Proceedings of the 12th International Conference of Control Systems, and Robotics (CDSR 2025)
July 13, 2025 - July 15, 2025 | Imperial College London Conference Center, London, United Kingdom
Paper No. 112
DOI: 10.11159/cdsr25.112

Digital Model-Based Motor Servo Control Considering a Sampling-Induced Time Delay

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Abstract **-** This paper proposes the use of by-design model-based digital control systems for motor speed control, while the motor is connected to a mechanical load. This research is motivated by the fact that most controllers are implemented digitally, but they are designed with techniques suiting analogue systems based on continuous mathematics. Such a discrepancy results in loss of performance. A communication delay occurring in digital implementation, practical torque limits and measurement noise were considered in this research. The proposed methodology leads to a simple, easy to implement, digital/discrete control system which outperforms a PID designed with Internal Model Control (IMC) method.

Keywords**:** Digital Control, Motor Control, Servo, Speed Control, Internal Model Control, IMC, PID

1. Introduction

This paper targets a core challenge in digitalisation of control systems, mismatch of analogue (continuous) methods to design control systems and digital devices to implement control systems. Nowadays, most control systems, including well-established Proportional-Integral-Derivative (PID) controllers, are formulated and designed with the methods developed in continuous mathematics, suiting analogue devices, but are implemented on digital devices [1, 2]. Digital implementation imposes sampling of signals; this limits the bandwidth of operation [3]. Moreover, in digital systems, a delay, at least equal to the sample time of the system, should exist to avoid algebraic loops [4]. These sampling-induced bandwidth limit and delay are widely neglected in continuous design methods as they do not appear in analogue systems. This mismatch of design and implementation have shown to influence the performance of control systems negatively[4]. Motor speed control is an area affected by this mismatch with wide use of analogue control methods implemented on digital drives [5]. Complicated reference filters have been employed to partly compensate this effect mostly at the cost of a slower response [6].

Preliminary research by the authors shows that using digital design methods, based on discrete mathematics, for digitally-implemented control systems inherently removes the issues caused by delay and bandwidth limit and leads to a superior performance[4]. As a main advantage, time delays can be easily integrated into model-based discrete control system design methods [3], while, they lead to nonlinearity in continuous models [7]. In addition, commonly used PIDs, which are essentially continuous/analogue, are often designed based on trade-off[8]; while, comparable digital controllers have straightforward optimal design methods [3, 4].

2. Problem Statement

Fig.1 shows a servo control system for motors connected to a mechanical load, based on the information presented in [6]. However, the current control (or internal) loop has a such a fast dynamics, compared to the speed (or external) loop which can be ignored [9]; in other words, it is reasonable to assume that any physically realisable demanded torque by the speed controller is easily met. The real issue is the mechanical part with slow dynamics shown in Fig.2.

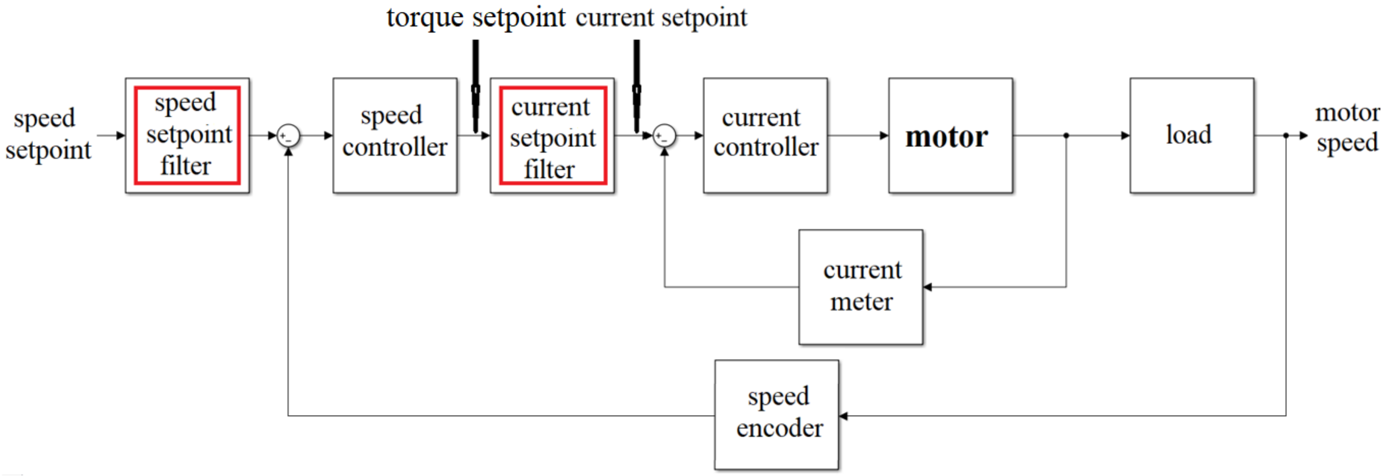

Fig.1 Schematic of a Servo Drive with an encoder on motor shaft

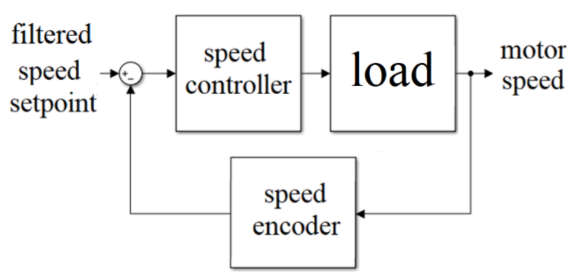

Fig.2 A simplified control system of a servo motor, where the current control system is so fast as to be neglected.

Eq. (1) is a fairly accurate model of the load in time domain [9],

$$J\dot{\omega}(t) + C\omega(t) = U(t - t_d). \qquad (1)$$

where U, J , C and ω represent the input torque (from the motor), moment of inertia, viscous damping coefficient and rotational speed, respectively. In addition, t_d is the communication/processing delay, which is widely ignored [4]. As previously mentioned, such a delay, which is at least equal to the sample time, must exist in all digital closed loop systems to preclude an algebraic loop. (1) practically demonstrates that the control law generates the control input, U, to be applied at time t, based on measurements at time t- t_d.

Eq. (1) can be re-written as the transfer function of (2) in Laplace domain assuming zero initial angular velocity:

$$G(s)=\frac{\Omega(s)}{U(s)}=\frac{e^{-st_d}}{Js+C}. \qquad (2)$$

In the following sections, a discrete model, equivalent of (1)/(2), is developed, then a discrete controller is designed based on this model. The closed loop response will be compared to the response of a closed loop system with continuous PID designed based on IMC.

3. Discretisation of the Model

$G_{wd}(s)$, presented by (3), represents the model of Eq. (2) without a delay in Laplace domain:

$$G_{wd}(s)=\frac{G(s)}{e^{-st_d}}=\frac{1}{Js+C}. \qquad (3)$$

Eq. (4) demonstrates the process of discretisation of Eq. (3) with zero order hold [10]:

$$G_{wd}(z)=\frac{z-1}{z}\mathcal{Z}\left\{\mathcal{L}^{-1}\left\{\frac{G_{wd}(s)}{s}\right\}\right\}. \qquad (4)$$

Partial fraction decomposition, presented in (5), is required to transform the model from Laplace domain to Z domain, as shown in (6), e.g. with the formulae listed in [11]:

$$\frac{G_{wd}(s)}{s}=\frac{1}{Js^2+Cs}=\frac{1/C}{s}-\frac{J/C}{Js+C}=\frac{1}{C}\left(\frac{1}{s}-\frac{J/C}{J/C\,s+1}\right). \qquad (5)$$

$$\mathcal{Z}\left\{\mathcal{L}^{-1}\left\{\frac{G_{wd}(s)}{s}\right\}\right\}=\frac{1}{C}\left(\frac{z}{z-1}-\frac{z}{z-e^{-\frac{t_sC}{J}}}\right)=\frac{z\left(z-e^{-\frac{t_sC}{J}}\right)-z(z-1)}{C(z-1)\left(z-e^{-\frac{t_sC}{J}}\right)}. \qquad (6)$$

Consequently, the discrete form of $G_{wd}(s)$ is presented in (7):

$$G_{wd}(z)=\frac{z-1}{z}\frac{z\left(z-e^{-\frac{t_sC}{J}}\right)-z(z-1)}{C(z-1)\left(z-e^{-\frac{t_sC}{J}}\right)}=\frac{\left(z-e^{-\frac{t_sC}{J}}\right)-(z-1)}{C\left(z-e^{-\frac{t_sC}{J}}\right)}=\frac{1-e^{-\frac{t_sC}{J}}}{C\left(z-e^{-\frac{t_sC}{J}}\right)}. \qquad (7)$$

Considering (3) and (7), (8) is the discrete form of the transfer function of (2):

$$G(z)=\frac{1-e^{-\frac{t_sC}{J}}}{C\left(z-e^{-\frac{t_sC}{J}}\right)}z^{-\frac{t_d}{t_s}}=\frac{1-e^{-\frac{t_sC}{J}}}{Cz^{\frac{t_d}{t_s}}\left(z-e^{-\frac{t_sC}{J}}\right)}=\frac{\frac{1-e^{-\frac{t_sC}{J}}}{C}}{z^{\frac{t_d}{t_s}}\left(z-e^{-\frac{t_sC}{J}}\right)}=\frac{\frac{1-e^{-\frac{t_sC}{J}}}{C}}{z^n\left(z-e^{-\frac{t_sC}{J}}\right)}. \qquad (8)$$

where n is the delay order, which is assumed to be a natural number in this research.

4. Model-based Digital Control

In this section, based on the model shown in (8), first, a discrete controller is designed for n=1, the most plausible value of n for motor control. Then, the design methodology is extended to greater values of n. A transfer function such as (2) or (8) with a long delay can be used to present a first order process with time delay [12].

4.1. Model Based Control for Unit Communication Delay

Due to availability of very fast processors, it can plausibly be assumed that the processing/communication delay time of the motor drive has its minimum value or t_d=t_s or n=1. Then,

$$G(z)=\frac{\frac{1-e^{-\frac{t_sC}{J}}}{C}}{z^{\frac{t_d}{t_s}}\left(z-e^{-\frac{t_sC}{J}}\right)}=\frac{\frac{1-e^{-\frac{t_sC}{J}}}{C}}{z^2-e^{-\frac{t_sC}{J}}z}=\frac{a}{z^2+bz}=\frac{a}{P(z)}, \tag{9}$$

where

$$a=\frac{1-e^{-\frac{t_sC}{J}}}{C}\ and\ b=-e^{-\frac{t_sC}{J}}. \tag{10}$$

The controller is presented as $C(z)$, a proper discrete transfer function:

$$C(z)=\frac{R(z)}{Q(z)}=\frac{r_m z^m+\cdots+r_1 z+r_0}{z^m+\cdots+q_1 z+q_0}. \tag{11}$$

Based on Eqs. (10)-(11), assuming that the encoder in Fig.2 has no dynamics, Eq, (12) shows the transfer function of the closed loop system of Fig.2:

$$T_{cl}(z)=\frac{a\,R(z)}{a\,R(z)+P(z)Q(z)}. \tag{12}$$

Assuming p_i as a closed loop pole, Eq. (13) relates these poles to the parameters of the transfer function and the controller:

$$\prod_i (z-p_i)=a\,R(z)+P(z)\,Q(z). \tag{13}$$

With adding desirable closed loop poles to the left side of Eq. (13), the parameters of R(z) and Q(z) can be found so as to realise these poles. In this section, Eq. (14), as a specific form of Eq. (11), is considered as the controller:

$$C(z)=\frac{R(z)}{Q(z)}=\frac{r_1 z}{z+q_0}. \tag{14}$$

Combination of Eqs. (9), (13) and (14) results in Eq. (15):

$$\prod_i (z-p_i)=a(r_1 z)\ +\left(z^2+bz\right)(z+q_0). \tag{15}$$

As to Eq. (15), the closed loop system evidently has a pole at zero. Let us assume the other two poles are unique (or repeated), shown as p. Then, Eq. (15) is converted to Eq. (16):

$$(z-p)^2=a r_1\ +(z+b)(z+q_0). \tag{16}$$

As a result,

$$z^2-2pz+p^2=z^2+(b+q_0)z+\ (a r_1\ +b q_0). \tag{17}$$

This leads to

$$\begin{cases} q_0=-2p-b, \\ r_1=\dfrac{p^2+2pb+b^2}{a}. \end{cases} \tag{18}$$

4.2. Model Based Control for Non-unit Communication Delays

The methodology presented in the previous subsection can be extended to a longer (or a higher order of) time delay in Eq. (8). Considering Eq. (8) and (10), Eq. (2) with a long delay can be presented as Eq. (19),

$$G(z)=\frac{a}{z^n(z+b)}. \tag{19}$$

The following controller is proposed to control the system represented by Eq. (19):

$$C(z) = \frac{U(z)}{E(z)} = \frac{r_n z^n}{z^n + \cdots + q_1 z + q_0}. \tag{20}$$

Eq. (21) is the extended version of Eq. (15), the characteristic equation of the closed loop system, for the system of Eq. (19) and the controller of Eq. (20):

$$\prod_{i}(z - p_i) = \square\left(r_n z^n\right) + z^n(z + \square)\left(z^n + \cdots + q_1 z + q_0\right). \tag{21}$$

The closed loop system, with the characteristic equation of Eq. (21), has $2n+1$ poles in total: n poles at 0, contributing to the stability of the system, and $n+1$ other poles of choice. After selection of these non-zero poles of choice, Eq. (21) leads to $n+1$ equations which can determine $n+1$ parameters of the controller, q_0 to q_{n+1} and r_n. This is a general discrete solution to any first order system with a time delay.

5. Results and Analysis

This section focuses on specifying the parameters of (18) and assessment of the control system with the use of the following realistic values for the system parameters and the sample time: J= 1kg.m^2, C=0.1 N.m/s and t_s=0.001 s.

5.1. The Proposed Control System

With the abovesaid parameters and n=1, Eqs. (2) and (9) can be presented as Eqs. (22)-(23):

$$G(s) = \frac{\Omega(s)}{U(s)} = \frac{e^{-0.001s}}{s + 0.1}. \tag{22}$$

$$G(z) = \frac{\Omega(z)}{U(z)} = \frac{9.9995 \times 10^{-4}}{z^2 - 0.9999z}. \tag{23}$$

With the use of Eq. (23) as the motor model and the aforementioned parameters, Eq. (18) is converted to Eq. (24):

$$\begin{cases} q_0 = -2p + 0.9999, \\ r_1 = 1000p^2 - 1.9999p + 999.85 \end{cases}. \tag{24}$$

which demonstrates how the controller parameters are obtained using pole placement. Absolute value of motor torque, as the control input, is limited to 300 N.m in this paper to have realistic simulations. This imposes nonlinearity in high torques and decelerates the convergence to the speed setpoint.

The proposed control system was created and run in a mixed discrete-continuous model in MathWorks Simulink version 24.2, as shown in Fig. 3, with a fixed step Runge-Kutta solver. The solver step size was set equal to the sample time, 1 ms. The discrete controller is a combination of (14) and (24). The unit of speed in Fig. 3 is rad/s, and the use of other units requires the addition of unit change blocks.

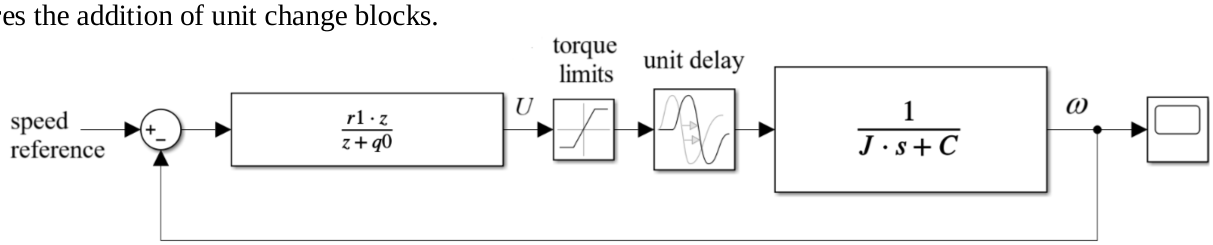

Fig.3 The control system without noise and unit change blocks

5.2. The Consequences of Torque Bound and Measurement Noise

With bounded torque value and without considering speed measurement noise, it was observed that, change of the placed pole, p in (24), from 0.1 to 0.8 then to 0.97 does not significantly change the convergence speed. 0.1 is comparably very close to zero. and without torque limits, would lead to a much faster response compared to 0.97. However, in the presence of torque bounds, the closed loop responses and the control input (torque) diagrams for p of 0.1, 0.8 and 0.97 nearly match;

thus, they are not shown in the paper. Use of slower poles, however, decreases the controller magnitude, shown in (25) as function of the excitation frequency (ω_e).

$$C_{r(z)}(\omega_e) = \frac{r_1}{\sqrt{(1 + q_0\cos(\omega_e))^2 + (q_0\cos(\omega_e))^2}}. \tag{25}$$

For instance, with p of 0.1, 0.8 and 0.97, r_1 in (14) becomes 809.86, 39.96, 0.894, respectively, while the absolute value of q_0 in (14) differs slightly across the aforementioned poles. A smaller controller magnitude reduces the negative effect of measurement noises, because the measurement noise is transferred to the control error.

As to [13], 0.1% is a reasonable error for a motor speed encoder. As a result, a random value between [-0.1% 0.1%] of the actual speed is added as the speed encoder noise to make the simulation further realistic. The impact of noise on the response is insignificant in simulations; however, it results in fluctuations of the control input (torque) and raises implantability concerns as depicted in Fig. 4. This figure shows the torque demanded by the proposed controller, presented by (14) and (24), to track the reference shown in Fig.6.

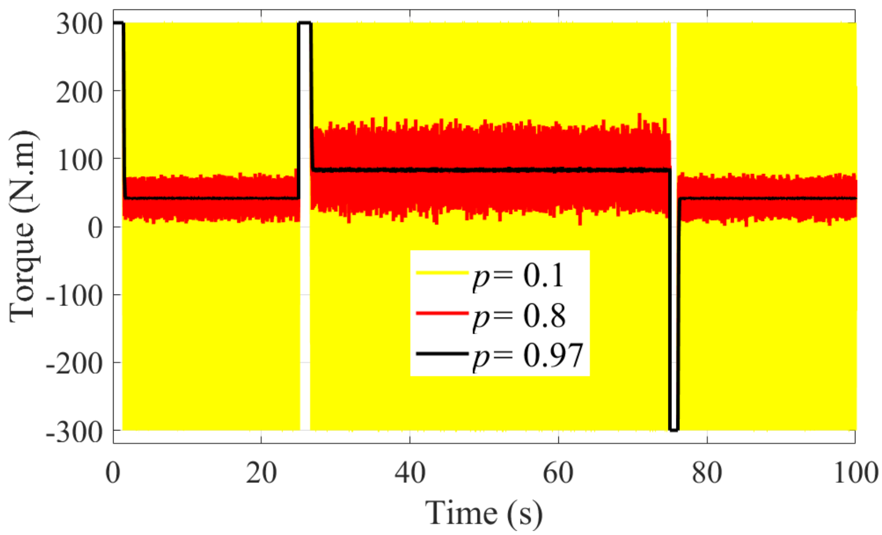

Fig.4 Torque demanded by the controller of (14) and (24) with different poles for tracking control problem of Fig.6

Fig.4 evidently demonstrates that the controller parameters, calculated in (24), which place the pole of 0.1 are not implementable, as they lead to full range, -300 to 300 N.m, fluctuations too frequently. Fluctuation for the pole of 0.8 are substantial and problematic too. However, p=0.97 leads to manageable fluctuations in torque and the following discrete controller:

$$C(z) = \frac{U(z)}{E(z)} = \frac{0.894z}{z - 0.940} = \frac{0.894}{-0.940z^{-1} + 1}. \tag{26}$$

or $$U(k)=0.940\, U(k-1)+ 0.894\, E(k), \tag{27}$$

where E stands for the control error. Advantageously, (27) is very easy to implement with any digital hardware.

5.3 IMC-PID Feedback Controller for Comparison

For comparison, with the use of Pade approximation, a feedback continuous IMC-based PID is designed on the basis of (2) and the parameter values presented in 5.1, according to the methodology detailed in the appendix of [14] and in [15]:

$$C(s) = \frac{U(s)}{E(s)} = \frac{0.005s^2 + 10.0005\, s + 1}{10(\lambda + 0.0005)s}. \tag{28}$$

where λ is the time constant of the controller. The smaller λ, the quicker convergence towards the reference, and the less robustness [14]. However, similar to fast poles in the proposed discrete control system, too small values of λ in IMC-PID lead to major torque fluctuations in the presence of measurement noise, as depicted in Fig. 5. Figures demonstrate the control input (torque) to track the reference shown in Fig. 6.

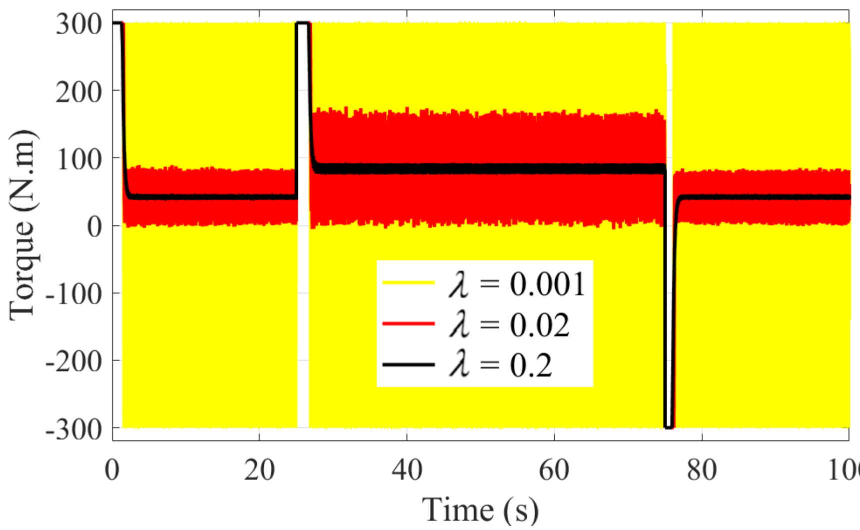

Figure 5 Torque demanded by the controller of (25) for different values of λ for tracking control problem of Fig.6

Figure 5 indicates that λ= 0.2 results in a manageable torque fluctuation and was chosen for (28):

$$\square(s) = \frac{\square(s)}{\square(s)} = \frac{0.005s^2 + 10.0005\, s + 1}{2.005s} = 4.988 + \frac{0.4988}{s} + \frac{s}{401}. \tag{29}$$

5.4 Control Performance

Fig. 6 demonstrates the control performance for the feedback controllers of (27) and (29) with the aforementioned torque bounds and sensor noise. Let us define settling time as the time duration from change of the reference till the response settles in a 100 rpm vicinity from the reference. With this definition, the longest settling time of the PID controller in Fig.6 is more 12s; whereas, the settling time for the discrete control system is always shorter than 2s. In addition, the PID controller presents noticeable overshoots up to 206 rpm.

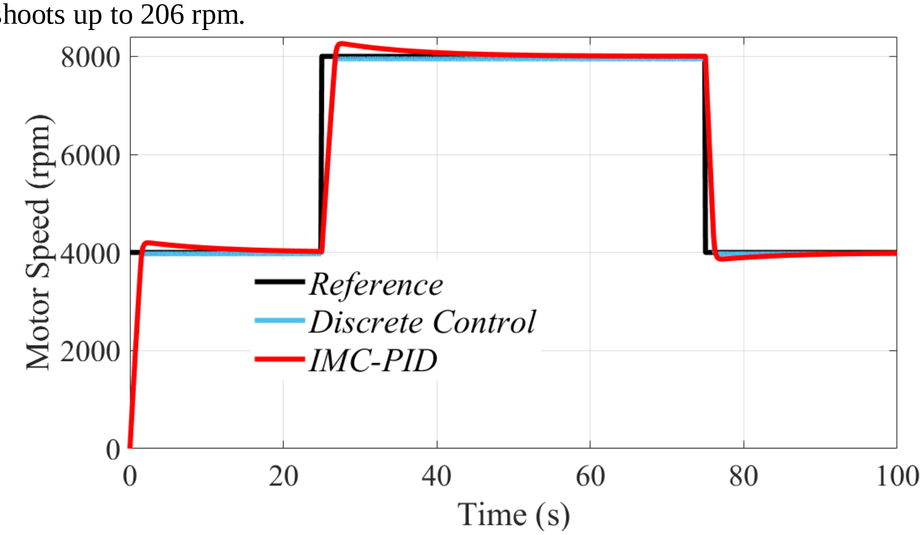

Figure 6 Control performance of (27) and (29)

4. Conclusion

This paper reports the design and validation of a model-based digital control system for servo/speed control of a motor connected to a mechanical load. Digital or discrete approach to control system design removes the issues of digital implementation of control systems designed with continuous mathematics, e.g. sampling-induced delay and bandwidth limit.

A control system for servo motors has two loops, one inside another. Current control or internal loop merely includes electrical components and is very fast compared to the speed or external loop, which has mechanical components; therefore, similar to many research works in the literature, this loop was not discussed in this paper.

In order to design the proposed control system, first, a discrete parametric model of the system was developed with the use of zero order hold, considering the communication time delay associated with digital implementation. With the most plausible value of the time delay, a simple discrete controller was proposed and designed based on pole placement. Due to the bounds of the control input (torque) and measurement noise, fast poles, with absolute values ≤ 0.9, lead to frequent and substantial fluctuations of torque; this phenomenon was found to be insubstantial with a repeated pole of 0.97. This apparently slow pole, in the presence of noise and torque bounds, provides a convergence speed close to the ones of fast poles and was used in design. The proposed controller outperforms a more complex feedback IMC-PID controller and provides very short (<2s) settling time with almost no overshoot

Acknowledgements

Research resulted in this paper was supported by German University of Technology in Oman, through Seed Grant #G2025/NR/ENG/MM.

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