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Quanser Self Driving Car Trajectory Tracking by employing a Bicycle's Model

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Abstract **–** This paper uses the bicycle's kinematic model to address the Ackerman (Self-Driving Car) mobile robot control. For this purpose, a previous kinematic model analysis was carried out in which only two wheels were considered. Such wheels are located in the vehicle's center and are parallel to the actual wheels. Then, a linearizing feedback controller is designed so that the Ackerman mobile robot follows a desired trajectory. Numerical simulations exhibit the performance of the designed control law in trajectory tracking.

Keywords**:** Ackermann configuration, Bicycle model, Feedback control

1. Introduction

Mobile robotics, a complex and fascinating field, is dedicated to modeling, designing, constructing, programming, and controlling robots that can navigate diverse and intricate environments. As [1] categorizes, mobile robots span three significant domains: terrestrial, aerial, and aquatic. Among the land mobile robots, those that utilize wheels for locomotion, such as the omnidirectional robot, the differential robot, and the Ackerman type, are particularly noteworthy. The literature offers many methods and approaches for regulating or tracking the trajectory of these robots.

Specifically, an Ackerman-type mobile robot has a degree of steerability and a degree of mobility and whose structure resembles that of a conventional car; that is, it has two rear wheels joined by an axle, and it has two front wheels which, using a steering wheel, its steering angle can be changed. A particular manner of modeling the motion of an Ackerman mobile robot is through the bicycle's kinematic model. This model, despite its simplicity, accurately represents the motion of the robot with just one rear wheel and one front wheel, which can change the steering angle.

Using this model, the robot's motion is described by the linear and angular velocities of the wheels. By utilizing a matrix transformation, it is possible to calculate the velocity and orientation of the robot at any time. This scheme, based on the bicycle's kinematic model, allows us to design control algorithms for the mobile robot. These algorithms are crucial in enabling the robot to perform a specific task, such as following a route or avoiding obstacles.

The bicycle's kinematic model, a crucial tool in understanding and controlling the motion of the Ackerman robot, is more than just a theoretical concept. Its simplicity and accuracy make it invaluable for practical applications, such as autonomous navigation in urban environments or exploring unknown terrains. This underscores its significance and relevance in the field of mobile robotics.

Some works that consider the control of the bicycle's kinematic model can be found in [2], [3], [4], [5]. For example, in [2], the authors designed a Model Predictive Control to follow a route considering speed limits and lateral acceleration parameters. In [3], a control law is designed for trajectory tracking, which considers actuator saturations and state constraints. Vehicle localization by implementation of a kinematic bicycle model has been developed in [4] using the MATLAB mobile.

At the same time, in [5], the optimal control problem is addressed where the trajectories are required to satisfy the safety constraints in the continuous-time sense.

On the other hand, some works that tackle the problem of control the Ackerman mobile robot are found in [6], [7], [8], [9]. Specifically, in [6], a path-tracking control method based on pole assignment, virtual Ackerman steering control, and forward-steering proportional control method based on the principle of pulse-width modulation were designed to solve the problems of the poor smoothness of steering control and low control accuracy of tracked vehicles. An Ackerman steering model is presented in [7] to determine the path of the towing vehicle using the vehicle's speed and acceleration data. In [8], the authors describe the mathematical modeling corresponding to the kinematics and dynamics of a vehicle with 4-wheels to determine that the vehicle design meets and guarantees better driveability on a local road or highway. At the same time, in [9], linear and non-linear control approaches are analyzed for reaching a goal position with the least possible error.

Building on the above, this article is dedicated to designing a feedback linearizing control for trajectory tracking of an Ackerman mobile robot. The practical implications of this research are significant, as it leverages the bicycle's kinematic model to enhance the control of self-driving vehicles.

The structure of the paper is as follows: Section 2 describes the kinematic model of the Ackerman mobile robot and how it can be simplified into a bicycle's kinematic model. Then, Section 3 presents the control law design to make the Ackerman robot follow a desired trajectory. Section 4 illustrates the simulation results, while Section 5 states the conclusions of this work.

2. Kinematic model

The kinematic model of the Ackerman model (Fig. 1), considering it as bicycle model (Fig. 2), is defined, according to [10], as

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} v\cos(\beta+\psi) \\ v\sin(\beta+\psi) \\ w \end{bmatrix} \tag{1}$$

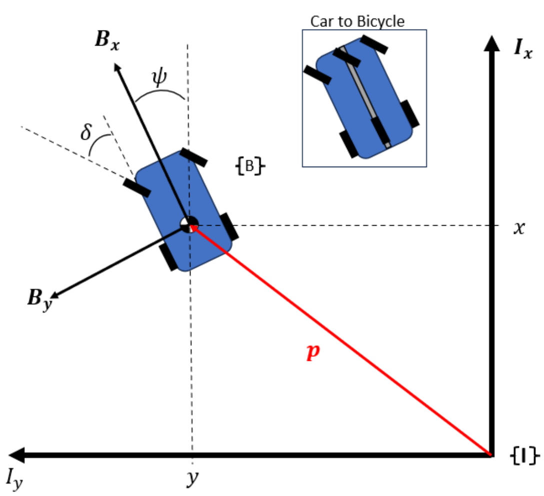


Fig. 1 Ackerman mobile robot.

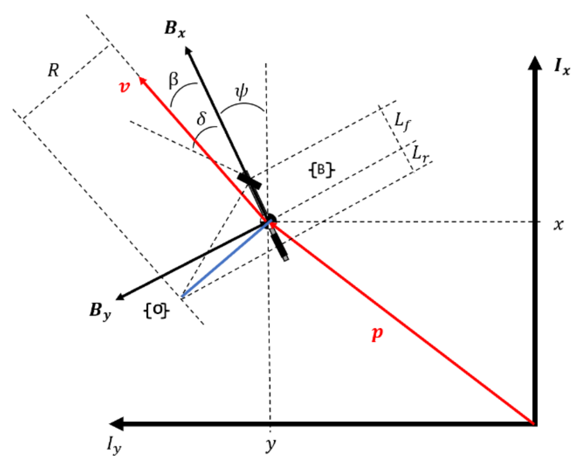


Fig. 2 Bicycle model.

Where x and y correspond to the postion in the plane of the vehicle's center mass, v is the longitudinal velocity and it is the control input, β is the sideslip angle that represents the orientation of the velocity vector v, ψ is the yaw angle and w is an auxiliary control which is defined as

$$w = \frac{v}{R} \tag{2}$$

with R as the turning radius. Based on [1], the angle β can be derived by analysing the triangle LNO in Fig. 3 as follows

$$\tan\delta = \frac{LN}{ON}, \quad \tan\beta = \frac{MN}{ON} \tag{3}$$

Where δ is the steering command.

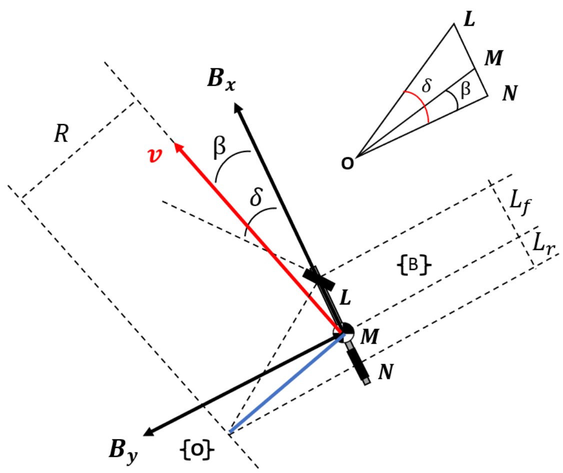


Fig. 3 Sideslip angle derivation.

By combining (3), it yields

$$\frac{1}{ON} = \frac{\tan\delta}{LN} = \frac{\tan\beta}{MN} \tag{4}$$

Replacing the lengths and isolating for β, one has

$$\beta = \tan^{-1}\left(\frac{L_r}{L_r + L_f}\tan\delta\right) \tag{6}$$

Furthermore, from triangle LNO, the turning radius R is equal to OM, therefore,

$$R = OM = \frac{OM}{\cos\beta} = \frac{\frac{LN}{\tan\delta}}{\cos\beta} = \frac{L_f + L_r}{\cos\beta\tan\delta} \tag{7}$$

3. Control Design

Consider the kinematic model using the the center of the rear wheels' position [11] without considering the sideslip angle

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\psi}\end{bmatrix} = \begin{bmatrix} v\cos(\psi)\\ v\sin(\psi)\\ \frac{V}{L}\tan(\delta)\end{bmatrix} \tag{8}$$

With $\dot{\psi} = w$, we obtain the derivative of $\dot{x}$ and $\dot{y}$ equations as follows

$$\begin{bmatrix}\ddot{x}\\ \ddot{y}\end{bmatrix} = \begin{bmatrix}\dot{v}\cos(\psi) - vw\sin(\psi)\\ \dot{v}\sin(\psi) + vw\cos(\psi)\end{bmatrix} \tag{9}$$

We propose the following control law

$$\begin{bmatrix}\dot{v}\\ w\end{bmatrix} = -\frac{1}{v}\begin{bmatrix} v\cos(\psi) & v\sin(\psi)\\ -\sin(\psi) & \cos(\psi)\end{bmatrix}\begin{bmatrix} k_{1x}(e_x) + k_{2x}(\dot{e}_x) - \ddot{x}_d\\ k_{1y}(e_y) + k_{2y}(\dot{e}_y) - \ddot{y}_d\end{bmatrix} \tag{10}$$

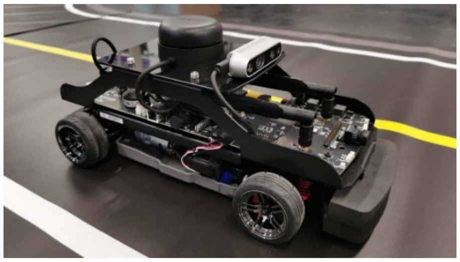
Where $e_x = x - x_d$, $e_y = y - y_d$, with x_d and y_d as the desired position and are at least twice differentiable, k_{ix} and k_{iy} are the control gains, for $i = 1,2$. Then, the steering angle is computed as

$$\delta = \tan^{-1}(\frac{(L_f + L_r)w}{v}) \tag{11}$$

Note that control (10) can only be implemented if the velocity $v \neq 0$.

4. Simulation Results on Quanser Self Driving Car

For the numerical simulations, the Qcar parameters are $L_f = L_r = 0.128$ m, the steering angle is in the range of $\left[-\frac{\pi}{6}, \frac{\pi}{6}\right]$rad. The center of gravity of Qcar is assumed to be in the center of the vehicle chassis, the sideslip angle using equation (6) is computed as $\beta = 0.2810$ rad, and the turning radius is computed using equation (7) with $R = 0.462$ m (see [1]). Figure 4 shows the Qcar prototype and the typical trajectory used to determine the maximum turning radius.



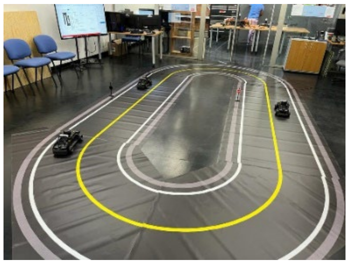


Fig. 4 Quanser car prototype and maximum turning radius.

The simulation results were implemented in Matlab Simulink using Runge-Kutta solver algorithm where the sample time was set to 0.005 s. The gains of proposed control were chosen in an heuristic way and were set as: $k_{1x} = k_{1y} = 30$ and $k_{2x} = k_{2y} = 6$.

The desired trajectory is a Lemniscate defined as $[x_d \quad y_d]^T = [1.5\cos(wt) \quad 0.6\sin(2wt)]^T$ with $w = \frac{\pi}{10}$, while the initial position is given by $[x(0) \quad y(0) \quad \psi(0)]^T = \left[1.5 \quad 0 \quad \frac{\pi}{2}\right]^T$. Figure 5 depicts the trajectory tracking performed by the mobile robot in different time instants. Note that the control point corresponds to the rear part of the vehicle. Furthermore, Fig. 6 displays the position error, which oscillates around zero. Such oscillations are presented when the vehicle turns.

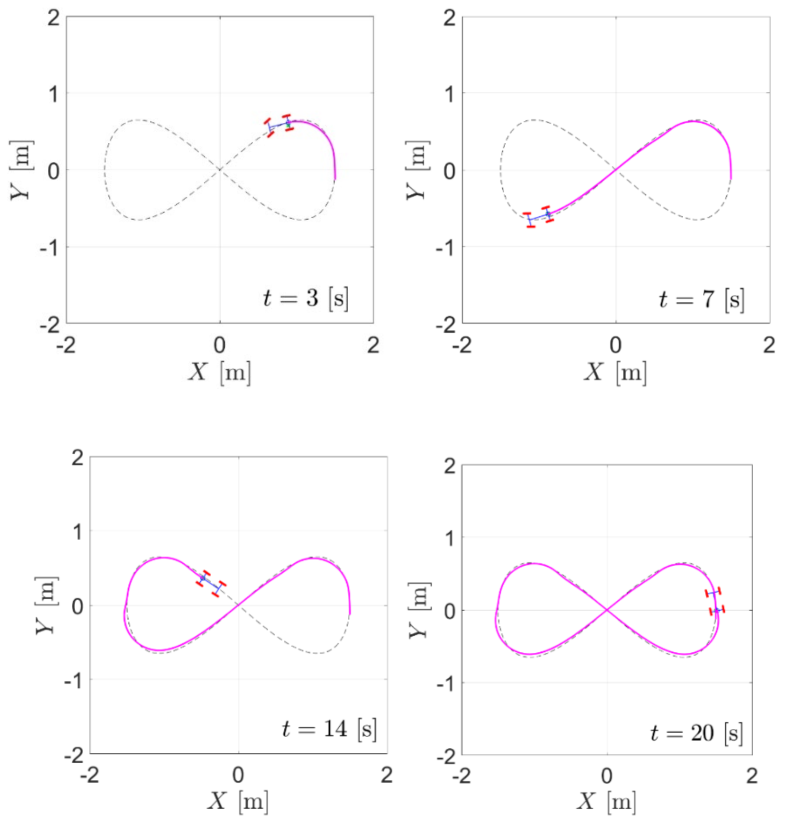


Fig. 5 Trajectory in the plane in different time instants.

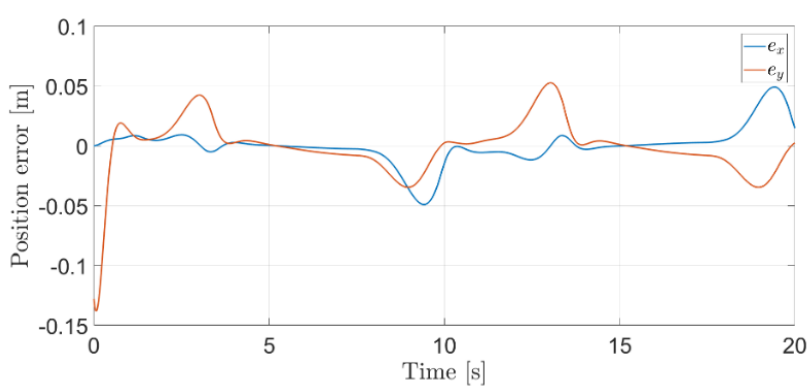


Fig. 6 Position error.

On the other hand, Fig. 7 illustrates the control inputs required for the vehicle to track the trajectory. In this sense, the linear velocity oscillates between 0.2 and 0.6 m/s in a steady-state, while the steering angle δ remains bounded between ± 0.5 rad.

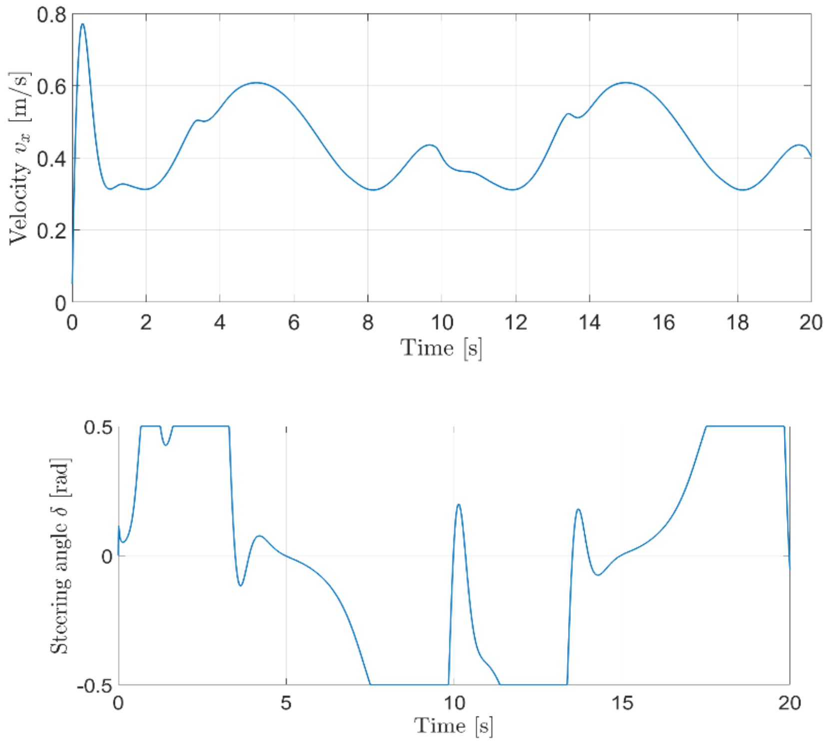


Fig. 7 Control inputs.

5. Conclusion

The proposed controller achieves acceptable trajectory tracking for Qcar. Although the Qcar kinematic model obtains the car position from the center of gravity of the car chassis and considers the sideslip angle, the proposed control law uses a simplified model considering the center of the rear wheels' position, which simplifies control and its implementation. In future work, we suggest implementing the proposed control with experimental results in the Qcar prototype.

Acknowledgements

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