Proceedings of the 10th International Conference of Control Systems, Dynamic Systems, and Robotics (CDSR'23)
Ottawa, Canada – June 01 - 03, 2023
Paper No. 217
DOI: 10.11159/cdsr23.217

First Order Dynamic Sliding Mode Control of a Wind Turbine with Optimized Tip Speed Ratio

Nishanth Padmanabhuni[1], Jeff Pieper[2]
University of Calgary
Calgary, Alberta
nishanth.padmanabhun@ucalgary.ca; pieper@ucalgary.ca

Abstract **-** This paper presents a novel sliding mode control method to enhance power generation from wind turbines, with a focus on power optimization. Generator torque only is used as an input since maximizing power using pitch and yaw control is not deemed worth decreasing the life of the turbine due to wear of the mechanical system. The controller is designed based on a 3rd-order model with rotor aerodynamic torque as a disturbance input. Simulation is done using a nonlinear wind turbine model. The first objective is to determine the optimal tip speed ratio for maximum power. To do this, Recursive Least Squares (RLS) is used to estimate a polynomial relating the Tip-Speed Ratio (TSR) and aerodynamic power coefficient. This gives the optimal operating point. To ensure that the system can adapt to changing environments, a forgetting factor is used. The second objective, a first-order dynamic sliding mode controller with integration (FODSMCI), is used to control the wind turbine and maintain it at the optimal TSR with good transient dynamics. The results show that the RLS with high forgetting factor is effective in determining the optimal TSR. FODSMCI allows the user to adjust trade-offs between controller performance and rotor speed tracking, resulting in a response without chattering.

***Keywords*:** Wind energy, Nonlinear control, Robust Control, Wind Turbines, Reference Tracking, Online Estimation

1. Introduction

In recent years, the Canadian government has made a commitment to reduce greenhouse gas emissions and achieving net-zero emissions by 2050[1], in an effort to address the negative impacts of climate change. To meet this goal, the adoption of renewable sources of energy, such as wind, solar, and hydro, has become increasingly important. Among these sources, wind energy has emerged as a particularly promising option, with the potential to provide up to 20% of global electricity production by 2050[2].

Wind turbines convert the wind kinetic energy into electricity through blades that efficiently capture the wind's energy. The rotor, composed of blades mounted on a shaft, turns due to the wind, causing the generator to rotate, producing electricity which is then sent to a transformer and distributed through power lines. Common wind turbine designs are 3 bladed horizontal axis wind turbines with blades parallel to the ground which is the focus of this paper.

The wind turbine operates within three specific regions, each characterized by the wind energy potential that can be captured. In Region I, the level of wind energy is insufficient to justify the operation of a wind turbine, making it an unviable area for wind energy production. In contrast, Region II is an area where wind energy is abundant and can be utilized to generate electricity. However, in Region III, the wind speed is so high that it poses a risk to the turbine's structure. To prevent damage, the turbine is controlled to ensure that it does not overproduce electricity that could damage the turbine. Pitch and yaw control are used in power regulation for wind turbines. However, to reduce wear, generator torque control is a better option for in the power optimization region(Region II)[3],to control rotor speed and maximize power production.

This paper proposes using recursive least squares with a forgetting factor as a reference tracking method to determine the optimal tip-speed ratio for wind turbine systems, in order to maximize power captured from the wind. The controller used will be a first-order dynamic sliding mode controller with integration (FODSMCI), an adaptation of the Pieper[4] method, for power optimization. Fig. 1 illustrates the implementation of the proposed method:

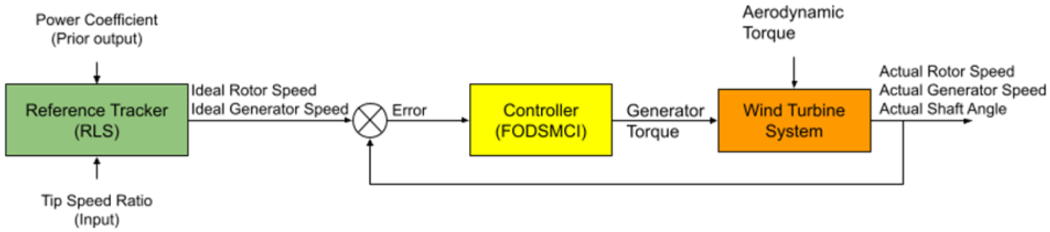

Fig. 1: Diagram of this system that will be implemented in this paper.

2. Dynamic Modelling of the Wind turbine

The reference tracker and controller will be evaluated using the wind turbine model developed by Bianchi et al. [5]. It should be noted that the model represents an idealized scenario and may not accurately reflect a real-world system. The wind turbine model was created using the dynamic equations of the drivetrain. These equations drive the system, based on Soltani et al. [6] and Bianchi et al. [5]:

$$\begin{bmatrix} \dot{\omega}_r \\ \dot{\omega}_g \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{-B_d}{J_r} & \frac{B_d}{NJ_r} & \frac{-K_d}{J_r} \\ \frac{B_d}{NJ_g} & \frac{-B_d}{N^2 J_g} & \frac{K_d}{NJ_g} \\ 1 & \frac{-1}{N} & 0 \end{bmatrix} \begin{bmatrix} \omega_r \\ \omega_g \\ \theta \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{-1}{J_g} \\ 0 \end{bmatrix} [T_g] + \begin{bmatrix} \frac{1}{J_r} \\ 0 \\ 0 \end{bmatrix} [T_r] \quad (1)$$

$$\dot{x} = \boldsymbol{A}x + \boldsymbol{B_c}u + \boldsymbol{B_d}d \quad (2)$$

Where $x \in R^n, u \in R$ and $d \in R$, are the wind turbine state, generator input torque (T_g) as input and rotor aerodynamic torque (T_r) as disturbance. $\boldsymbol{A}$, $\boldsymbol{B_c}$, and $\boldsymbol{B_d}$ are the state, control and disturbance matrix. ω_r denotes the rotor speed, ω_g denotes the generator speed, and θ denotes the torsion angle of the drivetrain.

T_r is a function of the wind speed and power coefficient, defined as:

$$T_r = \frac{1}{2}\rho\pi R^3 \frac{C_p(\lambda,\beta)}{\lambda} V^2 \quad (3)$$

The aerodynamic power coefficient (C_p) measures the power that the wind turbine extracts from the wind, it is the ratio of generated power to total power in the wind passing through the rotor. Its dependent on the tip speed ratio (λ) and pitch angle (β). Different techniques like exponential, sinusoidal, polynomial, or data-driven algorithms[7] can be used to model C_p. However, for this paper, this specific model will be used:

$$C_{\mathrm{p}}(\lambda,\beta) = 0.5176\left(\frac{116}{\lambda_i} - 0.4\beta - 5\right)\mathrm{e}^{\frac{-21}{\lambda_i}} + 0.0068\lambda$$
$$\frac{1}{\lambda_i} = \frac{1}{\lambda + 0.08\beta} - \frac{0.035}{\beta^3 + 1} \quad (4)$$

Table 1 below shows the wind turbine characteristics to develop the wind turbine model for simulation. These parameters are based on the 5MW onshore horizontal wind turbine[8].

Table 1: Wind Turbine Parameters used in Simulation for a 5MW wind turbine.

Symbol	Description	Value
B_d	Drivetrain damping coefficient	$6.215E6\frac{\text{Nms}}{rad}$
K_d	Drivetrain spring constant	$8.68E8\frac{\text{Nm}}{\text{rad}}$
J_r	Rotor inertia	$38677056\ \text{kg m}^2$
J_g	Generator inertia	$534.1\ \text{kgm}^2$
N	Gearbox ratio	97
R	Rotor blade radius	63 m
ρ	Air density	$1.225\frac{\text{kg}}{\text{m}^3}$

Fig. 2 below shows how the wind turbine system will be designed in Simulink. The generator torque will be used as the control input to make sure the rotor speed tracks the optimal rotor speed. The aerodynamic rotor torque is a function of the rotor speed, wind speed and power coefficient and will be used as the disturbance input. This paper will not incorporate pitch control to maximize power and will maintain the pitch at 0°.

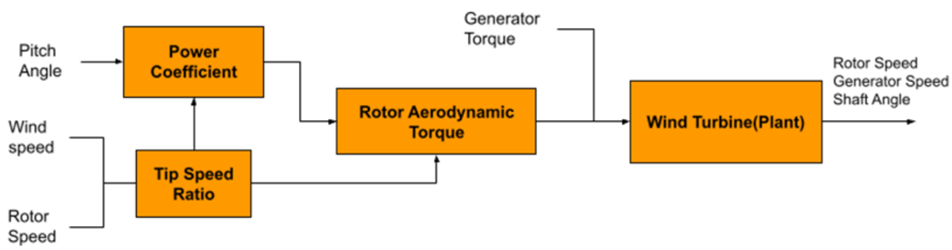

Fig. 2: Wind turbine System used to test the controller.

3. Reference Tracking with Recursive Least square with an forgetting factor

This paper assumes that wind speed and rotor speed are known. Measuring rotor speed is relatively simple with an encoder or tachometer, however, measuring wind speed can be difficult due to factors such as turbulence and sensor accuracy. Methods for measuring wind speed include the use of anemometers, lidar, and sodar.

Recursive least squares (RLS) is a linear model parameter estimation algorithm that updates estimates using recent data. It has fast convergence rate and is well suited for online learning. RLS is also robust to noise. It is useful in determining the optimal TSR for a wind turbine.

This paper assumes that the results obtained from the wind turbine system are in continuous time. However, RLS requires sampled data. To minimize computational complexity while preserving the accuracy of TSR estimation, a sampling period (T_s) will be chosen. The method of implementing RLS with a forgetting factor will be based on the work of Vahidi et al [9]. The input used in this method will be the TSR, and the output will be the Aerodynamic Power Coefficient (C_P) for the polynomial equation.

$$C_p(i) = \alpha_m TSR_i^m + \alpha_{m-1} TSR_i^{m-1} + \cdots + \alpha_2 TSR_i^2 + \alpha_1 TSR_i + \alpha_o \qquad (5)$$

Where α_m is the coefficients. Then, initialize the covariance matrix($\boldsymbol{P}$), input vector($\boldsymbol{\sigma}$), and estimated parameters vector($\widehat{\boldsymbol{\theta}}$):

$$\boldsymbol{P} = 100\begin{bmatrix}1 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & 1\end{bmatrix}, \boldsymbol{\sigma} = \begin{bmatrix}TSR_i^m\\ \vdots\\ 1\end{bmatrix}, \widehat{\boldsymbol{\theta}} = \begin{bmatrix}\alpha_m\\ \vdots\\ \alpha_o\end{bmatrix} = \begin{bmatrix}0\\ \vdots\\ 0\end{bmatrix} \tag{6}$$

Update the parameter gain value K:

$$\boldsymbol{K} = \frac{\boldsymbol{P}(i-1)\boldsymbol{\sigma}}{\mu + \boldsymbol{\sigma}^T\boldsymbol{P}(i-1)\boldsymbol{\sigma}} \tag{7}$$

Calculate the covariance matrix:

$$\boldsymbol{P}(i) = \frac{\boldsymbol{P}(i-1)}{\mu}\left(\boldsymbol{I}(m+1) - \boldsymbol{K}\boldsymbol{\sigma}^{\boldsymbol{T}}\right) \tag{8}$$

The forgetting factor (μ) in Eqs. (7) and (8) is a parameter that determines the weight given to new data. It typically ranges from 0.9 to 1, with lower values indicating that only more recent data should be trusted, and a value of 1 indicating that all data should be given equal weight.

$$\widehat{\boldsymbol{\theta}}(i) = \widehat{\boldsymbol{\theta}}(i-1) + \boldsymbol{K}(y(i) - \boldsymbol{\sigma}^{\boldsymbol{T}}\widehat{\boldsymbol{\theta}}(i-1)) \tag{9}$$

To find the optimal TSR that yields the maximum power, the derivative of the power coefficient polynomial (Eqs. (5)) with respect to TSR is taken and set equal to zero:

$$\frac{dC_p(i)}{dTSR} = \alpha_m m * TSR^{m-1} + \alpha_{m-1} * (m-1) * TSR^{m-2} + \cdots + \alpha_2 * 2 * TSR + \alpha_1 = 0 \tag{10}$$

In this study, $m = 3$ is utilized, simplifying the process of determining the optimal TSR that yields the maximum C_P value. The MATLAB 'roots' function will be used to accomplish this.

Repeat this entire process for the next sample data with the new $\widehat{\boldsymbol{\theta}}$ and next $\boldsymbol{\sigma}$.

4. Controlling the System with First Order Dynamic Sliding mode control with Integration

This paper will use sliding mode control (SMC) to maximize power generation from the turbine. SMC is a robust and finite-time converging control strategy for systems with uncertain dynamics and disturbances. However, a major issue with SMC is chattering, caused by the discontinuity of the sliding surface. This can lead to increased wear and tear on the system.

In this case, first-order dynamic sliding mode control with integration (FODSMCI) will be used. This modified model is based on the work of Pieper[4]. The first-order dynamics of the controller filter the error signal, eliminating chattering at the actuator as a result. This is achieved by incorporating an integral term directly into the sliding function, which eliminates steady-state errors and compensates for persistent disturbances. Additionally, linear quadratic optimal design conditions can be used to select the sliding surface vector parameter.

To start, the controllable portion Eqs. (2) is needed:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}_{\boldsymbol{c}}u \tag{11}$$

And then selected sliding surface:

$$s = \boldsymbol{C_1}\boldsymbol{x} + C_2\int \boldsymbol{C_1}\boldsymbol{x} + Du \tag{12}$$

Where $C_1$is the sliding gains of the state and $C_2$is the sliding gains for the integral terms. Then, to find the control model, take the derivative of Eqs. (12) and substitute in Eqs. (11). Finally, isolate for $D\dot{u}$.

$$D\dot{u} = -\boldsymbol{C_1 A x} - \boldsymbol{C_1 B_c} u - C_2 \boldsymbol{C_1 x} + \dot{s} \tag{13}$$

The sliding mode condition($\dot{s}$) defined by satisfying the reaching condition[10]($s\dot{s} < -\eta|(s)|$). Therefore, the sliding sliding mode condition will be:

$$\dot{s} = -\eta * \text{sign}(s) \tag{14}$$

It can be assuming that $D \neq 0$ and further, without loss of generality that $\text{D} = 1$. Therefore, the controller can be reformed as a first order differential equation as:

$$\dot{u} = -\boldsymbol{C_1 A x} - \boldsymbol{C_1 B_c} u - C_2 \boldsymbol{C_1 x} - \eta * sign(s) \tag{15}$$

4.1. Hyperplane Design for First Order Dynamic Sliding Modes with Integration

To find C_1 and C_2, the system can be assumed to be on the sliding surface which gives the closed loop equations of the system to be:

$$\begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{x}}_{\boldsymbol{i}} \\ \dot{u} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A} & 0 & \boldsymbol{B} \\ \boldsymbol{C_1} & 0 & 0 \\ -\boldsymbol{C_1 A} - C_2 \boldsymbol{C_1} & 0 & -\boldsymbol{C_1 B_c} \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{x_i} \\ u \end{bmatrix} \tag{16}$$

To use the Hyperplane design method in [11], Eqs. (16) above needs to be transformed:

$$\begin{bmatrix} \dot{\boldsymbol{x}}_{\boldsymbol{s}} \\ \dot{u} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A_a} & \boldsymbol{B_a} \\ -\boldsymbol{C_a A_a} & -\boldsymbol{C_a B_a} \end{bmatrix} \begin{bmatrix} \boldsymbol{x_s} \\ u \end{bmatrix} \tag{17}$$

Where:

$$\boldsymbol{A_a} = \begin{bmatrix} \boldsymbol{A} & 0 \\ \boldsymbol{C_1} & 0 \end{bmatrix}, \boldsymbol{B_a} = \begin{bmatrix} \boldsymbol{B}_c \\ 0 \end{bmatrix}, \boldsymbol{C_a} = [\boldsymbol{C_1} \quad C_2], \boldsymbol{x_s} = \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{x_i} \end{bmatrix} \tag{18}$$

Then quadratic performance index[4], [11] chosen as:

$$J = \int_0^\infty [\boldsymbol{x_s^T} \quad \boldsymbol{u^T}] \begin{bmatrix} \boldsymbol{H_{11}} & \boldsymbol{H_{12}} \\ \boldsymbol{H_{21}} & R \end{bmatrix} \begin{bmatrix} \boldsymbol{x_s} \\ u \end{bmatrix} dt \tag{19}$$

$\boldsymbol{H_{11}}$ and R are positive semi-definite matrices that is used to weigh the importance of the states and control inputs for the performance index. $\boldsymbol{H_{11}}$ determines how much the control strategy should try to minimize the deviation of the states from their desired values. R determines how much the control strategy should try to minimize the control effort.

In order to minimize Eqs. (19), cross product terms are considered since $\boldsymbol{H_{12}}$ and $\boldsymbol{H_{21}}$ are generally non-zero. First, a new state is defined and the transformed system is solved using the state transition matrix **($\mathbf{F}^*$)** and state weighting matrix **($\mathbf{R}^*$)** in the Algebraic Riccati Equation (ARE):

$$\begin{aligned} \mathbf{F}^* &= \boldsymbol{A_a} - \boldsymbol{B_a} R^{-1} \boldsymbol{H_{21}} \\ \mathbf{R}^* &= \boldsymbol{H_{11}} - \boldsymbol{H_{12}} R^{-1} \boldsymbol{H_{21}} \end{aligned} \tag{20}$$

Then solve ARE for $\boldsymbol{S}$:

$$\boldsymbol{S F^*} + \boldsymbol{F^{*T} S} - \boldsymbol{S B_a} R^{-1} \boldsymbol{B_a}^{\boldsymbol{T}} \boldsymbol{S} + \boldsymbol{R}^* = 0 \tag{21}$$

Once the solution has been obtained, the coordinates are transformed back to the original ones to obtain the pseudo-control as defined in Eq. (22), which serves as the choice of sliding mode defining vector:

$$\boldsymbol{C_a} = R^{-1}\left(\boldsymbol{H_{21}} + \boldsymbol{B_a}^{\boldsymbol{T}} \boldsymbol{S}\right) \tag{22}$$

However, since $\boldsymbol{A_a}$ is based on $\boldsymbol{C_1}$, which is based on $\boldsymbol{C_a}$, the system needs to be solved iteratively until the sliding surface($\boldsymbol{C_a}$) gain converges to the stopping criteria(i.e. $max(\boldsymbol{C_a}(i)) - max(\boldsymbol{C_a}(i-1)) > 0.01$). Then, to minimize the cost function along the sliding surface, redefine:

$$\boldsymbol{H_{11}} = \boldsymbol{C_a^T C_a} \tag{23}$$

Repeat the process from Eqs. (18) to (22) until C_a converges as shown in Fig. 3 below:

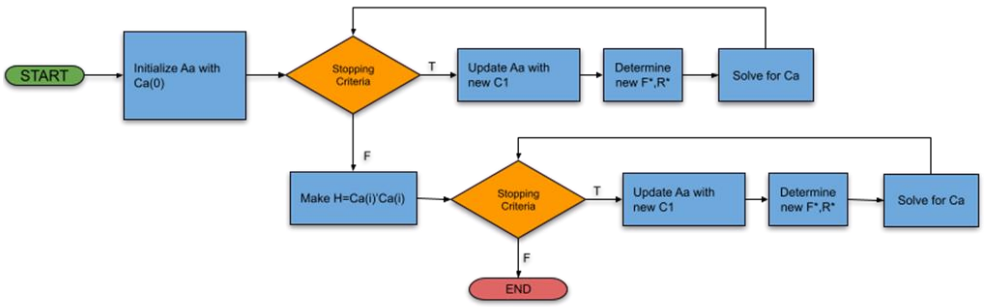

Fig. 3: Flow chart on solving for the Sliding surface gains.

Our aim is to make the rotor speed track the optimal rotor speed to maximize power generation, while also reducing the usage of generator torque to extend the life of the turbine. However, this introduces a trade-off as reducing the usage of generator torque will lower the rotor speed tracking, thus reducing power.

5. Simulations Results

The reference tracking and controller are tested using a model outlined in Section 2. The wind is generated using TurbSim[12], a software tool developed by NREL for simulating turbulent wind fields. This software allows users to generate time-series wind field data, which serves as input to represent wind disturbances in the system.

Before entering the controller, the wind data is filtered through a low-pass filter, as shown in Fig. 4. This effectively suppresses high-frequency noise and improves the control system's performance. By removing the rapid fluctuations, the filter leaves only the underlying steady-state wind. This enables the controller to make more accurate and stable decisions, thus reducing the risk of damage to the controller and other components.

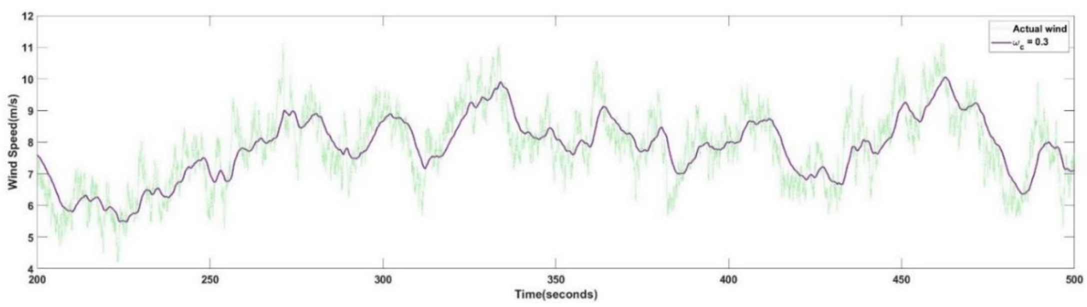

Fig. 4: Wind Speed Profile to be used a disturbance input for the aerodynamic rotor torque

5.1. Results from the Recursive least square method with a forgetting factor

The method outlined in Section 3 was used to find the optimal TSR. To simulate the change in optimal TSR, the pitch angle of the system was changed from 0° to 30° for Eqs. (4) after 300 seconds. A sample period of $T_s = .05$ seconds was also selected, and a third order ($m = 3$) polynomial was used.

As shown in Fig. 5, the RLS method successfully tracks the optimal TSR, but with a slow reaction time. Utilizing a high forgetting factor (μ) allows for a quicker response to changes in the system. However, when $\mu = 0.9$, tracking becomes unstable and less desirable.

A low forgetting factor can lead to less optimal system performance but changes in wind turbine characteristics occur gradually, so a higher forgetting factor allows better adaptation to these changes.

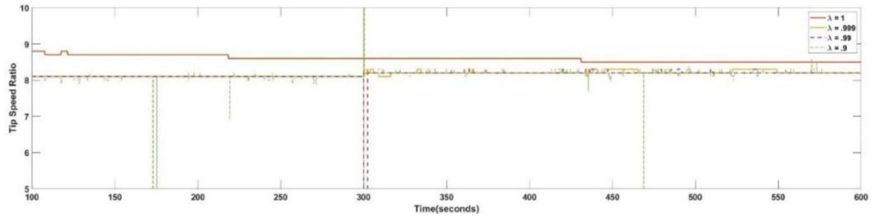

Fig. 5: Optimal TSR using RLS with a forgetting factor

5.2. Results from the First order dynamic sliding mode controller with Integration

The FODSMCI offers flexibility in the design of sliding surfaces and closed-loop dynamics to achieve specific performance objectives. In particular, the aim is to balance the conflicting goals of maximizing power generation and extending the life of the wind turbine. The design parameters selected for this case were:

$$\boldsymbol{H_{11}} = \begin{bmatrix} .1 & 0 & 0 & 0 \\ 0 & .01 & 0 & 0 \\ 0 & 0 & .01 & 0 \\ 0 & 0 & 0 & H_{11}(4,4) \end{bmatrix}, \boldsymbol{H_{12}^T} = \boldsymbol{H_{21}} = [0 \quad 0 \quad 0 \quad 0]$$

Table 2 below compares the power generated with different penalties on R for the performance index (Eqs. (19)).

Table 2 Power generated over 300 seconds with difference weights

Parameters	Energy Generated (MJ) for 300s	% Change from R=1	RMSE Rotor Speed
$R = 0.1, H_{11}(4,4) = 1$	658.30	0.1%	0.1%
$R = 1, H_{11}(4,4) = 1$	657.73	-	1.1%
$R = 10, H_{11}(4,4) = 10$	648.23	-1.4%	5.7%
PI control	656.76	-0.1%	1.2%

As seen in Fig. 6 and 7 below, the FODSMCI effectively filters the error signal before it is sent to the generator, resulting in a smooth response without chattering, eliminating the need for boundary layers. Additionally, it is observed that as the parameter R increases, controller performance in terms of smoothness improves, however, rotor speed tracking deteriorates. Conversely, as R decreases, the controller becomes more aggressive, resulting in an improvement in rotor speed performance. These findings demonstrate a trade-off between the smoothness of controller performance and rotor speed tracking.

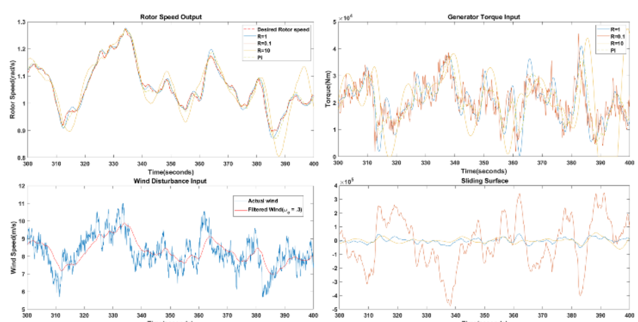

Fig. 6: Results of FODSMCI with different penalties

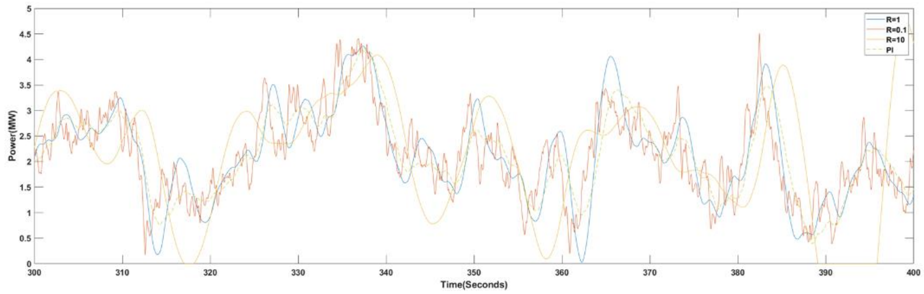

Fig. 7: Power Generated over time compared with different penalties.

4. Conclusion

This study aimed to improve the power optimization of wind turbines. Results from the reference tracking portion showed that RLS with a forgetting factor was successful in determining the optimal TSR. However, the use of the FODSMCI was found to have trade-offs between controller performance and rotor speed tracking. Increasing the control penalty improved controller performance in terms of smoothness, but negatively impacted rotor speed tracking. Conversely, decreasing the control penalty improved rotor speed performance but decreased controller performance. Additionally, the FODSMCI was effectively able to eliminate chattering. These findings provide useful insights for future research in the implementation of wind turbine models.

References

[1] E. and C. C. Canada, “Government of Canada legislates climate accountability with first net-zero emissions law,” Jun. 30, 2021. https://www.canada.ca/en/environment-climate-change/news/2021/06/government-of-canada-legislates-climate-accountability-with-first-net-zero-emissions-law.html (accessed Dec. 20, 2021).

[2] A. S. Darwish and R. Al-Dabbagh, “Wind energy state of the art: present and future technology advancements,” *Renew. Energy Environ. Sustain.*, vol. 5, p. 7, 2020, doi: 10.1051/rees/2020003.

[3] J. Chu, L. Yuan, Z. Chen, W. Xu, Z. Lin, and Y. Du, “A Torque Coordination Control Strategy for Wind Turbine Based on Pole Placement,” in *2018 37th Chinese Control Conference (CCC)*, Jul. 2018, pp. 7387–7393. doi: 10.23919/ChiCC.2018.8483512.

[4] Jeff Pieper, “First order dynamic sliding mode control,” in *Proceedings of the 37th IEEE Conference on Decision and Control (Cat. No.98CH36171)*, Tampa, FL, USA: IEEE, 1998, pp. 2415–2420. doi: 10.1109/CDC.1998.757768.

[5] F. D. Bianchi, H. De Battista, and R. J. Mantz, *Wind turbine control systems: principles, modelling and gain scheduling design*. in Advances in industrial control. London: Springer, 2007.

[6] M. N. Soltani, T. Knudsen, M. Svenstrup, and R. Wisniewski, “Estimation of Rotor Effective Wind Speed: A Comparison,” *IEEE Trans. Contr. Syst. Technol.*, vol. 21, no. 4, pp. 1155–1167, Jul. 2013, doi: 10.1109/TCST.2013.2260751.

[7] M. Carpintero-Renteria, D. Santos-Martin, A. Lent, and C. Ramos, “Wind turbine power coefficient models based on neural networks and polynomial fitting,” *IET Renewable Power Generation*, vol. 14, no. 11, pp. 1841–1849, 2020.

[8] J. Jonkman, S. Butterfield, W. Musial, and G. Scott, “Definition of a 5-MW Reference Wind Turbine for Offshore System Development,” NREL/TP-500-38060, 947422, Feb. 2009. doi: 10.2172/947422.

[9] A. Vahidi, A. Stefanopoulou, and H. Peng, “Recursive least squares with forgetting for online estimation of vehicle mass and road grade: theory and experiments,” *Vehicle System Dynamics*, vol. 43, no. 1, pp. 31–55, Jan. 2005, doi: 10.1080/00423110412331290446.

[10] Sarah Spurgeon, “Sliding mode control: a tutorial,” in *2014 European Control Conference (ECC)*, Strasbourg, France: IEEE, Jun. 2014, pp. 2272–2277. doi: 10.1109/ECC.2014.6862622.

[11] C. M. Dorling and A. Zinober, “Hyperplane Design in Model-Following Variable Structure Control Systems,” *IFAC Proceedings Volume*, vol. 18, no. 5, pp. 1901--1905, 1985.

[12] B. J. Jonkman, “Turbsim User’s Guide: Version 1.50,” NREL/TP-500-46198, 965520, Sep. 2009. doi: 10.2172/965520.