-1}(u_y) \tag{25}$$

4. Simulation Results

Simulations were performed to validate the trajectory tracking (with position and orientation) presented in this paper. The results show that the proposed control scheme can minimize tracking errors and enhance tracking precision. As an illustrative example, we show the results when requested the UAV to track a spiral trajectory while maintaining the heading and orientation of the aircraft as per Eq. (26) where the initial state of the system is set to $X_0 = [0\ 0\ 0\ 0\ 0]^T$.

$$\begin{cases}
 x_d(t) = 2 \sin(0.2t) \\
 y_d(t) = 2 \cos(0.2t) - 2 \\
 z_d = 0.1t \\
 \psi_d = 0 \\
 \theta_d = -pi/6
\end{cases}$$
(26)

While using a sample time of 0.01 seconds the results are shown in Figure 2. The drone's position and attitude (orientation) are shown in Figures 2a and 2b, respectively. Figure 3. depicts the trajectory achieved by the drone in 3D space.



Fig. 2. UAV response results in trajectory tracking: a) Position control. b) Attitude control.



Fig. 3. Nose-down maneuver while executing an upward spiral.

The results demonstrated that the developed trajectory tracking control approach is effective for both position and attitude control during trajectory tracking of complex aircraft systems which cannot be effectively controlled with simple linear controllers and poses highly coupled motions.

5. Conclusions

In this paper a nonlinear trajectory controller based on INDI mechanisms has been proposed. The formulation includes an outer NDI/INDI loop controller for position and an inner INDI loop controller for attitude. The proposed controller is effective in controlling aircraft system without the need to have an exact mathematical model of the system yet still using the nonlinear model. The controller is fast enough to control systems in real time and in real world conditions where disturbance might be experienced (although not shown in this paper).

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