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Data-Driven Adaptive Control, For Unknown Non-Affine Nonlinear Systems with Varying Control Direction.

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Abstract **-** Non-affine systems with variant control directions are often hard to control when no information on the system is available. Hence, this article proposes a data-based adaptive control for this type of system that does not require any information on the system's mathematical model. This article uses a model estimator and a model-free adaptive controller based on Multi-input Fuzzy Rules Emulated Network. The estimator helps to obtain an approximation of the unknown and varying control direction. The estimated control direction helps the adaptive controller to have a fast response when the system's control direction changes, with no previous information on the system. We provide closed-loop stability proof according to a Uniformly Ultimately Bounded function of Lyapunov. As validation, we provide experimental results for a switching gain circuit, where the system's control direction undergoes an abrupt change. The controller could obtain a mean absolute percentage tracking error of 2.59% throughout the experiment. This proves the proposed controller's performance for systems with varying control directions.

Keywords**:** Adaptive control; Non-affine systems; Data-based control; Fuzzy modelling; Variant control direction.

1. Introduction

Unlike affine systems, the output of non-affine systems has a nonlinear relationship regarding the control effort[1]. Wastewater treatment applications, permanent magnet linear motors, turbine systems, chemical reactions, and flight control systems are some examples of non-affine systems[2, 3, 4]. The nonlinear relationship of non-affine systems' output regarding their input makes it very difficult to obtain the mathematical model of the system. Hence, the most common approach to control this type of system is adaptive controllers. However, most of these approaches assume either a known or positive sign of the relationship between the system's output and its input[5, 6]. The sign of this relationship is often called the control direction of a system.

The first adaptive control for systems with unknown control direction was proposed by Roger D. Nussbaum[7] in 1983. He proposed an adaptive gain to reflect the unknown control direction of the system. This gain has a slow adaptation and reflects the correct control direction of a system. We find countless applications of the Nussbaum gain such as nonlinear systems[8, 9], industrial applications [10], and systems with time-varying control gain without changing the sign [11]. To our knowledge, very few papers address the problem of systems whose control direction has sign changes[12]. In these articles, the control approach is based on fuzzy observers and controllers with the help of the Nussbaum gain. Their problem formulation is for nonlinear affine systems with simulation results and stability analysis.

The following article proposes a data-driven adaptive controller for non-affine systems with varying control directions when the system mathematical model and its control direction are completely unknown and are time-variant. The controller is based on Multiple-input Fuzzy Rules Emulated Network (MiFREN), and relies on a system estimator Affine Equivalent Model[13] (AEM) to enhance the controller adaptation to the system's varying control direction. We present a closed-loop stability proof with the conclusion of a bounded tracking error according to Lyapunov. We also present experimental results with an electronic system of switching control direction, where the controller maintained a mean absolute percentage tracking error of 2.59% throughout the experiments.

The next section describes the class of systems that will be addressed in this article. The third section describes the proposed control law, alongside the stability proofs and conditions for stability. The fourth section describes the experimental setup and presents the validation results. Finally, the last chapter summarizes this article's results and future works.

2. Problem statement

For a non-affine, nonlinear, discrete-time, single input, single output system described as

$$\mathrm{y}(\mathrm{k}+1)=\mathrm{F}\big(\{\mathrm{y}(\mathrm{k}),\cdots,\mathrm{y}(\mathrm{k}-\mathrm{n_y}),\mathrm{u}(\mathrm{k}),\cdots,\mathrm{u}(\mathrm{k}-\mathrm{n_u})\}\big), \tag{1}$$

where $y(k+1)$ is the system's output, $u(k)$ the control law, and $F(\{\})$ is a non-linear function dependent on n_y and n_u, the unknown orders of the system output and control law, respectively. When non-affine systems, as (1), are Lipschitz continuous regarding the control input, have an affine representation as

$$\begin{gathered} y(k+1)=f(\nu\{k\})+g(\nu\{k\})u(k)+\varepsilon_h(\nu\{k\}), \\ \nu\{k\}=\{y(k),\cdots,y(k-n_y),u(k-1),\cdots,u(k-n_u)\}, \end{gathered} \tag{2}$$

where $f(\nu\{k\})$ and $g(\nu\{k\})$ are unknown nonlinear functions, and $\varepsilon_h(\nu\{k\})$ is the bounded residual error. This affine representation is estimated with AEM, proposed in [13] as

$$\hat{y}(k+1)=\hat{f}(k)+\hat{g}(k)u(k). \tag{3}$$

The control direction of a system represents the *motion* direction of the system under any control law. This direction can be represented by the sign of the system's output, regarding the control input[14]. On the non-affine system (1), the control direction is represented as $\operatorname{sign}\{\partial y(k+1)/\partial u(k)\}$. From (2)-(3), this article proposes to use the estimation of the control direction in (1) as

$$\operatorname{sign}\left\{\frac{\partial y(k+1)}{\partial u(k)}\right\}\approx\operatorname{sign}\left\{\frac{\partial\hat{y}(k+1)}{\partial u(k)}\right\}=\operatorname{sign}\{\hat{g}(k)\} \tag{4}$$

The proposed approximation will be used in the next section to improve the controller's adaptation to unknown and varying control directions.

3. Control law

In this section, we propose a model-free adaptive controller with MiFREN using AEM as an estimator of the system control direction. This modification aims to enhance the controller's adaptation to the known and varying control direction of the system. We propose a MiFREN model-free data-based adaptive controller as

$$u(k)=\varphi^T(k)\beta(k), \tag{5}$$

where $u(k)$ is the control law, $\varphi^T(k)$ the multidimensional membership-function vector, and $\beta(k)$ the weight vector. The weight vector's actualization, according to the gradient descent method, is calculated as

$$\beta(k+1)=\beta(k)-\eta\left[\frac{\partial E(k+1)}{\partial\beta(k)}\right], \tag{6}$$

where η is the controllers learning rate, and $E(k+1)$ the cost function. The cost function is established as

$$E(k+1)=\frac{1}{2}e^2(k+1), \tag{7}$$

where $e(k+1)$ is the system tracking error defined as

$$e(k+1)=r(k+1)-y(k+1), \tag{8}$$

with $r(k+1)$ as the desired trajectory. The partial derivative needed in (6) is obtained through the gradient descent method

$$\frac{\partial E(k+1)}{\partial\beta(k)}=\left[\frac{\partial E(k+1)}{\partial e(k+1)}\right]\left[\frac{\partial e(k+1)}{\partial y(k+1)}\right]\left[\frac{\partial y(k+1)}{\partial u(k)}\right]\left[\frac{\partial u(k)}{\partial\beta(k)}\right],$$

$$\frac{\partial E(k+1)}{\partial \beta(k)} = [e(k+1)][-1]\left[\frac{\partial y(k+1)}{\partial u(k)}\right][\varphi^T(k)], \tag{9}$$

The term $\frac{\partial E(k+1)}{\partial \beta(k)}$, according to AEM[13] with the assumptions of section 2, is estimated as

$$\frac{\partial y(k+1)}{\partial u(k)} \approx \frac{\partial\big(f(k) + g(k)u(k) + \varepsilon_h(k)\big)}{\partial u(k)} \approx \hat{g}(k), \tag{10}$$

hence, the weight vector is actualized as

$$\beta(k+1) = \beta(k) + \eta e(k+1)\hat{g}(k)\varphi(k). \tag{11}$$

Theorem 1. *A class of non-affine, discrete-time, unknown systems described in (1), with an affine representation (2), and estimated as (3) according to AEM, are controllable with the law (5), and its weight vector adapted as (11). The tracking error and weight vector are bounded according to Lyapunov UUB when the controller parameter meets the following condition:*

$$0 < \eta < \frac{1}{\hat{g}_{max}^2 \varphi^2 max} \leq \frac{1}{\hat{g}^2(k)\varphi^2(k)} - \delta_g.$$

Proof: To analyse the closed-loop tracking error convergence, a Lyapunov semi-definite positive function is proposed as

$$L(k+1) = \kappa_e e^2(k+1) + \kappa_\beta \tilde{\beta}^T(k+1)\tilde{\beta}(k+1), \tag{12}$$

where $\kappa_e$and κ_β are definite positive constants. The differentiation of (12) is obtained as

$$\Delta L(k+1) = \Delta L_e(k+1) + \Delta L_\beta(k+1), \tag{13}$$

where

$$\Delta L_e(k+1) \triangleq \kappa_e e^2(k+1) - \kappa_e e^2(k), \tag{14}$$

and

$$\Delta L_\beta(k+1) \triangleq \kappa_\beta \tilde{\beta}^T(k+1)\tilde{\beta}(k+1) - \kappa_\beta \tilde{\beta}^T(k)\tilde{\beta}(k). \tag{15}$$

If we begin the analysis of (14), we first require the closed-loop tracking error. To obtain the closed-loop tracking error, we use the affine representation (2) of the non-affine system (1), and substitute it into the tracking error (8) as

$$e(k+1) = r(k+1) - \big(f(k) + g(k)u(k) + \varepsilon_h(k)\big). \tag{16}$$

Assuming the existence of an ideal weight vector β^*, the vector actualization error is defined as

$$\varphi(k)\tilde{\beta}(k) = \varphi^T(k)\beta^* - \varphi^T(k)\beta(k), \tag{17}$$

hence, the difference between the ideal control law and the k^{th} iteration is

$$\tilde{u}(k) = \varphi(k)\tilde{\beta}(k) = u^*(k) - u(k). \tag{18}$$

In a similar sense, an ideal controller is expected to produce a zero error in the system as $e^*(k+1) = 0$, hence

$$r(k+1) = f(k) + g(k)u^*(k). \tag{19}$$

Therefore the closed-loop tracking error is rewritten as

$$e(k+1) = g(k)\tilde{u}(k) - \varepsilon_h(k). \tag{20}$$

Substituting (20) in (14)

$$\Delta L_e(k+1) = \kappa_e\big(g(k)\tilde{u}(k) - \varepsilon_h(k)\big)^2 - \kappa_e e^2(k), \tag{21}$$

where, considering that $(a-b)^2 < 2a^2 + 2b^2$ and (18), (21) is rewritten as

$$\Delta L_e(k+1) < 2\kappa_e\left(g(k)\tilde{\beta}^T(k)\varphi(k)\right)^2 + 2\kappa_e\varepsilon_h^2(k) - \kappa_e e^2(k). \tag{22}$$

On the other hand, the term (15) depends of the weight vector error $\tilde{\beta}(k+1) = \beta^* - \beta(k+1)$ with the substitution of (11) becomes

$$\tilde{\beta}(k+1) = \tilde{\beta}(k) - \eta e(k+1)\hat{g}(k)\varphi(k). \tag{23}$$

With (23), (15) becomes

$$\Delta L_\beta(k+1) = \kappa_\beta\left(\tilde{\beta}(k) - \eta e(k+1)\hat{g}(k)\varphi(k)\right)^2 - \kappa_\beta\tilde{\beta}^T(k)\tilde{\beta}(k), \tag{24}$$

is rearranged as

$$\Delta L_\beta(k+1) = -\kappa_\beta\eta\left(\left(g(k)\tilde{\beta}^T(k)\varphi(k)\right) - \varepsilon_h(k)\right)^2\left(2\frac{\hat{g}(k)}{g(k)} - \eta\hat{g}^2(k)\varphi^T(k)\varphi(k)\right) + 2\kappa_\beta\eta\frac{\hat{g}(k)}{g(k)}\varepsilon_h^2(k). \tag{25}$$

Considering that $(a-b)^2 < 2a^2 + 2b^2$, (25) is rearranged as

$$\begin{aligned}\Delta L_\beta(k+1) < {} & -2\kappa_\beta\eta\left(g(k)\tilde{\beta}^T(k)\varphi(k)\right)^2\left(2\frac{\hat{g}(k)}{g(k)} - \eta\hat{g}^2(k)\varphi^T(k)\varphi(k)\right) \\ & + 2\kappa_\beta\eta\left(4\frac{\hat{g}(k)}{g(k)} - \eta\hat{g}^2(k)\varphi^T(k)\varphi(k)\right)\varepsilon_h^2(k).\end{aligned} \tag{26}$$

According to terms (22) and (26), the differentiation of the proposed Lyapunov function (13) is rewritten as

$$\begin{aligned}\Delta L(k+1) < {} & -2\left(g(k)\tilde{\beta}^T(k)\varphi(k)\right)^2\left(2\kappa_\beta\frac{\hat{g}(k)}{g(k)}\eta - \kappa_\beta\eta^2\hat{g}^2(k)\varphi^T(k)\varphi(k) - \kappa_e\right) \\ & + 2\kappa_\beta\eta\left(4\frac{\hat{g}(k)}{g(k)} - \eta\hat{g}^2(k)\varphi^T(k)\varphi(k)\right)\varepsilon_h^2(k) - \kappa_e e^2(k) + 2\kappa_e\varepsilon_h^2(k).\end{aligned} \tag{27}$$

The term $2\kappa_\beta\frac{\hat{g}(k)}{g(k)}\eta - \kappa_\beta\eta^2\hat{g}^2(k)\varphi^T(k)\varphi(k) - \kappa_e$ needs to be definite positive, hence separating some terms in the inequality

$$2\kappa_\beta\frac{\hat{g}(k)}{g(k)}\eta - \kappa_\beta\eta^2\hat{g}^2(k)\varphi^T(k)\varphi(k) - \kappa_e > 0, \tag{28}$$

we obtain

$$\frac{1}{g^2(k)\varphi^2(k)} - \frac{\kappa_e}{\kappa_\beta} > \left(\eta\hat{g}(k)|\varphi(k)| - \frac{1}{g(k)|\varphi(k)|}\right)^2. \tag{29}$$

If we set $\frac{1}{g^2(k)\varphi^2(k)} - \frac{\kappa_e}{\kappa_\beta} = \delta^2$ where $0 < \delta \ll 1$ it is observed that

$$\delta > \eta\hat{g}(k)|\varphi(k)| - \frac{1}{g(k)|\varphi(k)|}, \tag{30}$$

and the learning rate boundaries for stability purposes are set as

$$0 < \eta < \frac{1}{\hat{g}_{max}^2 \varphi_{max}^2} \leq \frac{1}{\hat{g}^2(k)\varphi^2(k)} - \frac{\delta}{\hat{g}(k)\varphi^2(k)}, \tag{31}$$

where $|\hat{g}(k)| \leq \hat{g}_{max}$ y $|\varphi(k)| \leq |\varphi_{max}|$, therefore the inequality in (28) is met.

With those assumptions, we analyse the differentiation of the Lyapunov function in (27) as a definite negative function according to the tracking error boundary

$$-\kappa_e e^2(k) + 2\kappa_\beta \eta \left(4 \frac{\hat{g}(k)}{g(k)} - \eta \hat{g}^2(k) \varphi^T(k) \varphi(k) \right) \varepsilon_h^2(k) + 2\kappa_e \varepsilon_h^2(k) < 0, \tag{32}$$

with

$$e^2(k) > \Omega_e, \tag{33}$$

where $\Omega_e \triangleq \frac{2\kappa_\beta \eta \left(4\frac{\hat{g}(k)}{g(k)} - \eta \hat{g}^2(k)\varphi^T(k)\varphi(k) \right) \varepsilon_h^2(k) + 2\kappa_e \varepsilon_h^2(k)}{\kappa_e}$.

If we analyse the differentiation of the Lyapunov function in (27) as definite negative according to the weight vector error

$$\begin{aligned} 2\kappa_e \varepsilon_h^2(k) - 2\left(g(k)\tilde{\beta}^T(k)\varphi(k)\right)^2 \left(2\kappa_\beta \frac{\hat{g}(k)}{g(k)}\eta - \kappa_\beta \eta^2 \hat{g}^2(\mathrm{k})\varphi^T(k)\varphi(k) - \kappa_e\right) \\ + 2\kappa_\beta \eta \left(4 \frac{\hat{g}(k)}{g(k)} - \eta \hat{g}^2(k)\varphi^T(k)\varphi(k) \right) \varepsilon_h^2(k) < 0, \end{aligned} \tag{34}$$

with

$$\left|\tilde{\beta}(k)\right|^2 > \Omega_\beta, \tag{35}$$

as $\Omega_\beta \triangleq \frac{\kappa_\beta \eta \left(4\frac{\hat{g}(k)}{g(k)} - \eta \hat{g}^2(k)\varphi^T(k)\varphi(k) \right) \varepsilon_h^2(k) + \kappa_e \varepsilon_h^2(k)}{g^2(k)|\varphi(k)|^2}$.

This concludes the stability proof with the limits (35) and (33), where the estimation error and weight vector of the estimator are defined as *uniformly ultimately bounded* (UUB) according to the proposed Lyapunov function (12). For more information on UUB functions according to Lyapunov, please refer to Lyapunov Extension Theorem 2.5.7, [15]. □

4. Results

Experimental results are shown as validation of the proposed adaptive controller (5), the experiments are conducted on an electronic amplifier circuit with positive and negative switched gains. The circuit is further explained in figure 1 with the electronics diagram, and a picture of the experiment assembly is in figure 2. This electronic circuit was selected to show the controller adaptation to fast changes in the control direction, along with the estimation of the control direction. From figure 1, the red square shows the amplifier with a positive gain around R_1/R_0 and the blue square shows a negative amplifier with a gain of around -1. The yellow square shows the control direction switch. The $S_i(k)$ signal changes the system output from the positive gain amplifier to the positive gain amplifier with a signal change (thanks to the negative amplifier). All circuit components were selected according to table 1 with a dual power supply ± 12 [V] with the four-channel LM324N operational amplifiers. The switching device is a RAS-1210 relay. It is worth noticing that all components have a tolerance of ± 15 \%.

Table 1: Electrical diagram components value.

Parameter	R_0	R_1	R_2	C_0	L_0
Value	$10\ k\ \Omega$	$5.6\ k\ \Omega$	$15\ k\ \Omega$	$330\ \mu H$	$330\ nF$

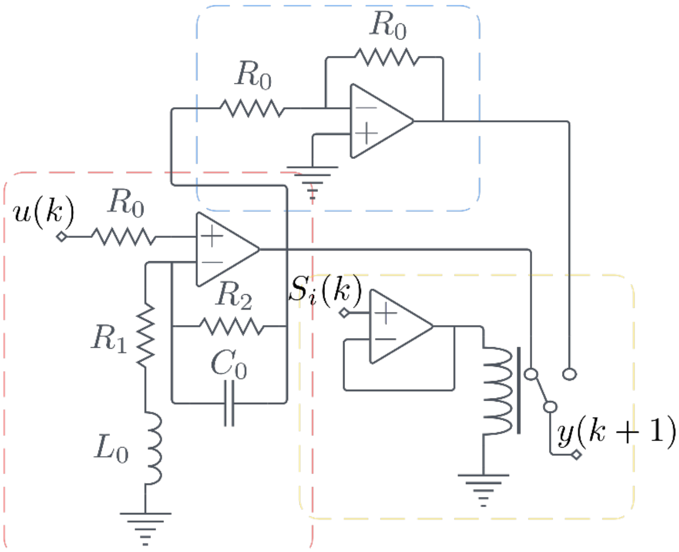

Fig. 1: Electrical diagram for experimentation.

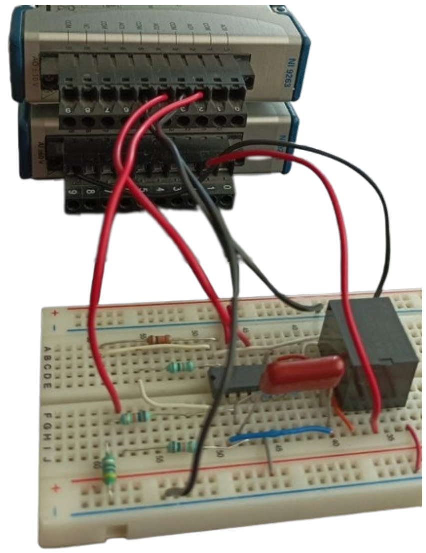

Fig. 2: Experiment assembly.

The switching signal $S_i(k)$ and control input $V_i(k)$ are generated via ports AO_1 and AO_2 of the NI-9263 analog output device. System output $V_o(k)$ is detected by the data input channel AI_1 of the NI-9221. The algorithm was designed in MATLAB 2021b, which generates communication with NI-9263 and NI-9221 devices. The computer running the algorithm has a 7 AMD Ryzen processor with 8 GB RAM. The average sampling time for the experiments is about 41 milliseconds.

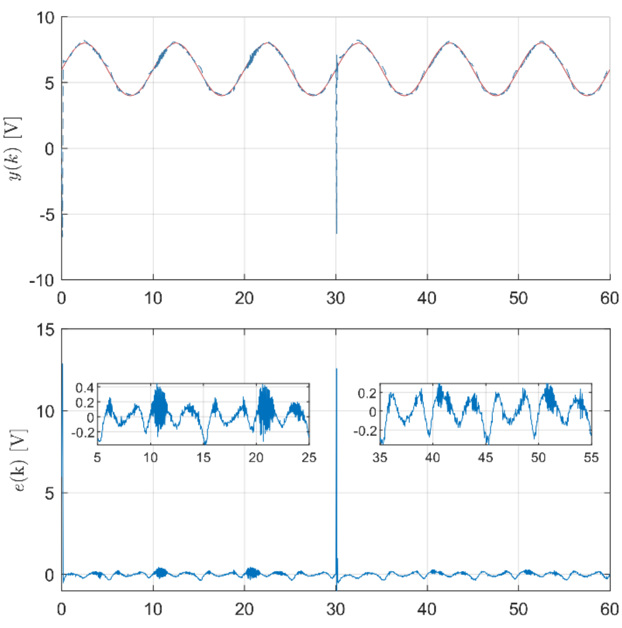

Fig. 3: System response, —$r(k)$, - -$y(k)$, —$e(k)$.

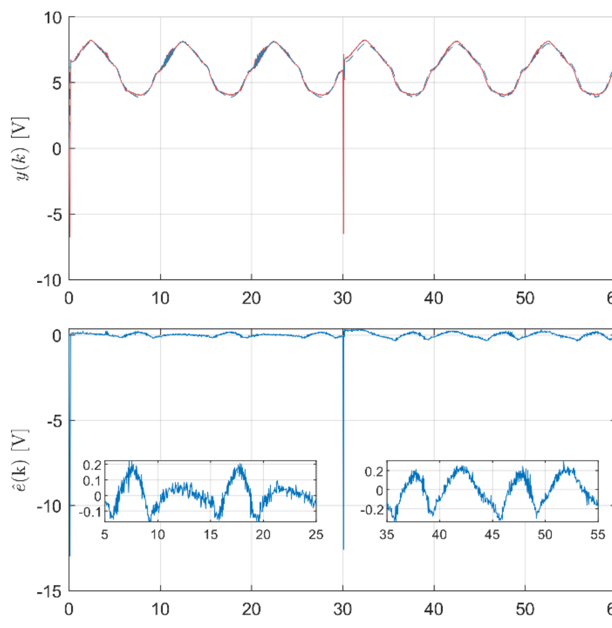

Figure 4: Estimator response, —$y(k)$, - -$\hat{y}(k)$, —$\hat{e}(k)$.

The desired trajectory was chosen as $r(k) = 2\sin(2\pi t(k)/10) + 6$, and the control direction switching signal as $S_i(k) = \mathrm{sqrt}(2\pi t(k)/60)$. The AEM estimation neural network has three inputs: the output of the system $y(k)$, the control input of the system $u(k-1)$, and the estimation of the parameter associated with the control address $\hat{g}(k-1)$. Each entry has three membership functions, creating the multidimensional membership vector with twenty-seven neurons. The inputs for the controller's neural network are the tracking error $e(k)$, the desired path $r(k+1)$, and the estimation of the parameter associated with the control address $\hat{g}(k-1)$. The first two entries ($e(k)$ and $r(k+1)$) have seven membership functions each, and the third entry $\hat{g}(k-1)$ has three membership functions, creating a multidimensional membership vector with one hundred and forty-seven neurons.

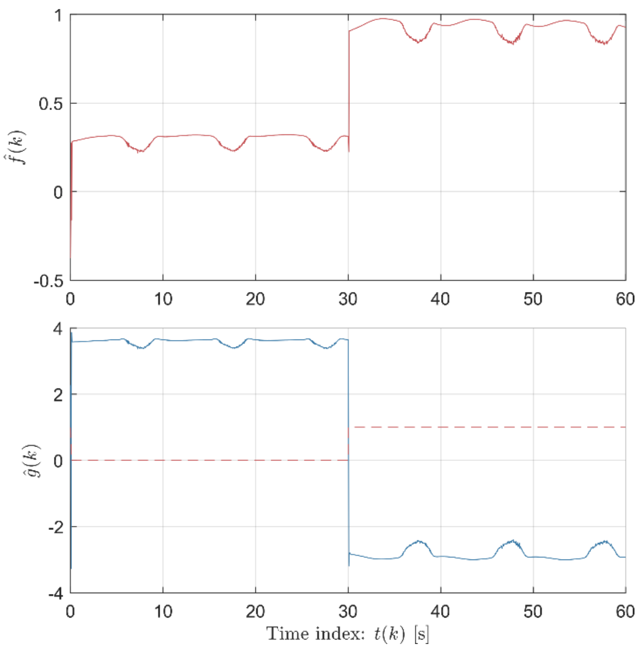

Fig. 5: Functions estimation (control direction), —$\hat{f}(k)$, —$\hat{g}(k)$, - -$\widehat{S}_\iota(k)$.

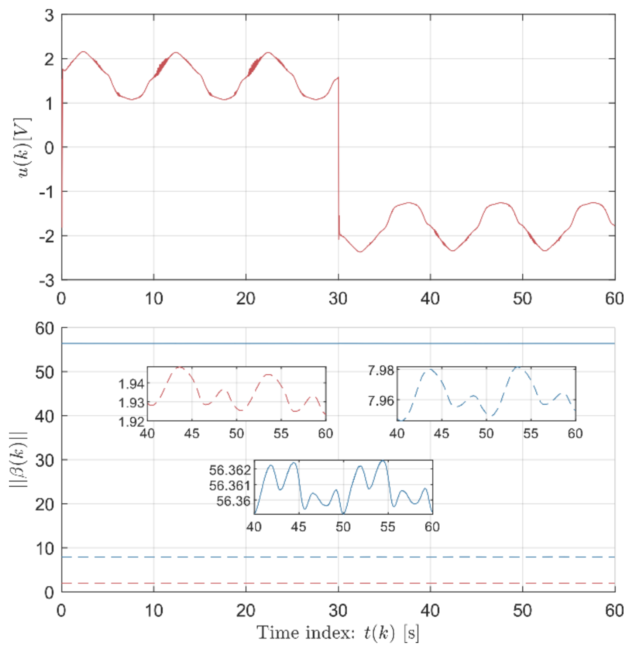

Figure 6: Control law and weight vectors norm, —$u(k)$, —$\|\beta_c(k)\|$, - -$\|\beta_f(k)\|$, $- -$ $\|\beta_g(k)\|$.

The experimental results are shown in figures 3-6. The tracking error is bounded at 0.5 V (Fig. 3), while the estimation error is bounded at [-0.4,0.3) V (Fig. 4). On the other hand, the mean absolute percentage tracking error ($MAPE = 1/n\sum_{k=1}^{n}|r(k)-y(k)|/|r(k)|$) is 2.59 % for the entire simulation, whereas the mean absolute percentage estimation error ($MAP\hat{E} = 1/n\sum_{k=1}^{n}|y(k)-\hat{y}(k)|/|y(k)|$) is of 3.58%. It is observed that the change of control direction produces a spike in the estimation and tracking error, this is to be expected since the controller is model-free and does not know the control direction of the system and the change times of the control direction. It is also observed that both errors (estimation and tracking) have a prompt recovery after the change of control direction. This adaptation to the change of control direction is also observed in the estimated functions (Fig. 5) where it is observed that the control direction of the system is correctly estimated by AEM (the sign of the function $\hat{g}(k)$). It is also observed the adaptation in the control law (Fig. 6), rapidly changes the signal sent to the system according to its control direction. In figure 6, the adaptation of the estimator and controller weight vectors is also observed.

5. Conclusion

The proposed controller (5), with the estimation of the control direction based on AEM, addresses non-affine discrete-time systems with varying control direction (1) when these systems are Lipschitz continuous regarding the control input. These types of systems have an affine representation (2) whose estimation (3) is used as an approximation of the system control direction. Hence, the data-based controller with no previous information of the mathematical model of the system or its control direction, is capable of a fast adaptation. The closed-loop system tracking error was probed bounded in the theorem 1, according to a uniformly ultimately bounded function of Lyapunov, when the parameter η follows the theorem design.

The experimental results show a mean absolute percentage tracking error of 2.59 % for the entire simulation. The estimation of the control direction, represented by the sign on the estimated function $\hat{g}(k)$, is correct throughout the simulation and reflects fast adaptation when the amplifier gain is switched. The estimator adaptation is also reflected in the controller's fast recovery to the control direction switch.

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References

[1] A. Boulkroune, M. M'Saad, and M. Farza, “Fuzzy approximation-based indirect adaptive controller for multi-input multi-output non-affine systems with unknown control direction," IET Control Theory & Applications, vol. 6, no. 17, pp. 2619-2629, 2012.

[2] D. Wang, M. Zhao, M. Ha, and J. Ren, “Neural optimal tracking control of constrained nonaffine systems with a wastewater treatment application," Neural Networks, vol. 143, pp. 121-132, 2021.

[3] Y. Zhan, S. Zou, X. He, and M. Sun, “Incremental adaptive control of a class of nonlinear nonaffine systems," Complexity, vol. 2022, 2022.

[4] T. Binazadeh and M. A. Rahgoshay, “Robust output tracking of a class of non-affine systems," Systems Science & Control Engineering, vol. 5, no. 1, pp. 426-433, 2017.

[5] Y. Hui, R. Chi, B. Huang, Z. Hou, and S. Jin, “Observer-based sampled-data model-free adaptive control for continuous-time nonlinear nonaffine systems with input rate constraints," IEEE Transactions on Systems, Man, and Cybernetics: Systems, vol. 51, no. 12, pp. 7813-7822, 2020.

[6] L. Tan, Z. Chen, Q. Gao, and J. Liu, “Performance recovery of uncertain nonaffine systems by active disturbance rejection control," Measurement and Control, vol. 57, no. 1, pp. 3-15, 2024.

[7] R. D. Nussbaum, “Some remarks on a conjecture in parameter adaptive control," Systems & control letters, vol. 3, no. 5, pp. 243-246, 1983.

[8] H. Ma, H. Ren, Q. Zhou, R. Lu, and H. Li, “Approximation-based nussbaum gain adaptive control of nonlinear systems with periodic disturbances," IEEE Transactions on Systems, Man, and Cybernetics: Systems, vol. 52, no. 4, pp. 2591-2600, 2021.

[9] H. Yue, A. Xue, and J. Li, “Adaptive fuzzy finite-time output feedback fault-tolerant control for mimo stochastic non-linear systems with non-affine nonlinear faults," Journal of Low Frequency Noise, Vibration and Active Control, p.14613484241238595, 2024.

[10] H.-P. Ren, X. Wang, J.-T. Fan, and O. Kaynak, “Adaptive backstepping control of a pneumatic system with unknown model parameters and control direction," IEEE Access, vol. 7, pp. 64 471-64 482, 2019.

[11] S. Wang, M. Fu, and Y. Wang, “Robust adaptive steering control for unmanned surface vehicle with unknown control direction and input saturation," International Journal of Adaptive Control and Signal Processing, vol. 33, no. 7, pp. 1212-1224, 2019.

[12] M. R. Askari, M. Shahrokhi, and M. K. Talkhoncheh, “Observer-based adaptive fuzzy controller for nonlinear systems with unknown control directions and input saturation," Fuzzy Sets and Systems, vol. 314, pp. 24-45, 2017.

[13] M. Flores-Padilla and C. Treesatayapun, “Affine equivalent model based on data-driven fuzzy rules for a class of discrete-time adaptive controller," IEEE International Conference on Fuzzy Systems, 2021.

[14] Y. Xudong and J. Jingping, “Adaptive nonlinear design without a priori knowledge of control directions," IEEE Transactions on Automatic Control, vol. 43, no. 11, pp. 1617-1621, 1998.

[15] F. Lewis, S. Jagannathan, and A. Yesildirak, Neural network control of robot manipulators and non-linear systems. CRC press, 1998.