Proceedings of the 11th International Conference of Control Systems, and Robotics (CDSR 2024)
Chestnut Conference Centre - University of Toronto, Toronto, Canada – 10 -12 June, 2024
Paper No. 124
DOI: 10.11159/cdsr24.124

Trajectory Tracking of Gantry Crane Differential Flatness Control

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Abstract **-** In this article, it is presented a trajectory tracking of gantry crane using a differential flatness control scheme. In order to the simply file the problem only a single pendulum gantry crane is considered, taking differential flatness approach allows found a flat out put and its derivatives using to control the positioning of the trolley while eliminating the swing angle of the load so that it can be minimal when a smooth trajectory is applied, ensuring stability and robustness of the closed loop system.

Keywords**:** Gantry; Underactuated Systems, Differential flatness.

1. Introduction

Currently, overhead cranes are one of the most used systems in the industry worldwide as a means of heavy transport operating in 2D or 3D work areas [1], also their control problem focuses on the balancing of the load when it is transported from one point to another, as is the case of the gantry crane where its dynamics are non-linear can be interpreted and delimited as a model It can be analysed as an under-actuated system because there is no magnitude of force that can be controlled in the support mechanism [2]. On the other hand, most stabilizing control schemes are based on equilibrium point linearization, which may require robust linear schemes for the approximate system stabilization problem [3], [4].

The problem of gantry crane control in simple pendulum mode can be approached from the perspective of differential flatness control [5], [6], [7], where the problem formulation focuses on the control of a simplified system, where the disturbance signal is a function containing unmodeled dynamics and/or perturbation functions [8].

2. Dynamic Model of the Gantry Crane

2.1. The nonlinear model

Fig 1 describes the block diagram of a gantry crane system [9]. Considering that the moving direction x_b of the bridge along the horizontal axis (x), l is the length of the hoisting, g is the gravitational acceleration, and θ is the angular displacement to payload, as shown in Fig. 1. The mass of the trolley and payload are denoted ass m_b and m respectively, and u is the trolley drive force. Its dynamic model can be expressed as denoted by the equation (1).

$$(m_b + m)\ddot{x}_b + lmcos\theta\ddot{\theta} - mlsin\theta\dot{\theta} = u$$
$$lmcos\theta\ddot{x}_b + l^2m\ddot{\theta} + mglsin\theta = 0 \quad (1)$$

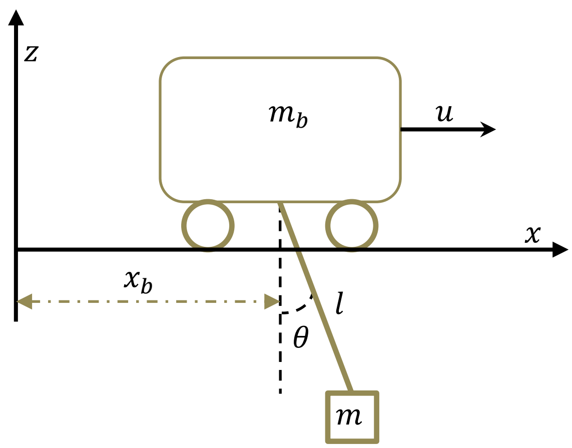

Fig. 1: Gantry Crane Model.

2.2. Differential flatness of the gantry crane

The tangent linearization of the system (1) around a desired stable, the equilibrium point, that is for $x_b = 0, \theta = 0$, l is constant, using the following approximations, $\dot{x}_b = 0$, $\dot{\theta} \approx 0$, $cos\theta \approx 1$, $sin\theta \approx \theta$, allow to obtain the state space representation,

$$\dot{x} = Ax + Bu \quad (2)$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{gm}{m_b} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{g(m_b+m)}{lm_b} & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ \frac{1}{m_b} \\ 0 \\ -\frac{1}{lm_b} \end{bmatrix}, x = [x_b \quad \dot{x}_b \quad \theta \quad \dot{\theta}]^T$$

In order to define the flat output, we first take the system controllability matrix of the linearized system by (2) which is given by $C_k = [B \quad AB \quad A^2B \quad A^3B]$. The system under consideration is controllable with det $C_k \neq 0$, hence it is flat, [8], [10]. The flat output can be obtained as follows:

$$F = [0 \quad 0 \quad 0 \quad 1]C_k^{-1}x$$
$$F = \frac{lm_b}{g}x_b + \frac{l^2m_b}{g}\theta \quad (3)$$

Considering that for linear systems [10], [11], the flat output (3) can be given in terms of a constant factor $F_\gamma = \gamma F$, where $\gamma = \frac{g}{lm_c}$ is a constant, thus obtaining the equation (4),

$$F_\gamma = x_b + l\theta \quad (4)$$

On the other hand, the parameter γ is selected in order to simplify the calculation of the flat output time derivatives, where

$$F_\gamma = \gamma[0 \quad 0 \quad 0 \quad 1]C_k^{-1}x \tag{5}$$

Let us define by (5) the next vector as $C_\gamma = \gamma[0 \quad 0 \quad 0 \quad 1]C_k^{-1}$ to new flat output F_γ and a finite number of its time its time derivatives can be obtained using the observability matrix $O = \begin{bmatrix} C_\gamma & C_\gamma A & C_\gamma A^2 & C_\gamma A^3 \end{bmatrix}$

$$\begin{bmatrix} F_\gamma \\ \dot{F}_\gamma \\ \ddot{F}_\gamma \\ F_\gamma^{(3)} \end{bmatrix} = \begin{bmatrix} C_\gamma \\ C_\gamma A \\ C_\gamma A^2 \\ C_\gamma A^3 \end{bmatrix} x \tag{6}$$

By computing (6), the flat output and its derivates are functions of the state variables of the system as:

$$F_\gamma = x_b + l\theta \tag{7}$$

$$\dot{F}_\gamma = \dot{x}_b + l\dot{\theta} \tag{8}$$

$$\ddot{F}_\gamma = -g\theta \tag{9}$$

$$F_\gamma^{(3)} = -g\dot{\theta} \tag{10}$$

The relative degree of the system (2) is $n = 4$. The flat output fourth order time derivative is obtained as follows:

$$F_\gamma^n = C_\gamma A^{n-1}Bu + C_\gamma A^n x$$

$$F_\gamma^{(4)} = \frac{g}{lm_b}u - \frac{g^2}{lm_b}(m_b + m)\theta \tag{11}$$

$$F_\gamma^{(4)} = \frac{g}{lm_b}u + \varphi \tag{12}$$

Where φ denotes a generalized disturbance function that involves some non-modeled dynamics, neglected linearization terms and possible external disturbance inputs these terms are known and ignored to this study case.

3. ADRC Control Design

The control input can be synthesized by including an active disturbance cancelling strategy of the total disturbance, φ which is estimated by $\hat{\varphi}$. The output feedback control is given as follows:

$$u = \frac{u_0 - \hat{\varphi}}{b} \tag{13}$$

The auxiliary control u_0 is computed as follows:

$$u_0 = \omega_c^4(F - F^*) + 4\omega_c^3(\dot{F} - \dot{F}^*) + 6\omega_c^2(\ddot{F} - \ddot{F}^*) + 4\omega_c(\dddot{F} - \dddot{F}^*) \tag{14}$$

Where F^* is the desired trajectory and $b = \frac{g}{lm_b}$ is the control gain. Let us propose the closed-loop poles through a stable Hurwitz polynomial $P_c = s^4 + 4\omega_c s^3 + 4\omega_c s^3 + 6\omega_c^2 s^2 + 4\omega_c^3 s + \omega_c^4$. Notice that the controller gains should be chosen using only one parameter, the controller bandwidth, ω_c.

4. Numerical Simulation

The control strategy was implemented in the MATLAB-Simulink Software using a sampling time of 0.001 [s] a fourth order Runge-Kutta algorithm with the following crane parameters $m_b = 2.2620\, Kg$, $m = 1\, Kg$, $l = 0.8\, m$, $g = 9.81\, m/s^2$. The control design parameter specified to be $\omega_c = 20$. The test starts with initial conditions of the crane at point $[x_b = 0, \theta = 0]$ when the flat output satisfies:$F_\gamma = 0$. Figure 2 shows the flat output $F_\gamma(t)$ performance using the scheme Differential flatness and this allows us to indirectly and simultaneously control the state variables $(x_b(t), \theta(t))$ of the under-actuated system, while the cart position $x(t)$ carries out a smooth rest to rest trajectory as shown in Figure 3, the angular position $\theta(t)$ remains near the equilibrium point as shown in Figure 4.

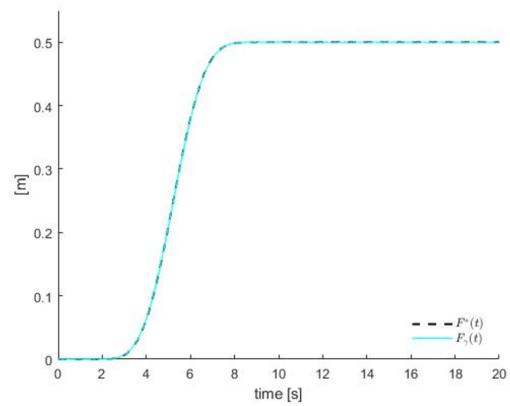

Fig. 2: Flat output closed loop trajectory tracking

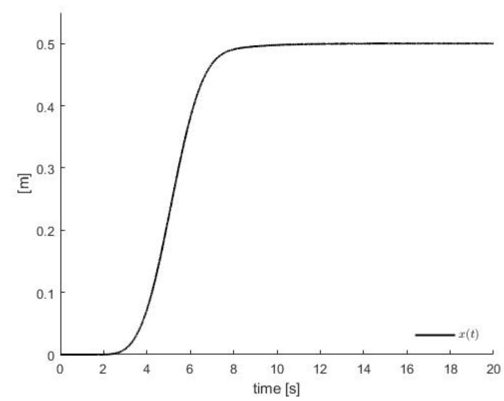

Fig. 3: Trolley position.

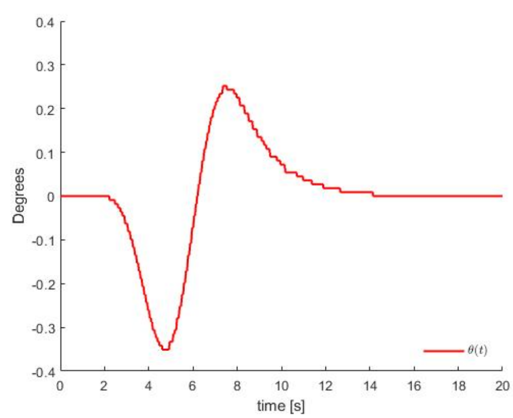

Fig. 4: Sway angle of Payload.

5. Conclusions

Some underactuated systems whose nonlinear dynamics may not be controllable can be locally linear in its equilibrium (approximated linearization) to be stabilized and, moreover, forced to track a reference trajectory through a flatness-based, where the combination of the flatness allows a trivialization of the trajectory tracking problem. The Gantry crane has been analyzed, finding that its differential flatness and its derivatives allowed it possible to control the positioning of the carriage and at the same time eliminate the angle of rotation.

Acknowledgements

This work was supported by the Universidad Iberoamericana Ciudad de México, Prolongación Paseo de la Reforma 880, Colonia Lomas de Santa Fe, Álvaro Obregón, Ciudad de México 01219, and by Secretaría de Investigación y Posgrado – Instituto Politécnico Nacional under grant 20240693.

References

[1] M. R. Mojallizadeh, B. Brogliato y C. Prieur, «Modeling and control of overhead cranes: A tutorial overview and perspectives,» *Annual Reviews in Control,* vol. 56, p. 100877, 2023.

[2] N. M. Alyazidi, A. M. Hassanine y M. S. Mahmoud, «An Online Adaptive Policy Iteration-Based Reinforcement Learning for a Class of a Nonlinear 3D Overhead Crane,» *Applied Mathematics and Computation,* vol. 447, p. 127810, 2023.

[3] Y. Cao y T. Li, «Review of antiswing control of shipboard cranes,» *IEEE/CAA Journal of Automatica Sinica,* vol. 7, pp. 346-354, 2020.

[4] L. Ramli, Z. Mohamed, A. M. Abdullahi, H. I. Jaafar y I. M. Lazim, «Control strategies for crane systems: A comprehensive review,» *Mechanical Systems and Signal Processing,* vol. 95, pp. 1-23, 2017.

[5] H. Zhang, S. Zhao y Z. Gao, «An active disturbance rejection control solution for the two-mass-spring benchmark problem,» de *2016 American Control Conference (ACC)*, 2016.

[6] J. Han, «From PID to Active Disturbance Rejection Control,» *IEEE Transactions on Industrial Electronics,* vol. 56, pp. 900-906, 2009.

[7] S. Zhao y Z. Gao, «An active disturbance rejection based approach to vibration suppression in two-inertia systems,» *Asian Journal of control,* vol. 15, p. 350–362, 2013.

[8] H. Sira-Ramírez, A. Luviano-Juárez, M. Ramírez-Neria y E. W. Zurita-Bustamante, Active disturbance rejection control of dynamic systems: a flatness based approach, Butterworth-Heinemann, 2017.

[9] K.-S. Hong y U. H. Shah, Dynamics and control of industrial cranes, Springer, 2019.

[10] H. Sira-Ramirez y S. K. Agrawal, Differentially flat systems, Crc Press, 2018.

[11] M. Ramírez-Neria, H. Sira-Ramírez, R. Garrido-Moctezuma y A. Luviano-Juárez, «On the linear control of underactuated nonlinear systems via tangent flatness and active disturbance rejection control: The case of the ball and beam system,» *Journal of Dynamic Systems, Measurement, and Control,* vol. 138, p. 104501, 2016.

[12] Keum-Shik Hong, Umer Hameed Shah, Dynamics and Control of Industrial Cranes, Springer, 2019.