

Resilience Analysis of H_2 and H_∞ Controllers for a class of Discrete-Time Nonlinear Stochastic Systems

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Abstract— An analysis scheme for state feedback control of a class of nonlinear stochastic systems with locally conic-type nonlinearities is developed, with a focus on the H_2 and H_∞ performance criteria. The proposed method effectively handles uncertainties in both the system's nonlinearity and the randomly perturbed feedback gains. By utilizing linear matrix inequality techniques, the study presents a unified framework capable of analyzing whether H_2 and H_∞ performance objectives can be realized by state feedback controllers for select systems. Example systems that are unstable and chaotic, and where perturbed control gains are used, are provided to demonstrate the effectiveness of the proposed approach.

Keywords: Resilient control, nonlinear systems, stochastic systems, H_2 performance, H_∞ performance, linear matrix inequalities.

1. Introduction

In this paper, a resilience analysis method for stochastic systems is proposed to evaluate whether a state-feedback controlled system satisfies multiple control criteria. The systems considered are discrete-time stochastic nonlinear systems. System parameter perturbations and nonlinearities are assumed to lie within a conic uncertainty region, whose boundaries are derived using a linear matrix inequality (LMI) formulation. By analyzing the system response in the presence of perturbations to the controller gains, the proposed method ensures resilient control.

Small deviations from the ideal controller gains can lead to significant performance degradation, resulting in a so-called “fragile” controller [1]. Examples of such fragility include numerical round-off errors during gain computation, which may arise in microprocessor implementations of controllers or observers. Therefore, ensuring resilience—defined as the system's tolerance to deviations in the control gain from its nominal value—is a desirable property. LMI techniques are employed to analyze resilient and robust controller properties, including asymptotic stability, H_2 and H_∞ performance, input strict passivity, output strict passivity, and very strict passivity [2], [3].

Several related works have addressed these concepts in different contexts. For instance, extensions to time-delay systems are presented in [4] and [5]; switched systems are studied in [6] and [7]; and mixed criteria control is explored in [8]. Singular systems are treated in [9] and [10], while fault-tolerant control for time-varying delay systems is analyzed in [11]. Resilient control for networked systems is considered in [12], and sliding mode control for discrete systems is discussed in [13]. The works in [14] and [15] investigate H_2/H_∞ control of state-dependent nonlinear systems with mixed criteria. The current paper extends the results in [16–19] to discrete-time uncertain nonlinear systems with uncertainty in the applied control gain. The key distinction of this work lies in the stochastic nature of the nonlinear system, which enables the proposed design to guarantee resilient properties in the presence of randomness.

The remainder of this paper is organized as follows. Section II introduces the system model and performance criteria, specifically H_2 and H_∞ norms. Section III presents the derivation of the analysis LMI. Section IV includes several simulation examples demonstrating the effectiveness of the proposed method. Finally, conclusions are drawn in Section V.

The following notation is used in this work: $x \in R^n$ represents an n -dimensions vector with real elements. $A \in R^{m \times n}$ represents an $m \times n$ matrix with real elements. $A > 0$ implies that matrix A is positive definite. The minimum and maximum eigenvalue of the symmetric matrix A is represented by $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ respectively.

In $\begin{bmatrix} A & B \\ * & C \end{bmatrix}$, $*$ represents the transpose or B^T . $E\{x\}$ denotes the expectation conditional on x . Lastly the Schur complement lemma is used often and is shown below.

Lemma 1. Schur complement lemma

$$\begin{bmatrix} A & B \\ * & C \end{bmatrix} > 0 \text{ is true if and only if } C > 0 \text{ and } A - BC^{-1}B^T > 0$$

2. Problem Formulation

Let us consider a discrete time nonlinear system,

$$x_{k+1} = f(x_k, u_k, w_k) \quad (1)$$

where $x_k \in R^x$ is the state, $u_k \in R^u$ is the input, and $w \in R^w$ is an ℓ_2 disturbance input.

A linear state feedback controller is used where it is assumed that there are uncertainties associated with the control gain K , the stochastic gain is perturbed as

$$u_k = \tilde{K}x_k = \sum_{i=1}^N \zeta_i K_i \quad (2)$$

where the white noise is defined as zero mean and σ_i^2 variance $\zeta_i \sim (0, \sigma_i^2)$ and the performance output is

$$z_k = C_z x_k + D_z u_k + E_z w_k \quad (3)$$

It is assumed that the linear part of the system can be extracted as

$$x_{k+1,lin} = Ax_k + Bu_k + Fw_k \quad (4)$$

So, the nonlinear part of the system \mathcal{F} is

$$\mathcal{F}_k = f(x_k, u_k, w_k) - (Ax_k + Bu_k + Fw_k) \quad (5)$$

It is also assumed that the nonlinear part of the system satisfies the conic condition

$$\|\mathcal{F}_k\|^2 \leq \alpha (C_{fx_k} + D_{fu_k} + E_{fw_k})^T (C_{fx_k} + D_{fu_k} + E_{fw_k}) \quad (6)$$

for every $x \in D^x, u \in D^u$, and $w \in D^w$, where $D^x \in R^x, D^u \in R^u$, and $D^w \in R^w$ are domains which include the origin.

We will consider the inequality

$$E_{x_k}\{V_k - V_{k+1} - \delta \|z_k\|^2 + \epsilon \|w_k\|^2\} > 0 \quad (7)$$

where V_k is a quadratic energy function $V_k = x_k^T P x_k$ and $P > 0$. The additional terms in the Lyapunov inequality are used to incorporate the H_2 and H_∞ performance criteria. This allows for the design of various performance criteria, for example, setting $\delta = 0$ and $\epsilon = 0$ will lead to mean square asymptotic stability. Setting $\delta > 0$ and $\epsilon = 0$ will yield a bound on the energy of the performance output with regards to the initial state x_0 ,

$$E\left\{\sum_i \|z_i\|^2\right\} \leq \frac{1}{\delta} \lambda_{max}(P) E\{\|x_0\|^2\} \quad (8)$$

Through the maximization of δ , a bound on the energy of the performance output will be found, which is defined as H_2 control.

Similarly, by setting $\delta = 1$ and $\varepsilon < 0$ will yield a bound on the energy of the performance output,

$$E\left\{\sum_i ||z_i||^2\right\} < -\varepsilon E\{||w_i||^2\} \quad (9)$$

Through the minimization of ε a bound on the energy of the performance output will be found, which is defined as H_∞ control. By differing values for δ and ε each performance criteria can be selected and each type of controller listed in Table 1 can be analyzed. In the next section, LMIs are formed to analyze controllers to see if the performance of the controllers meets the design criteria shown here.

3. Main Results

The following are the main results of this work:

Theorem 1. There exists a resilient state feedback controller $u = Kx$ for the discrete-time system described by (1) and (6) with the performance output (3), if the LMI for each case in Table 1 is feasible for some nonlinear bound α , a positive definite matrix $P > 0$ and controller gain K . If this LMI is satisfied than the controller will be resilient and tolerate an uncertainty on the applied gain.

$$H_1 = \begin{bmatrix} H_{11} & H_{12} & (A+BK)^T & 0 \\ H_{12}^T & H_{22} & F^T & 0 \\ (A+BK) & F & P & P \\ 0 & 0 & P & \alpha^{-1}I \end{bmatrix} > 0 \quad (10)$$

$$\begin{aligned} H_{11} &= P - \delta(C_z + D_{zK})^T(C_z + D_zK) - (C_f + D_{fK})^T(C_f + D_fK) \\ &\quad - \sum_{i=1}^N \sigma_i^2 K_i^T D_z^T DK - \sum_{i=1}^N \sigma_i^2 K_i^T D_f^T DK - \sum_{i=1}^N \sigma_i^2 K_i^T B^T (P - \alpha^{-1})BK \\ H_{12} &= \delta(C_z + D_{zK})^T E_z - (C_f + D_{fK})^T E_f \\ H_{22} &= \varepsilon I - \delta E_z^T E_z - E_f^T E_f \end{aligned}$$

Case 1. H_∞ controller (noise present, $\delta \neq 0$)

$$S_1 > 0$$

where $H_1 > 0$

Case 2. H_2 controller (non-noisy, $\delta \neq 0$)

$$S_2 > 0$$

where S_3 is obtained from H_1 by canceling the second row and column matrices.

Case 3. Asymptotic Stability (non-noisy, $\delta = 0$)

$$S_3 > 0$$

where S_3 is obtained from H_1 by canceling the second and third row and column matrices.

Proof. Substitute V_k and V_{k+1} in (7) to get,

$$E\{x_k^T P x_k - \delta ||z_k||^2 + \varepsilon ||w_k||^2 - (x_{k+1,lin} + \mathcal{F})^T P (x_{k+1,lin} + \mathcal{F})\} > 0 \quad (11)$$

Considering the terms inside (11) using Schur's complement

$$\begin{bmatrix} x_k^T P x_k - \delta \|z_k\|^2 + \epsilon \|w_k\|^2 & x_{k+1,lin}^T \\ x_{k+1,lin} & P^{-1} \end{bmatrix} > \begin{bmatrix} 0 & -\mathcal{F}^T \\ -\mathcal{F} & 0 \end{bmatrix} \quad (12)$$

Since,

$$\begin{bmatrix} \alpha^{-0.5} \mathcal{F}^T \\ \alpha^{0.5} I \end{bmatrix} [\alpha^{-0.5} \mathcal{F}^T \quad \alpha^{0.5} I] = \begin{bmatrix} \alpha^{-1} \mathcal{F}^T \mathcal{F} & \mathcal{F}^T \\ \mathcal{F} & \alpha I \end{bmatrix} \quad (13)$$

we have

$$\begin{bmatrix} \alpha^{-1} \mathcal{F}^T \mathcal{F} & 0 \\ 0 & \alpha I \end{bmatrix} > \begin{bmatrix} 0 & -\mathcal{F}^T \\ -\mathcal{F} & 0 \end{bmatrix} \quad (14)$$

So, a sufficient condition for (11) is

$$E \left\{ \begin{bmatrix} x_k^T P x_k - \delta \|z_k\|^2 + \epsilon \|w_k\|^2 - \alpha^{-1} \mathcal{F}^T \mathcal{F} & x_{k+1,lin}^T \\ x_{k+1,lin} & P^{-1} - \alpha I \end{bmatrix} \right\} > 0 \quad (15)$$

Then using Schur's complement, we have

$$E \{ x_k^T P x_k - \delta \|z_k\|^2 + \epsilon \|w_k\|^2 - \alpha^{-1} \mathcal{F}^T \mathcal{F} - x_{k+1,lin}^T (P^{-1} - \alpha I)^{-1} x_{k+1,lin} \} > 0 \quad (16)$$

Using (6) a sufficient condition is

$$E \{ x_k^T P x_k - \delta \|z_k\|^2 + \epsilon \|w_k\|^2 - (C_f x_k + D_f u_k + E_f w_k)^T (C_f x_k + D_f u_k + E_f w_k) - x_{k+1,lin}^T (P^{-1} - \alpha I)^{-1} x_{k+1,lin} \} > 0 \quad (17)$$

Expand z_k and $x_{k+1,lin}$

$$E \{ x_k^T P x_k - \delta [(C_z + D_z \tilde{K}) x_k + E_z w_k]^T [(C_z + D_z \tilde{K}) x_k + E_z w_k] + \epsilon \|w_k\|^2 - (C_f x_k + D_f u_k + E_f w_k)^T (C_f x_k + D_f u_k + E_f w_k) - [(A + B \tilde{K}) x_k + F w_k]^T (P^{-1} - \alpha I)^{-1} [(A + B \tilde{K}) x_k + F w_k] \} > 0 \quad (18)$$

Substitute for \tilde{K} and take expectation.

$$\begin{aligned} & x_k^T P x_k - \delta [(C_z + D_z K) x_k + E_z w_k]^T [(C_z + D_z K) x_k + E_z w_k] \\ & + \epsilon \|w_k\|^2 - (C_f x_k + D_f u_k + E_f w_k)^T (C_f x_k + D_f u_k + E_f w_k) \\ & - [(A + B \tilde{K}) x_k + F w_k]^T (P^{-1} - \alpha I)^{-1} [(A + B \tilde{K}) x_k + F w_k] \\ & - \sum_{i=1}^N \sigma_i^2 K_i^T D_z^T D K - \sum_{i=1}^N \sigma_i^2 K_i^T D_f^T D K - \sum_{i=1}^N \sigma_i^2 K_i^T B^T (P - \alpha^{-1}) B K > 0 \end{aligned} \quad (19)$$

Separate into its quadratic form.

$$\begin{bmatrix} x_k & w_k \end{bmatrix} H \begin{bmatrix} x_k \\ w_k \end{bmatrix} \quad (20)$$

where

$$H = \begin{bmatrix} h_{11} & h_{12} \\ h_{12}^T & h_{22} \end{bmatrix}$$

with

$$\begin{aligned} h_{11} &= P - \delta(C_z + D_{zK})^T(C_z + D_{zK}) - (C_f + D_{fK})^T(C_f + D_{fK}) - (A + BK)^T B^T (P - \alpha^{-1}) \\ &\quad - \sum_{i=1}^N \sigma_i^2 K_i^T D_z^T D K - \sum_{i=1}^N \sigma_i^2 K_i^T D_f^T D K - \sum_{i=1}^N \sigma_i^2 K_i^T B^T (P - \alpha^{-1}) B K \\ h_{12} &= \delta(C_z + D_{zK})^T E_z - (C_f + D_{fK})^T E_f \\ h_{22} &= \epsilon I - \delta E_z^T E_z - E_f^T E_f + F^T (P - \alpha^{-1})^{-1} F \end{aligned}$$

Using Schur's complement to get,

$$\begin{bmatrix} h_{11}^* & h_{12}^* & (A + BK)^T \\ h_{12}^{*T} & h_{22}^* & F^T \\ (A + BK) & F & P^{-1} - \alpha I \end{bmatrix} > 0 \quad (21)$$

where

$$\begin{aligned} h_{11}^* &= P - \delta(C_z + D_{zK})^T(C_z + D_{zK}) - (C_f + D_{fK})^T(C_f + D_{fK}) - (A + BK)^T B^T (P - \alpha^{-1}) \\ &\quad - \sum_{i=1}^N \sigma_i^2 K_i^T D_z^T D K - \sum_{i=1}^N \sigma_i^2 K_i^T D_f^T D K - \sum_{i=1}^N \sigma_i^2 K_i^T B^T (P - \alpha^{-1}) B K \\ h_{12}^* &= \delta(C_z + D_{zK})^T E_z - (C_f + D_{fK})^T E_f \\ h_{22}^* &= \epsilon I - \delta E_z^T E_z - E_f^T E_f \end{aligned}$$

Then pre and post multiplying by $\begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & P \end{bmatrix}$ and performing one more Schur's complement we get

$$\begin{bmatrix} h_{11}^* & h_{12}^* & (A + BK)^T & 0 \\ h_{12}^{*T} & h_{22}^* & F^T & 0 \\ (A + BK) & F & P & P \\ 0 & 0 & P & \alpha^{-1} I \end{bmatrix} > 0 \quad (22)$$

End of the Proof.

In the case of $i = 1$, the expectation of (22) is evaluated followed by two Schurs complements to separate the $(P - \alpha^{-1})$ term.

$$\begin{bmatrix} h_{11} & h_{12} & (A + BK)^T & 0 & \sigma_1 K_1^T B^T & 0 \\ h_{12}^T & h_{22} & F^T & 0 & 0 & 0 \\ (A + BK) & F & P & P & 0 & 0 \\ 0 & 0 & P & \alpha^{-1} I & 0 & 0 \\ \sigma_1 K_1 B & 0 & 0 & 0 & P & P \\ 0 & 0 & 0 & 0 & P & \alpha^{-1} I \end{bmatrix} > 0 \quad (23)$$

where

$$h_{11} = P - \delta(C_z + D_{zK})^T(C_z + D_{zK}) - (C_f + D_{fK})^T(C_f + D_{fK}) - (A + BK)^T B^T (P - \alpha^{-1})$$

$$\begin{aligned}
& -\sigma^2 K_1^T D_z^T D_z K_1 - \sigma^2 K_1^T D_f^T D_f K_1 - \sigma^2 K_1^T B^T (P - \alpha^{-1}) B K_1 \\
& h_{12} = \delta (C_z + D_{zK})^T E_z - (C_f + D_{fK})^T E_f \\
& h_{22} = \epsilon I - \delta E_z^T E_z - E_f^T E_f
\end{aligned}$$

4. Simulations Studies

Two systems will be used in simulation to show the effectiveness of the technique. The first system is a simple second order unstable system. The second system is Chua's circuit which exhibits chaotic behavior. The applied control is analyzed using the provided method to check if the performance criteria are met. The design parameters are given in Table 2.

Table 2: Design Parameters

δ	ϵ	$C_z \& C_z$	$C_z \& C_z$	$C_z \& C_z$
1	-3	$0.1 * I_2$	$[0.1; 0.1]$	$0.1 * I_2$

Example 1 - Unstable System: Below is the state space model of an unstable second order system

$$\begin{bmatrix} x_{1,k+1} \\ x_{2,k+1} \end{bmatrix} = \begin{bmatrix} 1 & 1.0101 \\ 1.0101 & 1 \end{bmatrix} \begin{bmatrix} x_{1,k} \\ x_{2,k} \end{bmatrix} + \begin{bmatrix} 0 \\ -0.01 \sin(x_{1,k}) \end{bmatrix} + \begin{bmatrix} 0.02 \\ 0.02 \end{bmatrix} u_k + \begin{bmatrix} 0.01 \\ 0 \end{bmatrix} w_k \quad (24)$$

with the finite energy term having the form of $w_k = 2e^k$.

Next, we use pole placement to stabilize the known linear part of the system. Placing the poles of the system very close to zero results in a gain of $K = [-127.17 \ -199.]$. Then we solve the LMI to determine if our performance criteria are still met with an increasing value of σ . Once the LMI is no longer feasible we have found the maximum value of σ . The maximum value results are shown in Table 3. Then applying control where we use the perturbed gain Kp we get the response shown in Fig 1b.

Table 3: Unstable System Performance Criteria

Criteria	σ_{max}^2	Performance Criteria Check
Asy. Stb.	9.62	$\lim_{k \rightarrow \infty} E\{\ x_k\ ^2\} = 0$
H_2 Ctrl.	8.77	$\sum_i E\{\ z_i\ ^2\} = 2.51 \leq \frac{\lambda_{max}(P)\ x_0\ ^2}{\delta} = 4.11$
H_∞ Ctrl.	9.27	$\sum_i E\{\ z_i\ ^2\} = 1.22 \leq -\epsilon \sum_i \ w_i\ ^2 = 112.02$

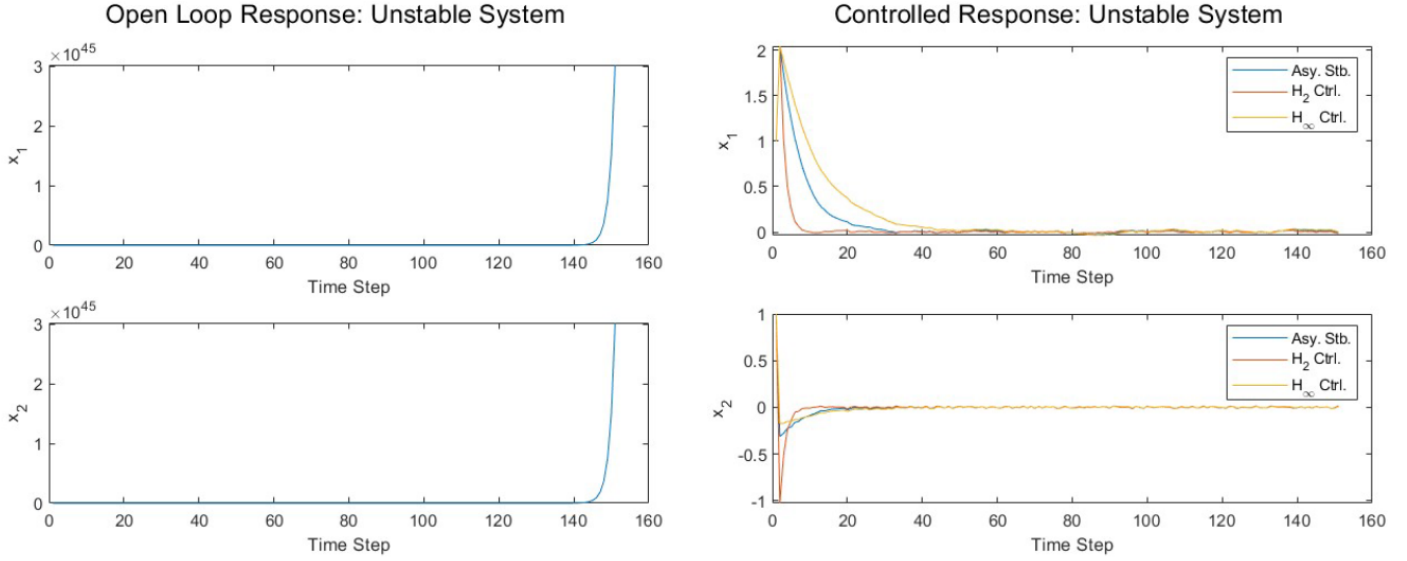


Figure 1: Comparison of Open and Closed Loop Responses of an Unstable System

Example 2 – Chua’s Circuit: The second system used is Chua’s circuit and the state space model is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0.909 & 0.091 & 0 \\ 0.01 & 0.99 & 0.01 \\ 0 & -0.1658 & 0.9986 \end{bmatrix} \begin{bmatrix} x_{1,k} \\ x_{2,k} \\ x_{3,k} \end{bmatrix} - \begin{bmatrix} 0.091f(x_1) \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix} u_k + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} w \quad (25)$$

where

$$f(x_1) = b x_1 + 0.5(a - b)(|x_1 + 1| - |x_1 - 1|)$$

with the parameters $\alpha c = 8.9, \beta c = 17, \mu = 0.15, a = 1.4, b = 0.76$ The discretized Chua’s circuit with a sampling time of $T = 0.01s$ is

$$\begin{bmatrix} x_{1,k+1} \\ x_{2,k+1} \\ x_{3,k+1} \end{bmatrix} = \begin{bmatrix} 0.909 & 0.091 & 0 \\ 0.01 & 0.99 & 0.01 \\ 0 & -0.1658 & 0.9986 \end{bmatrix} \begin{bmatrix} x_{1,k} \\ x_{2,k} \\ x_{3,k} \end{bmatrix} - \begin{bmatrix} 0.091f(x_1) \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix} u_k + \begin{bmatrix} 0.1 \\ 0 \\ 0 \end{bmatrix} w_k \quad (26)$$

where

$$w_k = e^{-k}$$

The open loop response of Chua’s circuit is shown in Fig 2a and is chaotic. The initial values for the state variables are chosen to be $[1; 1; 1]$.

Placing the poles of the system very close to 0.8 results in a gain of $K = [-29.6652 \ -49.1540 \ 32.0592]$. Then we solve the LMI to determine if our performance criteria are still met with an increasing value of σ . Once the LMI is no longer feasible we have found the maximum value of σ . The maximum value results are shown in Table 4. Then applying control where we use the perturbed gain Kp we get the response shown in Fig 2b.

Table 4: Unstable System Performance Criteria

Criteria	σ_{max}^2	Performance Criteria Check
Asy. Stb.	9.62	$\lim_{k \rightarrow \infty} E\{\ x_k\ ^2\} = 0$
H_2 Ctrl.	8.77	$\sum_i E\{\ z_i\ ^2\} = 2.51 \leq \frac{\lambda_{\max}(P)\ x_0\ ^2}{\delta} = 4.11$
H_∞ Ctrl.	9.27	$\sum_i E\{\ z_i\ ^2\} = 1.22 \leq -\epsilon \sum_i \ w_i\ ^2 = 112.02$

4. Conclusion

This work has presented resilience analysis of state feedback controllers are analyzed for a class of uncertain discrete-time nonlinear stochastic systems, focusing on both H_2 and H_∞ performance criteria. The proposed method accounts for uncertainties in both the system's nonlinearity and the randomly perturbed feedback gain, providing a resilient approach to control systems even when the systems gains are imprecise. By leveraging linear matrix inequality techniques, we have demonstrated how a unified framework can be developed to address various performance objectives, ensuring improved resilience in the control of complex systems.

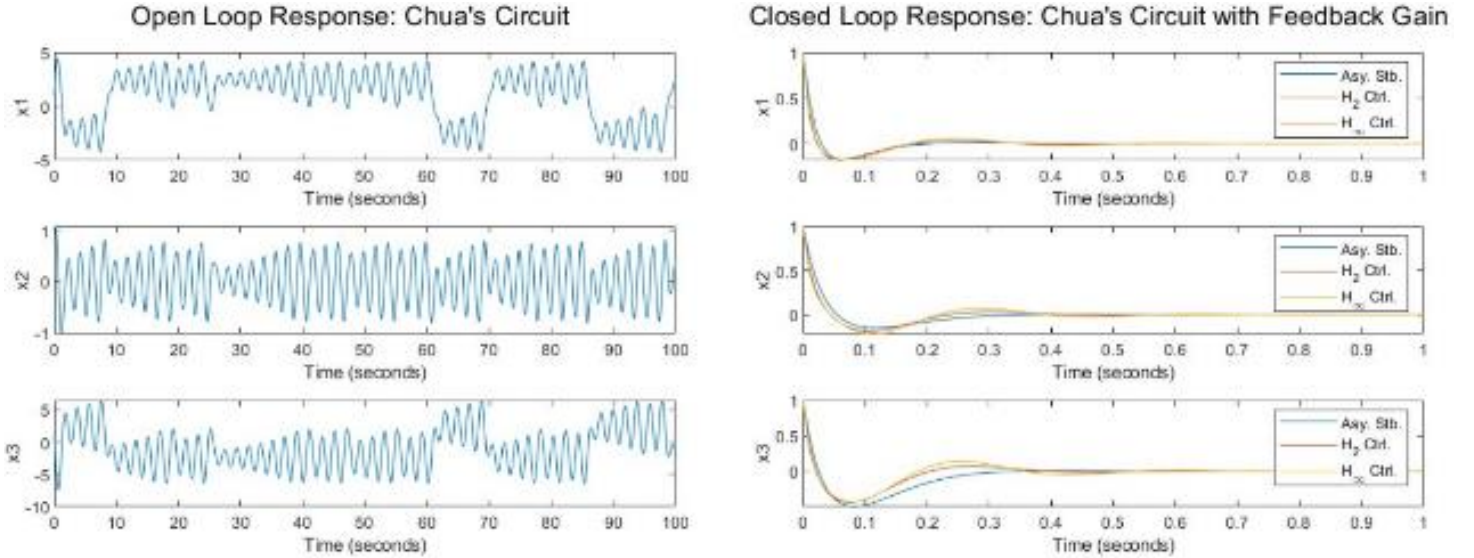


Figure 1: Comparison of Open and Closed Loop Responses of an Unstable System

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