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Dynamic Analysis of a Head-On Sedan Automobile Collision

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Abstract **-** There are thousands of automobile accidents each day, with none being exactly alike. Everyone is familiar with the crash tests automobile manufacturers perform to see how their vehicles will behave in the event of a crash, but it is impossible for an automobile manufacturer to test and analyze each type of accident that occurs on roadways today. Oftentimes, only a few tests are run, each having a different impact point on the vehicle (front, rear, or sides). This gives a vague idea of what to expect during a crash but cannot provide a proper analysis for every scenario. In the analysis presented within this paper, the temperatures are assumed to be below freezing, with snow on the road, replicating a crash that occurs quite often in the northern parts of the United States. By considering the reduced friction factor due to frozen roads, the properties of the materials of the vehicle at sub-freezing temperatures, as well as the behavior of the vehicle after the crash; this scenario is unique and is rarely, if not ever tested by auto manufacturers. This research provides strong evidence and gives a depiction of how vehicles behave in a head on collision in Winter driving conditions. During this simulation, the mass of the front crash bar had a maximum displacement of 0.52 meters, while the mass of the engine components only moved 0.16 meters. The fact that the front crash bar moved 0.52 meters towards the engine shows that the frontal engine components would have sustained damage during this crash because the crash bar and the engine are initially less than 0.5 meters apart. There were also substantial forces seen within the springs and damper, with a maximum value of approximately 90 kN being found in the spring representing the crash bar.

Keywords**:** Accident, forces, dynamic, system, snow, ice, engine, transportation

1. Introduction

Automobile accidents are very unpredictable and can happen in a number of different ways with an infinite number of factors and contributors, making them one of the harder circumstances to study and analyze. This is why it is essential to have many different forms of tests and models to learn about the behavior of not only the vehicles involved in the crash, but also some of the main components that comprise each vehicle. Most mathematical crash models do not consider the individual components of a vehicle, rather they focus on the car as a whole, modeling it as a single mass or other object [1]. During the simulation and analysis discussed in this paper, the components in front of the sedan's firewall were included because the front bumper assembly, engine, and engine components can displace greatly during a frontal crash, often causing expensive and sometimes irreparable damage to the vehicle. The engine components being considered are items such as the transmission, camshaft, and similar items within the engine block. These were taken to be mass M_1. The front crash bar, bumper, and front body paneling was taken to be M_2. These masses were the main objects of interest during the simulation. Oftentimes, in other vehicle to vehicle collisions, these components are not considered due to their complexity to model [2].

For this study, there were two separate vehicles involved, one being the vehicle that was considered the bullet vehicle (faster vehicle), and the other being the target vehicle, which was moving at a much slower speed. According to the National Safety Council, approximately 70 percent of automobile accidents involve multiple vehicles, rather than one vehicle and a stationary object [3]. This is the main reason two vehicles were considered, rather than just a single vehicle colliding with a stationary object such as a tree. The bullet vehicle was taken to be a 2007 Audi A4 sedan with a mass of 1,600 kilograms. This specific vehicle was used because it is a vehicle that the author is familiar with and understands how the vehicle behaves during a crash. The target vehicle was a much larger vehicle, like a truck, and its slower speed was added to the speed of the bullet vehicle to determine the total force acting upon the Audi's component system. Another purpose of this simplification was that the speed of the bullet vehicle was much faster than that of the target vehicle (11 m/s to 2.25 m/s), and by adding the speeds together, the calculation of the force would be more straightforward for the entire system.

Another reason that the crash study in this paper is different from similar crash studies is that it considers the effects of subzero temperatures on the friction coefficient of the road surface [2, 4]. This variable changes both how the car behaves when the brakes are applied, and how the vehicles displace after the crash. Combining these factors with those listed above makes this dynamic collision model very unique.

2. Collision Model

In order to obtain an accurate result for the two governing equations that were formulated for this system, a proper diagram of the system needed to be created showing the masses, springs, dampers, and any stationary objects within the system. This model is shown below in figure 1.

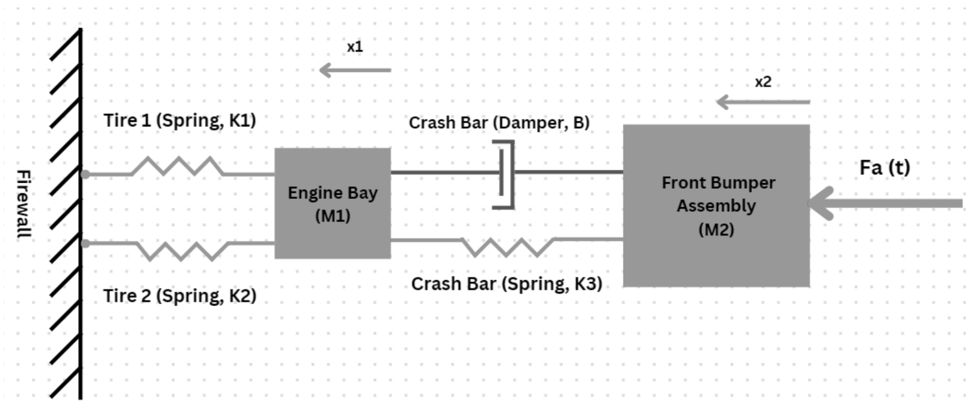

Fig. 1: Overall system diagram

Looking at the figure, it can be seen that there are two different masses within the system, each with their own respective displacements x_1 and x_2. Mass M_1 is attached to the stationary firewall of the vehicle by the two tires that are modeled as springs in this system (more on this later). The tires on the Audi A4 are located slightly behind the engine, so it is acceptable to take them as the connecting component between the engine and the firewall for the sake of simplicity. The mass M_1 is connected to mass M_2 by the supports located between the crash bar and the engine. These supports are made of mostly aluminum and a small amount of steel, so they can be modeled as both a spring and a damper due to aluminum being a highly ductile material. The final force that is acting within the system is the input force from the actual collision between the two vehicles. Due to the fact that this was a frontal crash, the crash bar, M_2, is the mass that is directly affected by force caused by the collision.

2.1 Forces Within the System

Each of the individual components contained within the system exhibit their own forces on other objects in the system besides the masses M_1 and M_2. This is because the masses are what the forces act on and do not have their own source of force. Using figure 1, it can be speculated that the displacement of mass M_2 will be greater than the displacement of mass M_1 because the outside force is acting directly on mass M_2. The forces for the spring equations are all similar but use different displacements depending on which of the masses they are connected to. These are shown below as equations 1 through 3.

$$F_{k1} = K_1 x_1 \tag{1}$$

$$F_{k2} = K_2 x_1 \tag{2}$$

$$F_{k3} = K_3 (x_2 - x_1) \tag{3}$$

The damping force of the damper B is given below in equation 4. It is important to note that the damping force uses the velocity of the masses, not the displacement, as the spring modelling equations had.

$$F_B = B(\dot{x_2} - \dot{x_1}) \tag{4}$$

2.2 Free Body Diagram

Free body diagrams are commonly used when modelling dynamic systems to display how forces act on objects within the system using blocks and arrows. The two objects of interest within this system are M_1 and M_2 which are shown as blocks in figure 2. The forces that are acting on them come from the external force during the collision and the spring and damping forces from the other components of the vehicle. Figure 2 shows the free body diagram of the system with all forces included.

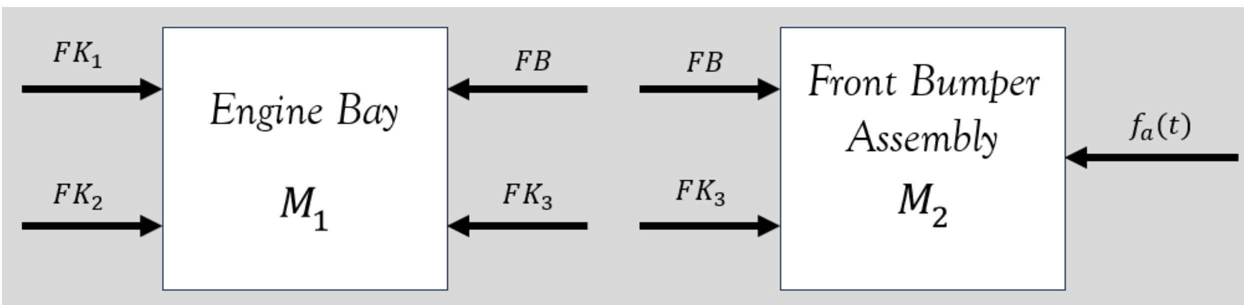

Fig 2: Free body diagram showing forces acting on each mass.

3. Dynamics

After modeling the system using the free body diagram, the direction of each force was determined, allowing for the formulation of modeling equations that represent each mass element, M_1 and M_2, respectively. The equation relating to mass M_1 is shown below as equation 5, and the equation relating to mass M_2 is shown as equation 6.

$$\boldsymbol{M_1\ddot{x_1} - B(\dot{x_2} - \dot{x_1}) - K_3(x_2 - x_1) + (K_1 + K_2)x_1 = 0} \tag{5}$$

$$\boldsymbol{M_2\ddot{x_2} + B(\dot{x_2} - \dot{x_1}) + K_3(x_2 - x_1) = f_a(\mathrm{t})} \tag{6}$$

3.1 State Variable Equations

The state variables are determined from equations 5 and 6 above, looking at the masses, M, and the springs, K. Masses have variable values of velocity ($\dot{x}_1,\ \dot{x}_2 = v_1, v_2$), while springs have values of displacement ($x_1,\ x_2$). Therefore, in this system, between the two masses and three springs, the state variables are x_1, x_2, v_1, and v_2. Next, these state variables needed to be combined into state variable equations. This is done by taking the derivative of each of the four state variables above. For x_1 and x_2, the state variable equations are very simple. Taking the derivative of each displacement leads to the velocities of the masses as shown below in equations 7 and 8.

$$\dot{x}_1 = v_1 \tag{7}$$

$$\dot{x}_2 = v_2 \tag{8}$$

The process to obtain the derivatives of v_1 and v_2 is slightly more complex. The modeling equations 5 and 6 need to be manipulated in order to calculate $\ddot{x}_1$ and $\ddot{x}_2$, which are equal to $\dot{v}_1$ and $\dot{v}_2$, and are shown below as equations 9 and 10.

$$\dot{v}_1 = \frac{B}{M_1}v_2 - \frac{B}{M_1}v_1 + \frac{K_3}{M_1}x_2 - \frac{K_3}{M_1}x_1 - \frac{(K_1+K_2)}{M_1}x_1 \tag{9}$$

$$\dot{v}_2 = \frac{B}{M_2}v_1 - \frac{B}{M_2}v_2 + \frac{K_3}{M_2}x_1 - \frac{K_3}{M_2}x_2 + \frac{f_a(t)}{M_2} \tag{10}$$

It is also possible to obtain a matrix relation for the state variable equations 7 through 10, and it is shown in equation 11. These matrices make it much easier to enter the equations into a computing software such as Matlab and utilize them in plots or for other coding, which is what was done to obtain the plots and results that are shown later in this research.

$$\begin{bmatrix} \dot{x_1} \\ \dot{v_1} \\ \dot{x_2} \\ \dot{v_2} \end{bmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -\frac{(K_3+K_2+K_1)}{M_1} & -\frac{B}{M_1} & \frac{K_3}{M_1} & \frac{B}{M_1} \\ 0 & 0 & 0 & 1 \\ \frac{K_3}{M_2} & \frac{B}{M_2} & -\frac{K_3}{M_2} & -\frac{B}{M_2} \end{pmatrix} \begin{bmatrix} x_1 \\ v_1 \\ x_2 \\ v_2 \end{bmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{M_2} \end{pmatrix} \cdot f_a(t) \tag{11}$$

3.2 Magnitudes of Component Forces

In order to utilize the matrix shown in equation 11 within the Matlab software, the magnitudes of each force variable ($FK_1, FK_2, FK_3, FB, M_1, M_2$, and the external force $F_a(t)$) needed to be determined. The first set of forces (FK_1 and FK_2) are the tires on the Audi A4. These tires are made of rubber, so they behave as springs when any outside force is applied to them, whether it be horizontal or vertical. The tires were both of the same compound, size, and brand, so their spring forces were assumed to be the same at a value of 475 KN/m, which is a value from *Advanced Tire Mechanics*, and was determined using the proper tire size (225/45/R17) [5]. The final spring within the system has a force of FK_3, which is labeled as the crash bar of the vehicle. The crash bar on the Audi A4 is made of aluminum and boasts properties similar to that of a spring in order to prevent deformation during a low-speed crash. To obtain the spring force that the crash bar exerts on the system, figure 3 was used. The plot on the left side of the figure shows the relationship of the spring constant of the front bumper to displacement during an automobile crash. The red circle shows which value was used in this simulation, 215,000 N/m [6].

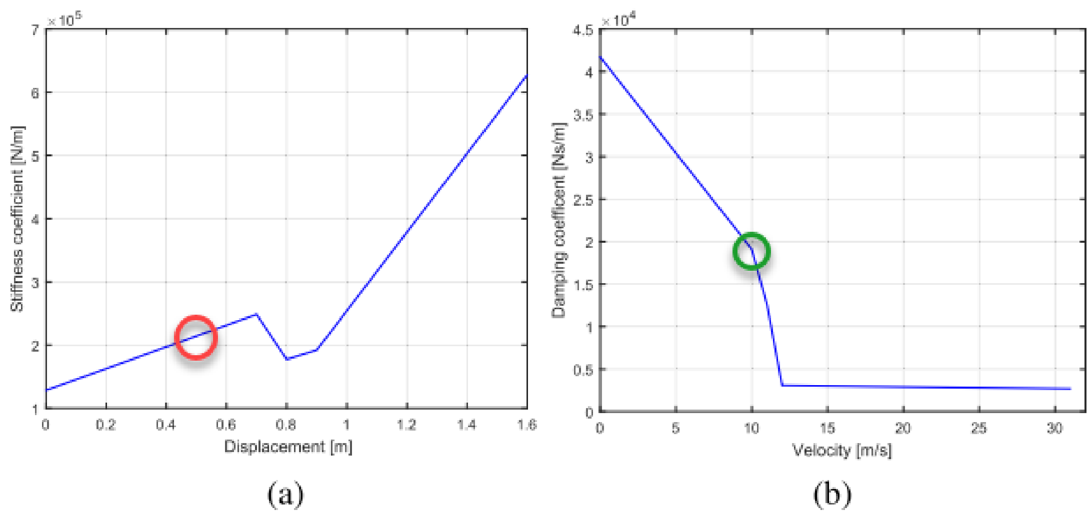

Fig 3: Spring and damper forces of a vehicle chassis during a crash [6].

The singular damper within the system has a damping force with a magnitude of FB. There is very little data involving the use of metals as dampers, so after thorough research, it was determined that a damping force for the whole front end of the car would be used, which had been calculated during a previous study of a crash using computer software [6]. The right plot in figure 3 shows the resulting plot of damping force in Ns/m in relation to the velocity of the vehicle during the crash.

For simplicity, this value was estimated at 10 meters per second because it is very clear where the plot crosses the 10 m/s line on the grid. This value was taken to be 18,000 Ns/m.

Next, the two masses, M_1 and M_2 were to be determined. Mass M_1 was referred to as the mass of the engine bay, which means that it is the sum of the engine components within the Audi A4. The engine, transmission, and similar engine related components were all considered within this mass. This value was determined to be 550 kg and was larger than the mass M_2. For mass M_2, the mass of the entire front end of the car (crash bar, bumper, quarter panels, and radiator) were determined using a simple ratio for all-wheel drive cars. Most all-wheel drive cars have a 60/40 ratio, meaning that 60 percent of the overall mass of the car is contained in the front of the vehicle, supported by the front axles. Because the total mass of the 2007 Audi A4 was close to 1600 kg, using the 60/40 rule, as well as excluding the mass that was already taken as mass M_1, the mass M_2 was approximated as 410 kg [7].

3.3 Magnitude of the External Force

The last, and arguably most important force that needed to be determined was the external force from the two vehicles colliding. This force was labeled as $F_a(t)$ and was calculated using equation 12, then subtracting the total braking force of the front and rear brakes to simulate the brakes locking up on the vehicle, which is the case right before most crashes.

$$F = \frac{2mv}{t} \tag{12}$$

Using equation 12, the overall force due to the two vehicles colliding was calculated to be 93,800 N. Now it was important to subtract the total braking force for an AWD car. This was done using figure 4, along with the friction coefficient of pavement during near freezing conditions. As mentioned in the introduction, the crash being modelled occurred on a snowy night at near freezing temperatures. From a chart located in [8], rubber (the tires) in contact with wet snow have a friction coefficient between 0.30 and 0.60. For the sake of this experiment, and because the snow had been on the road for a substantial amount of time at near freezing temperatures, a friction coefficient on the lower side of the range was used. This value was taken to be 0.4 and was then used in figure 4, which shows the relationship between braking force and friction coefficient [9].

Fig 4: Relationship between road friction coefficient and front and rear braking force [9]

In the figure, the x-axis is the front braking force, and the y-axis is the rear braking force. By summing these two forces, the total braking force could be determined. The point of intersection along the line with a friction coefficient of 0.4 is marked with a red circle within the figure. The total braking force was determined to be 8,500 N, and when subtracted from the total impact force of 93,800 N, the force $F_a(t)$ was found to be 85,300 N.

3.4 Modeling the System in Matlab Simulink

Matlab was the primary program used for the coding and plotting portion of this collision analysis. The Matlab sub-program, Simulink, was used in conjunction for the modelling portion of the analysis. The Simulink model for this system is shown below in figure 5.

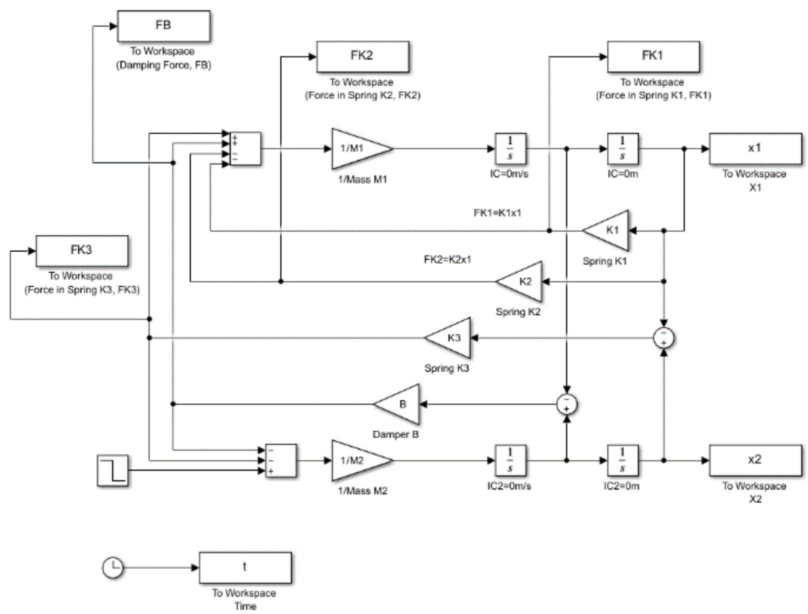

Fig 5: Simulink model created for Matlab representation of the system.

This model makes it much easier to create plots of individual forces over a specified time period because the plot commands are specified within the Matlab code, and the code can grab values from the Simulink model for each increment of time that the code is running. The Simulink model represents the same system as shown in figure 1, just displayed and connected in a way that the program can more efficiently interpret.

4. Simulation Results and Discussion

After the code was finalized in Matlab, and the Simulink model was created and properly linked to this code, the program was executed and produced the following diagrams and results.

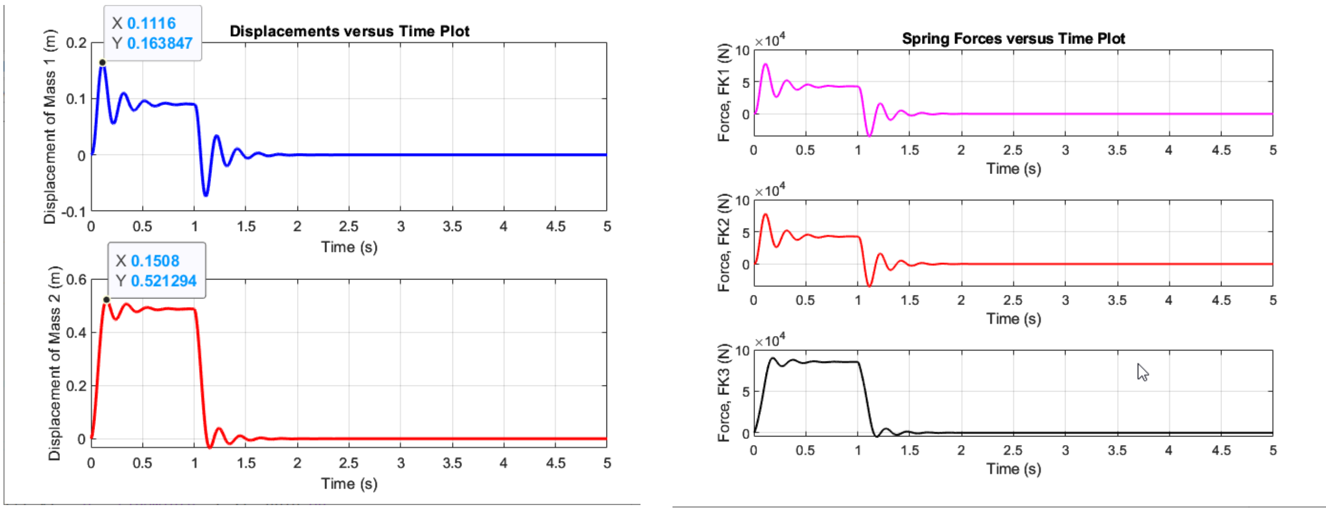

Fig 6: Plots of mass displacements and spring forces during the simulation.

Comparing the displacements of M_1 and M_2 shown in figure 6, it can be seen that the maximum displacement of M_2 is much greater than that of M_1. This aligns with what was expected because mass M_2 has the external force $F_a(t)$ acting directly on it, with no springs or dampers in between, unlike mass M_1, which has a spring and damper between it and mass M_2. K_1 and K_2 apply such a large spring force during the simulation that it prevents a large amount of displacement on the engine bay, M_1 as well. The fact that the springs are not directly attached to the front of the car causes the external force to act more aggressively on M_2, contributing to its larger displacement. It would be interesting to conduct a follow up study that remodels the system, so the tires (springs) are shown as connected to the front crash bar to see if it makes a large difference on the displacement and oscillation of the two masses. Adding more components and making the model more complex will also lead to a rework of the simulation, and most likely make it more accurate. One other unique characteristic of the displacement plot is that the oscillation of mass M_1 is larger than that of mass M_2. This is also attributed to the springs K_1 and K_2. Their large spring forces resist initial displacement, but once moved, tend to oscillate for a longer period due to the small magnitude of the damping force within the system.

The plot of the three spring forces, K_1, K_2, and K_3 in figure 6, shows that they also have some oscillation, very similar to that of their associated displacements. The oscillation for springs K_1 and K_2 is larger than that of K_3, due to the fact that the spring forces of springs K_1 and K_2 are both much larger than spring K_3, so they absorb more of the energy from the crash. This means that they will oscillate more aggressively after the force is removed in relation to K_3 that only oscillates slightly after the force from the crash is removed from the system. Another interesting part of the spring plots in figure 6 is that both K_1 and K_2 apply negative forces for a short time during the simulation. This was caused by the force of the crash being so large that when the springs rebounded after the initial oscillation, they were stretched by the oscillation of the masses, and exerted a negative force on the masses in order to return to their original "zero" state.

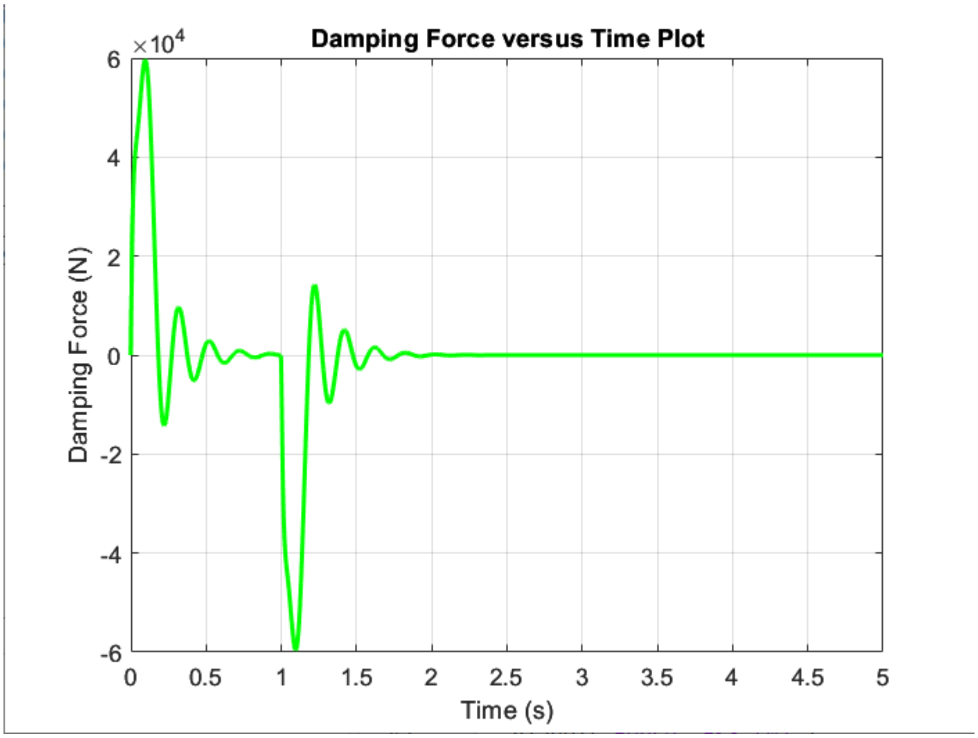

Fig 7: Damping force versus time graph for the duration of the simulation

The final plot that was generated from this collision simulation is shown in figure 7 and displays the damping force versus time for the duration of the simulation. This plot behaves the typical way that a damper would, beginning with a large amount of oscillation, but quickly steadying out to a damping force of zero newtons. The damper also copies the behavior of springs K_1 and K_2 by oscillating a large amount as the force is applied, then switching and oscillating at a much lower value after the force is removed. This behavior matches what was expected because it proves that the damper is fulfilling its purpose in helping reduce the oscillation within the springs. It is also important to note that the maximum damping force on the system matches what was expected because it falls slightly below that of the three springs but is large enough that it is able to efficiently stop the motion of the springs, as well as the displacement of the masses.

5. Conclusion

This research illustrates a unique type of crash test, one considering a vehicle and its components at near freezing temperatures, as well as roads with reduced friction conditions. With this combination of variables, this simulation becomes a unique starting point for anyone looking to analyze a crash outside of typical parameters. By introducing new variables and different types of models into the collision dynamic system, more accurate, in depth, or situation specific results can be obtained using the already written code and model within Matlab and Simulink. Future work would include modifying the dynamic analysis model, and in turn, the associated equations to obtain more accurate results.

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