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$\mathcal{H}_2$ Optimal Control of a Class of Discrete-Time Nonlinear Stochastic Systems Using Static Output Feedback

Justin J. Kennah, Edwin E. Yaz, Susan C Schneider
Marquette University
1515 W. Wisconsin Ave., Milwaukee, USA
{justin.kennah, edwin.yaz, susan.schneider}@marquette.edu

Abstract **–** This paper considers static output feedback control for a general class of discrete-time stochastic nonlinear systems. A linear matrix inequality is presented which is used to determine a constant feedback gain matrix. If a solution to the linear matrix inequality exists, then the closed-loop response of the system is guaranteed to satisfy $\mathcal{H}_2$ optimality in addition to achieving asymptotic stability in the mean square, and almost sure senses. Output feedback is used to eliminate the need to measure and/or estimate all of the states of the system. In this formulation, precise knowledge of the stochastic nonlinearity, or its statistics, are not needed. Rather, it is only required that an upper bound to the second moment of the stochastic nonlinearity can be determined.

Keywords**:** Output feedback, H2, Optimal control, Nonlinear systems, Stochastic systems, Discrete-time systems, Linear matrix inequality

1. Introduction

Most systems are nonlinear, to some degree, and are subject to stochastic disturbances, such as noise. Designing a stabilizing, or optimal, controller for such systems continues to be a challenging problem. Several well-known methods addressing this challenge, including feedback linearization and passivity-based methods, have been proposed over the years; however, these methods typically require the designer to have an accurate model of the system [1]. In practice, it may be difficult, or even impossible, to obtain a suitably accurate model of the system. Additionally, these methods typically require that the states of the system can be measured or estimated. Since it is common to have incomplete knowledge of the states of the system, it is often necessary to design an estimator which may add complexity to the design of a control system for a nonlinear stochastic system.

The work presented in [2] considered the stabilization of a general class of discrete-time nonlinear stochastic systems in which precise knowledge of the form of the nonlinearity was not needed. Rather, this formulation required only that the second moment of the stochastic nonlinearity be known. The need to have an accurate system model is somewhat relaxed since an optimal stabilizing control law is derived without needing to know the exact form of the nonlinearity. However, it is necessary that an exact expression describing the second moment of the nonlinearity can be determined which is a limitation of this work. Another limitation is that this method relies on the solution to a backwards running generalized Riccati equation which must be obtained off-line. This method also requires the availability of all states for measurement.

A suboptimal version of the previous work was presented in [3] where a static finite horizon state variable feedback control law was proposed. The benefit of this approach is that the need to obtain an off-line solution to a backwards running generalized Riccati equation is eliminated; however, an exact expression describing the second moment of the stochastic nonlinearity is required, and full availability of the states for measurement is again needed.

The infinite horizon case was considered in [4], and an observer was proposed in [5] to overcome the requirement that all states be measurable. A reduced order observer was given in [6]. Similar to previously cited works, an exact expression describing the second moment of the stochastic nonlinearity was required.

A General Performance Criteria (GPC) was considered in [7] which allowed for several control objectives (including $\mathcal{H}_2$, $\mathcal{H}_\infty$, and several passivity results) to be achieved. This work relaxed the requirement that an exact expression describing the second moment of the stochastic nonlinearity be known. Instead, with this formulation, it was only necessary that an upper bound to the second moment of the stochastic nonlinearity can be determined. Again, perfect knowledge of the states of the system was required.

The work in [8] considered a similar GPC which allowed for $\mathcal{H}_2$, $\mathcal{H}_\infty$, and several passivity design criteria to be met under a unified framework. In this formulation, a class of discrete-time nonlinear systems in which the nonlinearity is conically bounded was considered. This work was limited in the sense that perfect state knowledge was required. Further, the results presented are valid only for systems with nonlinearities containing finite energy, ℓ_2, disturbances.

A static output feedback controller was proposed in [9] to stabilize a general class of discrete-time nonlinear stochastic systems first presented in [2]. In this formulation, the use of output feedback eliminated the need to have perfect knowledge of the states, or to estimate any unknown system states. Additionally, this work did not require precise knowledge of the form of the stochastic nonlinearity or its second moment. Rather, it is only required that an upper bound to the second moment of the stochastic nonlinearity can be determined. Further, the nonlinearity was not restricted to include only finite energy, ℓ_2, disturbances. A linear matrix inequality (LMI) was presented which, when solved, provided a static feedback gain which guaranteed that the closed-loop response was asymptotically stable in the mean square (M.S.) and almost sure (A.S.) senses.

The work presented in this paper can be viewed as both an extension [8] to stochastic systems, and an extension of [9] from stabilizing to optimal control. In particular, the GPC in [8] is specified for the purposes of designing a static output feedback control law such that the closed-loop response satisfies the $\mathcal{H}_2$ property. An LMI is presented which is used to determine an unknown static control gain that will accomplish this objective. Since output feedback is used, it follows that all the states of the system do not need to be measured or estimated. Rather, it is only necessary that the output can be measured. Additionally, this formulation does not require precise knowledge of the form of the stochastic nonlinearity. It is only required that an upper bound to the second moment of the stochastic nonlinearity can be determined. Further, it will be shown through the results in [9] that in addition to satisfying the $\mathcal{H}_2$ property, the proposed control method will also be asymptotically stable in the M.S. and A.S. senses.

The remainder of this paper is organized as follows. First, we provide mathematical preliminaries which will be useful for understanding the results of this work. Next, we define the system of and derive an LMI which will be used to determine a static output feedback gain matrix. It will be shown that if a solution to the LMI exists, then the closed-loop response of the system will satisfy the $\mathcal{H}_2$ property and be asymptotically stable in the M.S. and A.S. senses. A simulation study is then presented which demonstrates controller design under the proposed framework. Lastly, we conclude the paper by summarizing our results.

1.1. Mathematical Preliminaries

The following notation is used in this paper: $x \in \mathbb{R}^n$ denotes an n-dimensional vector of real elements and $\mathrm{E}_{x_k}\{\cdot\}$ is the expectation of the argument conditioned on x_k. The 2-norm of the vector x_k is denoted as $\|x_k\| = (x_k^{\mathrm{T}} x_k)^{1/2}$. Positive (negative) definite and positive (negative) semidefinite matrix, P, is represented as $P > 0$ ($P < 0$) and $P \geq 0$ ($P \leq 0$) respectively. The trace of matrix T is given by $\mathrm{Tr}[T]$. An n-dimensional identity matrix, and an $n \times m$ null matrix, are represented by I_n and $[0]_{n\times m}$ respectively. The symbol $*$ is used to represent an element, or block, in a matrix needed to render the matrix symmetric. The square root of matrix N is denoted as $N^{1/2}$, and $\lambda_{max}(P)$ is used to denote the largest eigenvalue in the symmetric matrix P. A diagonal matrix, denoted $Diag(x, y, z)$, is a matrix with diagonal elements equal to $x, y,$ and z and off-diagonal elements equal to 0.

This paper also makes frequent use of Schur's lemma which is stated as follows; given appropriately sized matrices A, B, and C, the following statements are equivalent:

$$i)\ \begin{bmatrix} A & B \\ B^{\mathrm{T}} & C \end{bmatrix} > 0, \qquad ii)\ A - BC^{-1}B^{\mathrm{T}} > 0, \qquad iii)\ C - B^{T}A^{-1}B > 0$$

The following definitions related to stochastic stability will be useful:

Definition 1: A system is *M.S.) stable* if for any initial state, x_0, $\sup_k \mathrm{E}\{\|x_k\|^2\} < \infty$ holds for all k.

Definition 2: A system is *M.S. asymptotically stable* if in addition to being M.S. stable, $E\{\|x_k\|^2\} \to 0$ as $k \to \infty$.

Definition 3: A system is *A.S. stable* if for any initial state, $\|x_k\|^2 < 0$ holds for all k with probability equal to 1.

Definition 4: system is *A.S. asymptotically stable* if it is A.S. stable, and $\|x_k\|^2 \rightarrow 0$ as $k \rightarrow \infty$ with a probability of 1.

3. Main Result

Consider the discrete-time nonlinear stochastic system given by:

$$x_{k+1} = Ax_k + Bu_k + f_k \tag{1}$$
$$y_k = Cx_k \tag{2}$$
$$z_k = C_z x_k + D_z u_k \tag{3}$$

Where $x_k \in \mathbb{R}^n$ is the state vector, $u_k \in \mathbb{R}^m$ is the control input, $x_k \in \mathbb{R}^n$, $y_k \in \mathbb{R}^q$ is the output of the system, and $z_k \in \mathbb{R}^s$ is a performance output. The nonlinear function, $f_k \triangleq f(x_k, u_k, w_k): \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^p \rightarrow \mathbb{R}^n$, has the following properties:

$$f(0,0,w_k) = 0 \tag{4}$$
$$\mathrm{E}_{x_k}\{f_k\} = 0 \tag{5}$$
$$\mathrm{E}_{x_k}\{f_k f_j^{\mathrm{T}}\} = 0 \; \forall k \neq j \tag{6}$$
$$\mathrm{E}_{x_k}\{f_k f_k^{\mathrm{T}}\} \leq \sum_{i=1}^{r} T^i (x_k^{\mathrm{T}} M^i x_k + u_k^{\mathrm{T}} N^i u_k) \tag{7}$$

where $w_k \in \mathbb{R}^p$ is a timewise uncorrelated zero-mean noise sequence, and $r = n(n+1)/2$. Additionally, since (7) represents an upper bound to the second moment of the stochastic nonlinearity, it follows that all matrices on the right-hand side of the inequality are symmetric and at least positive semidefinite. It is further assumed that these matrices are known.

The nonlinearity described by (4) - (7) is quite general, and includes several well-known systems such as:

- Linear systems with state and control multiplicative noise:

$$f_k = w_k^1 \, G^{\mathrm{T}} x_k + w_k^2 \, D^{\mathrm{T}} u_k \tag{8}$$

- State and control norm dependent random vectors:

$$f_k = \|x_k\| G w_k^1 + \|u_k\| D w_k^2 \tag{9}$$

- Random vector dependent on the sign of a nonlinear function of the state and control input:

$$f_k = \mathrm{sgn}[h(x_k, u_k)] G w_k \tag{10}$$

as well as many others which can be found in [2].

Allowing the control law to be of the form $u_k = Ky_k = KCx_k$, where $K \in \mathbb{R}^{m \times q}$ is an unknown static gain matrix, we can express (1), (3), and (7) respectively as:

$$x_{k+1} = A_c x_k + f_k \tag{11}$$
$$z_k = C_c x_k \tag{12}$$
$$\mathrm{E}_{x_k}\{f_k f_k^{\mathrm{T}}\} \leq \sum_{i=1}^{r} T^i x_k (M^i + C^{\mathrm{T}} K^{\mathrm{T}} N^i K C) x_k \tag{13}$$

where $A_c \triangleq A + BKC$ and $C_c \triangleq C_z + D_z KC$.

Next, we derive an LMI which will allow for the determination of a static gain matrix, K, such that the closed-loop response of the system in Eqn. (11) satisfies the $\mathcal{H}_2$ property of optimality. As shown in [8], a system that satisfies the $\mathcal{H}_2$ property must satisfy the following inequality:

$$\mathrm{E}_{x_k}\left\{ x_k^T P x_k - x_{k+1}^{\mathrm{T}} P x_{k+1} - \delta \|z_k\|^2 \right\} > 0 \tag{14}$$

Where $P \in \mathbb{R}^{n\times n}$, $P = P^{\mathrm{T}} > 0$, and $\delta \in \mathbb{R} > 0$. Additionally, it was shown through the application of the Rayleigh inequality in [8], that the inequality in (14) could be equivalently expressed as:

$$\sum\nolimits_i \|z_i\|^2 < \delta^{-1}\lambda_{max}(P)\|x_0\|^2 \tag{15}$$

which will be useful later for considering the $\mathcal{H}_2$ gain of the system. Substituting (11) and (12) into (14) yields:

$$\mathrm{E}_{x_k}\left\{x_k^{\mathrm{T}} P x_k - (A_c x_k + f_k)^{\mathrm{T}} P\,(A_c x_k + f_k) - \delta\,(C_c x_k)^{\mathrm{T}} C_c x_k\right\} > 0 \tag{16}$$

Using (5) – (7) and exploiting the properties of the trace operator, the above inequality can be expressed as:

$$x_k^{\mathrm{T}}\left(P - A_c^{\mathrm{T}} P A_c - \delta C_c^{\mathrm{T}} C_c - \sum\nolimits_{i=1}^{r} \mathrm{Tr}\left[P\,T^i\right]\left(M^i + C^{\mathrm{T}} K^{\mathrm{T}} N^i K C\right)\right) x_k > 0 \tag{17}$$

which implies:

$$P - A_c^{\mathrm{T}} P A_c - \delta C_c^{\mathrm{T}} C_c - \sum\nolimits_{i=1}^{r} \mathrm{Tr}\left[P\,T^i\right]\left(M^i + C^{\mathrm{T}} K^{\mathrm{T}} N^i K C\right) > 0 \tag{18}$$

A computational problem arises due to the fact that the decision variables (P, K, and δ) appear nonlinearly in (18). An additional computational issue arises from the decision variable, P, appearing inside the trace operator. To overcome these difficulties, we define an upper bound to the matrix P as:

$$\gamma I_n \geq P > 0 \tag{19}$$

where $\gamma \in \mathbb{R} > 0$. Substituting (19) into (18) and rearranging terms, we can write a sufficient condition for (18) to hold as:

$$P > \gamma A_c^{\mathrm{T}} A_c + \delta C_c^{\mathrm{T}} C_c + \gamma \sum\nolimits_{i=1}^{r} \mathrm{Tr}\left[T^i\right]\left(M^i + C^{\mathrm{T}} K^{\mathrm{T}} N^i K C\right) \tag{20}$$

Dividing both sides of the above inequality by γ, and rearraigning terms, we obtain the following matrix inequality:

$$\tilde{P} - A_c^{\mathrm{T}} A_c - \frac{\delta}{\gamma} \mathrm{C}_c^{\mathrm{T}} C_c - \sum\nolimits_{i=1}^{r} \mathrm{Tr}\left[T^i\right](M^i + C^{\mathrm{T}} K^{\mathrm{T}} N^i K C) > 0 \tag{21}$$

where:

$$\tilde{P} \triangleq \frac{1}{\gamma} P > 0 \tag{22}$$

After repeated use of Schur's lemma, (21) can be expressed as:

$$\begin{bmatrix} \tilde{P} - \sum_{i=1}^{r} \mathrm{Tr}[T^i] M^i & A_c^{\mathrm{T}} & C^{\mathrm{T}} K^{\mathrm{T}} \left(\sum_{i=1}^{r} \mathrm{Tr}[T^i] N^i \right)^{\mathrm{T}/2} & C_c^{\mathrm{T}} \\ * & I_n & [0]_{n \times m} & [0]_{n \times s} \\ * & * & I_m & [0]_{m \times s} \\ * & * & * & \sigma I_s \end{bmatrix} > 0 \tag{23}$$

where:

$$\sigma \triangleq \frac{\gamma}{\delta} > 0 \tag{24}$$

Equation (23) is an LMI that can be solved for $\tilde{P}$, K, and σ with the restrictions given in (22) and (24). It is noted that if (23) has a solution, then the static output gain matrix, K, will yield a closed-loop response that satisfies the $\mathcal{H}_2$ property given in (15).

3. Stability

It was shown in [9] that if a solution exists to the LMI given by:

$$\Omega = \begin{bmatrix} \tilde{P} - \sum_{i=1}^{r} \mathrm{Tr}[T^i] M^i & A_c^{\mathrm{T}} & C^{\mathrm{T}} K^{\mathrm{T}} \left(\sum_{i=1}^{r} \mathrm{Tr}[T^i] N^i \right)^{\mathrm{T}/2} \\ * & I_n & [0]_{n \times m} \\ * & * & I_m \end{bmatrix} > 0 \tag{25}$$

then, the closed-loop system, with K as a static output feedback gain matrix, will be asymptotically stable in the M.S. and A.S. senses. Substituting, the inequality in (25) into (23) yields:

$$\begin{bmatrix} \Omega & \Phi^{\mathrm{T}} \\ \Phi & \sigma I_s \end{bmatrix} > 0 \tag{26}$$

where $\Phi = [C_c \quad [0]_{s \times n} \quad [0]_{s \times m}]$. By Schur's lemma, (26) can be expressed as $\Omega - \sigma^{-1} \Phi^{\mathrm{T}} \Phi > 0$, which implies $\Omega > 0$. Thus, if (23) has a solution, then it follows the closed-loop response will be asymptotically stable in the M.S. and A.S. senses in addition to satisfying the $\mathcal{H}_2$ property of optimality.

4. Simulation Results

This section presents simulation results which demonstrates controller design under the proposed framework. For the purposes of demonstration, we consider Chua's circuit, which is a well-known nonlinear circuit that exhibits chaotic behavior [10]. Continuous-time equations describing Chua's circuit are given below with parameters obtained from [8], [10], and [11]:

$$\dot{x}(t) = \begin{bmatrix} -\alpha & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & \mu \end{bmatrix} x(t) + \begin{bmatrix} 7 \\ 1 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} -\alpha f(x_1(t)) \\ 0 \\ 0 \end{bmatrix} \tag{27}$$

$$y(t) = [1 \quad 0 \quad 0]\, x(t) \tag{28}$$

where $\alpha = 10.0063$, $\beta = 16.5811$, and $\mu = 0.138083$ are system parameters [8], and:

$$f(x_1(t)) = bx_1(t) + 0.5(a-b)(|x_1(t)+1| - |x_1(t)-1\,|) \tag{29}$$

is a nonlinear function in $x_1(t)$, with $a = -1.39386$ and $b = -0.75590$. The nonlinearity in (29) can be equivalently piecewise defined as:

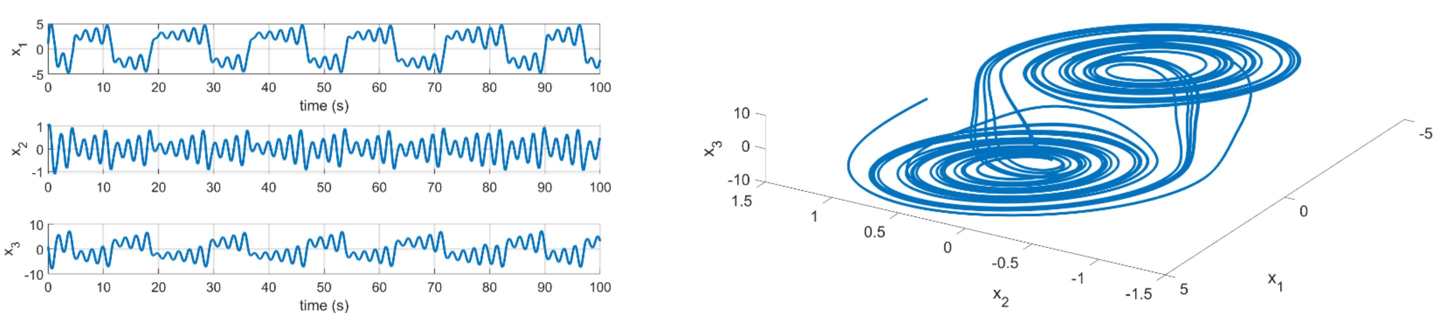
$$f(x_1(t)) = \begin{cases} bx_1(t) + (b-a), & x_1(t) \leq -1 \\ ax_1(t), & |x_1(t)| < 1 \\ bx_1(t) + (a-b), & x_1(t) \geq 1 \end{cases} \tag{30}$$

It is noted that in this example, the form of the nonlinearity is known. In general, it is not required that the nonlinearity be known. Rather, it is only necessary that an upper bound to the second moment of the nonlinearity can be determined. The choice to consider a system with a known nonlinearity was intentional and made for the purposes of simulation.

The continuous-time equations are discretized using forward Euler discretization with time step $T_s = 0.01s$. The discrete-time state space equations are given as follows:

$$\begin{bmatrix} x_{1,k+1} \\ x_{2,k+1} \\ x_{3,k+1} \end{bmatrix} = \begin{bmatrix} 0.8999 & 0.1001 & 0.0000 \\ 0.0100 & 0.9900 & 0.0100 \\ 0.0000 & -0.1658 & 0.9986 \end{bmatrix} \begin{bmatrix} x_{1,k} \\ x_{2,k} \\ x_{3,k} \end{bmatrix} + \begin{bmatrix} 0.07 \\ 0.01 \\ 0.01 \end{bmatrix} u_k + \tilde{f}_k \tag{31}$$

where $\tilde{f}_k = \begin{bmatrix} -0.1001 f_k(x_{1,k}) & 0 & 0 \end{bmatrix}^{\mathrm{T}}$. Choosing the initial conditions of the system to be $x_0 = [1 \quad 1 \quad 1]^{\mathrm{T}}$, a simulation of the uncontrolled system is shown in Fig. 1. While the evolution of the system states Fig. 1(a) may appear to be periodic, the chaotic nature of the circuit is evident in the phase portrait in Fig. 1(b) by noting that the trajectories never overlap.



(a) Evolution of system states

(b) Phase portrait showing chaotic behavior

Fig. 1: Simulation of uncontrolled Chua's circuit

By treating the discrete nonlinear function, $\tilde{f}_k$, as a discrete stochastic process, we can express the conditional covariance as follows:

$$E_{x_k}\{\tilde{f}_k \tilde{f}_k^{\mathrm{T}}\} = Diag(0.0100 f^2(x_{1,k}), \quad 0, \quad 0) \tag{32}$$

An upper bound to the second moment of the nonlinearity can be easily determined by considering Eqn. (30). Since $|a| > |b|$, it follows that:

$$\sup_{x_{1,k}} f^2(x_{1,k}) = (ax_{1,k})^2 \tag{33}$$

which, from (7), implies that the second moment of the discrete nonlinearity can be bounded above by:

$$\sum_{i=1}^{r} T^i\left(x_k^{\mathrm{T}} M^i x_k + u_k^{\mathrm{T}} N^i u_k\right) \leq Diag(1, \quad 0, \quad 0)\, x_k^{\mathrm{T}}\, Diag(0.0100a^2, \quad 0, \quad 0)\, x_k \tag{34}$$

Which further implies that:

$$\sum_{i=1}^{r} \mathrm{Tr}\left[T^i\right] M^i = Diag(0.0100a^2, \quad 0, \quad 0), \qquad N^i = 0\ \forall i \tag{35}$$

Since an upper bound to the second moment of the nonlinearity exists, and can be determined, (35) can be used in (23) and the LMI can be solved for $\tilde{P}$, K, and σ with the restrictions given in (22) and (24). A solution to (23) which satisfies these restrictions is given below where $C_z = 0.01I_3$, and $D_z = [0.1 \;\; 0.1 \;\; 0.1]^{\mathrm{T}}$.

$$\tilde{P} = Diag(1.9310, \quad 1.9212, \quad 1.9212), \qquad \sigma = 1.9212, \qquad K = -11.7029 \tag{36}$$

A plot showing the simulated response of the closed-loop system is shown in Fig. 2.

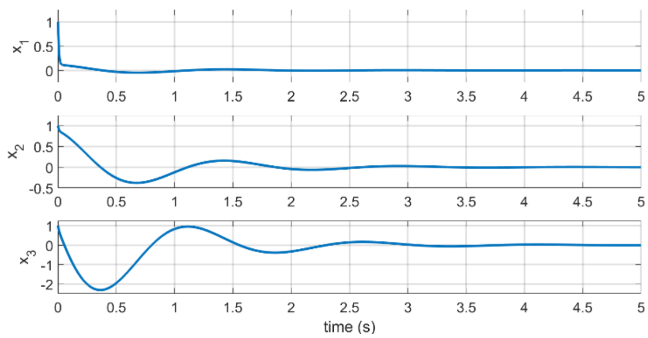


Fig. 2: Simulation of controlled Chua's circuit

We now consider the $\mathcal{H}_2$ property of the controller. To begin, we first use inequalities (22) and (24) to write:

$$\tilde{P} = \frac{1}{\sigma\delta} P \tag{37}$$

which, after substitution of $\tilde{P}$ and σ in (36), implies $P = Diag(3.7809\delta, \quad 3.6911\delta, \quad 3.6911\delta)$. Since by definition $\delta > 0$, it follows $\lambda_{max}(P) = 3.7809\delta$. Thus, the right side of (15) can be expressed as:

$$\delta^{-1}\lambda_{max}(P)\|x_0\|^2 = \delta^{-1}\,(3.7809\delta)(3) = 11.3427 \tag{38}$$

Where $\|x_0\|^2 = 3$ has been substituted into the above expression. It can be shown that for this simulation $\sum_i \|z_i\|^2 = 0.0695$, which clearly satisfies the inequality in (15) thus showing the $\mathcal{H}_2$ property is satisfied.

5. Conclusion

This work considered a general class of discrete-time nonlinear stochastic systems. An LMI was presented which can be used to determine a static output feedback gain which ensures that closed-loop system is not only asymptotically stable in the M.S. and A.S. senses, but also satisfies the $\mathcal{H}_2$ property of optimality. In this formulation, exact knowledge of all states of the system was not required. Rather, it was only necessary that the output of the system be available for measurement. Further, the proposed method does not require that the exact form of the nonlinearity be known. Instead, it is only necessary that an upper bound on the second moment of the stochastic nonlinearity can be determined. Chua's circuit was used to demonstrate controller design under the proposed framework. It was shown that the associated LMI had a solution which guaranteed that the closed-loop response would satisfy the $\mathcal{H}_2$ criteria in addition to being asymptotically stable in the M.S. and A.S. senses. Simulation results showed that the static output gain determined from the solution to the LMI indeed regulated the states of the system to the origin.

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